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A Reliability-Based Capability Approach

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This article proposes a rigorous mathematical approach, named a reliability-based capability approach (RCA), to quantify the societal impact of a hazard. The starting point of the RCA is a capability approach in which capabilities refer to the genuine opportunities open to individuals to achieve valuable doings and beings (such as being mobile and being sheltered) called functionings. Capabilities depend on what individuals have and what they can do with what they have. The article develops probabilistic predictive models that relate the value of each functioning to a set of easily predictable or measurable quantities (regressors) in the aftermath of a hazard. The predicted values of selected functionings for an individual collectively determine the impact of a hazard on his/her state of well-being. The proposed RCA integrates the predictive models of functionings into a system reliability problem to determine the probability that the state of well-being is acceptable, tolerable, or intolerable. Importance measures are defined to quantify the contribution of each functioning to the state of well-being. The information from the importance measures can inform decisions on optimal allocation of limited resources for risk mitigation and management.

KEY WORDS: Capability approach; poverty assessment; probability; risk analysis; well-being

1. INTRODUCTION

Risk is commonly described in terms of the probability of occurrence of a hazardous scenario and the associated consequences.[3] The determination and evaluation of the relevant consequences are thus crucial steps for risk mitigation and management.[(1,2)] For example, to justify the necessity of a risk mitigation program in a given region, it is critical to understand and evaluate the impact a given hazard might have upon the well-being of individuals within the affected communities. An accurate and complete assessment of the potential consequences can be an important source of information for decisions about how and where to optimally invest limited resources.

There is no consensus on the best way to define and evaluate the consequences of hazards; these questions remain the subject of ongoing debate between, for example, utilitarians and capability theorists. A review of different approaches can be found in Refs. 1 and 3. The purpose of this article is not to resolve the disputes over which approach to adopt when defining consequences. Our starting point is the definition of consequences used in a capability approach,[(4–6)] a definition first introduced by Murphy and Gardoni.[(7)] While the argumentation in support of conceptualizing consequences using capabilities is beyond the scope of this article, we do provide a brief overview of some reasons for defining consequences in terms of capabilities in the next section.[4]

A capability approach assesses consequences in terms of the impact on what individuals do or

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[3]Gardoni and Murphy[(3)] argued that this definition of risk is not sufficient and it should be expanded to include the source/cause of a risk besides the probability of occurrence and the consequences of a given hazard. Then, they proposed a new scale of risk that categorizes the risks along a multidimensional ranking.

[4]Those interested in such a defense can see Refs. 3, 7, and 8.

become that they have reason to value, called functionings. Examples of functionings include being mobile, being healthy, being adequately nourished, and being educated. The genuine opportunity to achieve a particular functioning is called a capability.[5] Genuine opportunities and actual achievements are influenced by what individuals have and what they can do with what they have. What they can do with what they have is a function of the structure of social, legal, economic, and political institutions and of the characteristics of the built environment (i.e., infrastructure). For example, consider the functioning of being mobile. The number of times an individual travels per week can be an indicator of mobility achievement. When explaining a given individual's achievement or lack of achievement, a capability approach takes into consideration the conditions that must be in place for the individual to be mobile. For instance, the possession of certain resources, like a bike, may influence mobility. However, possessing a bike may not be sufficient to guarantee mobility. If the individual has physical disabilities, then the bike will be of no help to travel. Similarly, if there are no paved roads or if societal culture imposes a norm that women are not allowed to ride a bike, then it will become difficult or even impossible to travel by means of a bike. As this example makes clear, different factors will influence the number of times the individual travels.

The fundamental aim of this article is to develop a rigorous mathematical formulation to assess and predict functionings and thereby determine the state of well-being in terms of the predicted functionings. Because the proposed approach is based on the theory of reliability analysis,[(15,16)] we call it a reliability-based capability approach (RCA). The proposed RCA can be used in the context of risk analysis to quantify the broad societal impact of hazards on individuals' functionings. Disruptive events can impact the value of each functioning by changing the values of its influencing factors (those factors that reflect what individuals have and what they can do with what they have). For example, an earthquake can impact mobility by causing damage to the transportation network.

In RCA, we propose probabilistic predictive models that relate the value of each functioning (as measured by an indicator) to its influencing factors. The proposed models account for the various sources of uncertainties in predicting the values of functionings.[(17–19)] The predicted values of different functionings for an individual collectively determine his/her state of well-being, which could be acceptable, tolerable, or intolerable. Due to the uncertainty in predicting the values of functionings, the state of well-being has to be determined in a probabilistic manner. The RCA uses the methods of system reliability analysis to determine the probability associated with each state of well-being. In the system reliability problem, we treat the well-being of each individual as a system, where the indicators of considered functionings define the components of the system.

Following this introduction, the next section briefly discusses some of the advantages of using a capability approach to societal risk assessment. The section focuses on introducing the terms and variables needed in the proposed RCA and on reviewing current formulations for assessing functionings that have been proposed across a broader range of applications (also outside of risk analysis). In the third section, we present a detailed description and evaluation of two of the most advanced and rigorous mathematical formulations of the capability approach. The purpose of the review of these two formulations is to motivate the need for an alternative formulation of the kind proposed in this article. The first formulation is the capability approach to risk analysis developed by Murphy and Gardoni[(20)] that tracks the possible changes in the capabilities of individuals due to the impact of a hazard. The second is a capability approach to multidimensional poverty measurement developed by Alkire and Foster,[(21)] where poverty is understood as capabilities deprivation. Though not directly focusing on risk, discussion of the multidimensional poverty analysis is valuable because it highlights the significance of accounting for the uncertainty in well-being quantification and demonstrates the need for a probabilistic formulation. In the fourth section, we discuss the proposed RCA. Actually assessing and predicting the functionings of individuals is challenging. The existing formulations for operationalizing the capability approach typically create a composite index that aggregates the measured or predicted values of the considered (achieved) functionings for each individual to determine his/her state of well-being.[(20,21)] Instead, the RCA focuses the attention on the role of each achieved functioning in the state of well-being. In addition, we propose an importance measure that uses the results of system reliability analysis to rank the functionings on the basis of their contributions

[5]For an overview of different approaches of risk analysis and consequence evaluation see Refs. 1, 3, 7, and 9–14.

to the state of well-being. Such information is particularly useful to optimally allocate limited resources for risk mitigation and management. The proposed RCA is a rigorous mathematical formulation that can be used for the implementation of a capability approach in any application (e.g., risk analysis and development economics). In the last section, we present a specific numerical example to illustrate the proposed formulation in the context of risk analysis.

2. THE BENEFITS OF USING A CAPABILITY APPROACH TO SOCIETAL RISK ASSESSMENT

The capability approach provides theoretical resources for defining the broad range of effects of a hazard on the well-being of individuals, thereby providing a comprehensive picture of its societal impact. It does not simply look at immediately evident effects, such as fatalities or physical damage. Rather, the impact can be defined to include the effects of a hazard on, for example, mobility, nutrition, and security, doings and beings that are constitutive elements of well-being. In a capability approach, functionings capture distinctive, valuable dimensions of well-being. The overall capability of each individual is shaped by his/her opportunity to achieve a set of distinctive doings and beings. The capability approach rejects the utilitarian assumption that all goods or dimensions of well-being are commensurable, comparable, and substitutable. That is, one does not compensate for a deprivation in nutrition by an improvement in opportunities for housing; a deprivation in being adequately nourished requires an improvement in an opportunity for nourishment.

Second, the capability approach does not quantify consequences using a monetary metric that has well-known conversion challenges, e.g., in the definition of the monetary value of a human life. Instead, the capability approach uses nonmonetary indicators, defined as proxies for specific functionings,[22] to quantify the level of achievement for a given functioning. For example, a hazard can impact the functioning of living a long and healthy life, which can be measured by the indicator health-adjusted life expectancy.[23] We discuss indicators in more detail in later sections. For now, we only want to note that the capability approach is not vulnerable to concerns about the appropriateness of monetizing loss of human life or damage to the environment.[24] Nor is it vulnerable to critiques of utility measures that focus on concerns about the accuracy of surveys or market information for capturing the losses associated with hazardous events given, for example, asymmetries in bargaining power or limits on knowledge.[8,25,26]

There are further reasons to find a capability approach to the consequences evaluation attractive. A capability approach to assessing well-being has already been adopted in a wide range of applications. It is currently being used by the United Nations to quantitatively measure the degree of development of countries around the world.[27] Multidimensional poverty measurement is another area in which a capability approach has been used extensively.[21,28,29] Formulations have been proposed to identify the least advantaged in a society and guide the focus of public policy toward the promotion of distributive justice.[30] A capability approach has also been developed to assess the impact of natural and anthropogenic hazards.[7,31–35] Thus, an additional benefit to using a capability approach is that it makes it possible to assess, for example, the risk of natural hazards and the impact of development policies using the same theoretical framework. This aids the process of policy and decision making. The possibility of creating a consistent theoretical framework is increasingly important given that risk management of natural hazards is recognized by the United Nations and broader community of development policymakers as critical to the success of sustainable development initiatives, especially given climate change.

3. CAPABILITY APPROACH IN PRACTICE: CURRENT FORMULATIONS AND THEIR LIMITATIONS

The current formulations for operationalizing the capability approach generally include two major steps.[27,34]

1. Quantification of functionings. For this, a set of properly selected indicators is typically defined to measure the relevant dimensions of well-being.
2. Aggregation of achievements in various indicators to create an overall measure of well-being through a composite index.

This section discusses two general formulations that operationalize these two steps: the one proposed by Murphy and Gardoni[20] and the one by Alkire and Foster.[21]

3.1. Capability Approach to Risk Analysis

Murphy and Gardoni[7,20,31–35] proposed a novel risk analysis approach that quantifies the possible consequences of hazardous scenarios in terms of functionings, achievements, and capabilities of individuals. For the purposes of quantification, they developed a hazard impact index, *HII*, as an aggregate measure that summarizes the overall impact of hazards.

In constructing the *HII*, first, the relevant capabilities are selected. The primary concern is to provide a collectively exhaustive list of capabilities that represent all aspects of well-being relevant to the problem. On the other hand, careful attention should be given to prevent selecting similar capabilities that provide redundant information and overemphasize particular dimensions of well-being, in a sense causing double or multiple counting.[34] Indicators are then selected to quantify the level of achieved functionings. Next, each indicator is converted into an index on a scale from 0 (minimum achievement) to 1 (maximum achievement). Fig. 1 (adapted from Ref. 20) shows that an individual i (out of n) might achieve functioning v_j, $j \in \{1, \ldots, J\}$ at level $l \in \{1, \ldots, L\}$, so that the achieved functioning is equal to v_{jl}. The achieved functionings are then converted into the corresponding indicator indices, $I_j^{(i)}$s.

Murphy and Gardoni[20] also discussed the issue of interdependence of functionings. That is, an individual's choice to achieve one functioning influences his/her opportunity to achieve other functionings, therefore creating interdependence among functionings. For example, there might be a genuine opportunity for an individual to have a well-paid full-time job or to complete higher education but not possible to achieve both at the same time. Because of the interdependence of the achieved functionings, an individual i can only choose a vector of the achieved functionings, $\mathbf{V}^{(i)}$, among a set of possible vectors (which might not include all possible combinations of v_{jl}). For example, individual i can choose $\mathbf{V}^{(i)} := (v_{12}, v_{2L}, \ldots, v_{J1})$ that includes v_1 achieved at level 2, v_2 at level L, and so on, up to v_J at level 1. Each vector $\mathbf{V}^{(i)}$ is then converted into a vector of indicator indices $\mathbf{I}^{(i)} := (I_1^{(i)}, \ldots, I_J^{(i)})$, as shown in the right plot of Fig. 1 (adapted from Ref. 20).

In the second step, the elements of $\mathbf{I}^{(i)}$ are combined to create an aggregate measure for individual i, $HII^{(i)}$, defined as the statistical average

$$HII^{(i)} := \frac{1}{J} \sum_{j=1}^{J} I_j^{(i)}. \tag{1}$$

The average of the $HII^{(i)}$'s over the sampled population, Avg.$[HII^{(i)}]$, is then used as an estimate of individuals' functionings achievement in an average sense across the sample of size n. The standard deviation of the $HII^{(i)}$s over the sampled population, St.Dev.$[HII^{(i)}]$, captures the breadth of freedom in functionings achievement. The uncertainty in the value of *HII* can be described using a probability density function (PDF) with a mean value of Avg.$[HII^{(i)}]$ and a standard deviation of St.Dev.$[HII^{(i)}]$.

The societal impact of a hazard is then explained by comparing the predicted value of the *HII* with the acceptability and the tolerability thresholds as defined in Ref. 32. Threshold levels are set for each distinctive dimension of well-being. The thresholds capture demands of justice; as Nussbaum[5] writes, a "necessary condition of justice for a public political arrangement is that it deliver to citizens a certain basic level of capability." Moreover, the thresholds provide critical information for policymakers, who need to know not only what is the case about levels of well-being, but what they should think about the information they have and whether the level of well-being is such that it requires policy intervention. When the level of well-being is acceptable, policy intervention is not necessary. However, when it is unacceptable, policy intervention is urgent to determine how to bring individuals to the acceptable level within a specified period of time. Similarly, intolerable levels of capability require immediate action to bring individuals to above at least the tolerable threshold. If there is an aspect of individuals' lives that will be intolerable or unacceptable, then that should be the priority from a public policy perspective. An evaluation of the predicted level of well-being is useful in the definition of policies and resource allocations that are designed to best mitigate the possible consequences of undesirable events. Most policies targeted to risk mitigation are based on the evaluation of the possible consequences.

According to Murphy and Gardoni,[32] the acceptable threshold, T_{acc}, is defined as the minimum value of *HII* below which individuals ideally should not fall. For example, it is not acceptable that individuals lack permanent and adequate shelter in the aftermath of a hazard. A risk is acceptable if

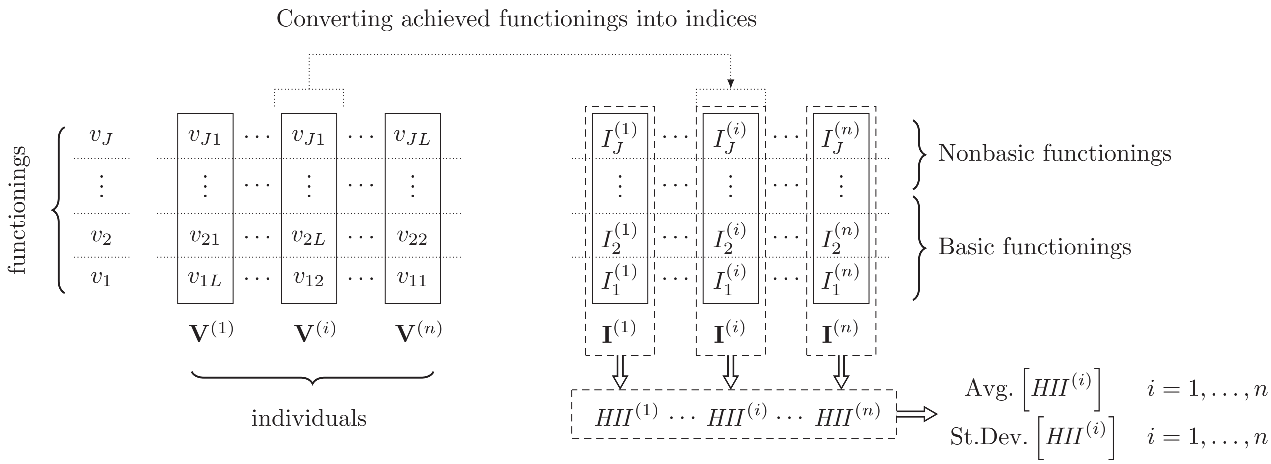

Fig. 1. Illustration of the *HII* formulation (adapted from Ref. 20).

the probability that any $HII^{(i)}$ will be less than T_{acc} in the aftermath of a hazard is sufficiently small. As discussed in Ref. 32, the precise specification of T_{acc} can be done through an internal democratic process.

However, it is not always feasible to keep the $HII^{(i)}$s of all individuals above T_{acc} in the immediate aftermath of a hazard. For example, some individuals may lose homes and need to be settled in temporary housing in the aftermath of a severe earthquake. In such circumstances, Murphy and Gardoni[32] defined a tolerable threshold, T_{tol}, that is an absolute minimum value of *HII* "below which no individual in a society should ever fall." For instance, in the shelter example, it would be neither tolerable nor acceptable for individuals to be left homeless. Accordingly, a risk is not acceptable, but only tolerable, if the $HII^{(i)}$s are likely to only temporarily be less than T_{acc} in the aftermath of a hazard and the probability of being less than T_{tol} is sufficiently small. Similar to the acceptability threshold, an internal democratic process can determine the value of T_{tol}.[32]

The formulation in Refs. 20 and 32 has four significant advantages, both theoretical and in terms of implementation, with respect to the other approaches that use capabilities. These advantages are: (1) it acknowledges the difference between achieved functionings and capabilities; (2) it accounts for the interactions of the achieved functionings using a vector of the achieved functionings instead of the isolated functionings; (3) it captures the variability in the achieved functionings among individuals by estimating the standard deviation of the achieved functionings in addition to the average value; and (4) it introduces the acceptability and tolerability thresholds for both the indicator indices and aggregate measure. However, it also has the following two limitations: (1) the definition of the $HII^{(i)}$, as the statistical average of the elements of $\mathbf{I}^{(i)}$, might be too simplistic, specifically, it does not distinguish between two individuals with the same averaged achieved functionings but different achievements in their specific indicator indices; and (2) it does not account for the uncertainty in the actual values of the $I_j^{(i)}$s.

In relation to the first limitation, the formulation of the $HII^{(i)}$ allows substitutability of the achieved functionings. That is, the high values of a subset of indices can outweigh the low values of the others. This fails to account for the incommensurability of capabilities. Thus, a more nuanced formulation is needed that also considers the performance of each $I_j^{(i)}$.

The second limitation is that the formulation treats the $I_j^{(i)}$s in a deterministic manner, and thus, does not account for their uncertainties. In this regard, Murphy *et al.*[19] noted that mathematical formulations should appropriately treat the prevailing uncertainties like measurement error, statistical uncertainty, and model error.[17,18] Specifically, in the context of risk analysis, measurement error is associated with the estimates of the values of indicators. For example, an indicator capturing the economic losses might underestimate the actual losses of an individual. Statistical uncertainty arises from the scarcity of data. For example, to formulate the *HII* of a society, the information for every single individual/household in the society may not be available.

Thus, a sample of the society is selected in practice as representative of the entire society. As the size of the sample increases, the confidence in the estimated *HII* increases as well. Finally, the mathematical models (like those used to predict the values of the indicators after future events as functions of regressors) might have errors due to missing variables (here indicators) and/or inaccurate model form. This type of uncertainty is called the model error. For example, there might be influencing indicators that are not included in the models or there might be a more appropriate form of the model. These uncertainties are epistemic uncertainties. In addition, when the formulation is being used for predicting the future values of the indicator indices, there is additional inherent variability/randomness that should be included as well.[36] Therefore, there is a need for a probabilistic formulation that properly accounts for different sources of uncertainty. Further discussion on the treatment of uncertainty in mathematical modeling can be found in Ref. 19.

3.2. Capability Approach to Poverty Measurement

In responding to the widespread recognition of the insufficiency of income as the sole measure of poverty,[37] Alkire and Foster[21] developed an approach that uses functionings achievements to measure poverty in a multidimensional way. A counting-based method is developed to identify the poor and measure dimensions of poverty, like education, health, and standard of living. This approach includes an identification step to define and quantify the number of individuals who are poor, based on counting the number of (weighted) deprivations, and an aggregation step to summarize the degree of poverty experienced by the poor.

The formulation defines indicators for considered functionings and scales the indicators to create indicator indices. The indicator indices take values between 0 and 1, where 0 represents the minimum possible achievement and 1 the maximum achievement. For the identification step, Alkire and Foster[21] define a deprivation threshold for each indicator index to identify if the corresponding functioning has been sufficiently achieved. Mathematically, let i be an individual and I_j be one of the J selected indicator indices; then, a functioning has been sufficiently achieved if $I_j^{(i)} \geq T_{1,j}$, where $I_j^{(i)}$ is the jth indicator index of individual i, and $T_{1,j}$ is the corresponding deprivation threshold.

Then, the number of deprived indicators (i.e., for which $I_j^{(i)} < T_{1,j}$) is counted for each individual and the number of deprived indicators divided by J is compared with a poverty threshold, T_2 (which is a selected number between 0 and 1). If such count is greater than T_2, then individual i is labeled poor. A measure of deprivation of indicator index j is computed as $d_{ij} := [T_{1,j} - I_j^{(i)}]^{\alpha}$ if $[T_{1,j} - I_j^{(i)}] > 0$ and $d_{ij} = 0$, otherwise; where α is a controlling parameter such that larger values of $[T_{1,j} - I_j^{(i)}]$ are under – or overemphasized, depending on the value of α. If the considered functionings are not equally important, Alkire and Foster[21] suggested using a weighting vector of the indicator indices, here denoted as $\mathbf{w} = (w_1, \ldots, w_J)$ (such that their sum equals 1), which captures their relative importance. Once the poor are identified, the degree of poverty of individual i is calculated as:

$$D_i := \sum_{j=1}^{J} w_j d_{ij} = \sum_{\forall j : I_j^{(i)} < T_{1,j}} w_j d_{ij}. \qquad (2)$$

For the nonpoor, $D_i = 0$. Finally, the D_is are combined over the sampled population to construct an aggregate measure defined as:

$$D := \frac{1}{n} \sum_{i=1}^{n} D_i = \frac{1}{n} \sum_{\forall i : D_i > 0} D_i, \qquad (3)$$

where n is the size of the sampled population.

Alkire *et al.*[38] discuss the sensitivity of D with respect to $(\mathbf{T}_1, T_2)$, where $\mathbf{T}_1 := (T_{1,1}, \ldots, T_{1,J})$. In particular, they examine how different choices of $(\mathbf{T}_1, T_2)$ may affect the ranking of the poverty measure across different groups. They also account for the statistical uncertainty in estimating D, which arises from using a sample of society as representative of the entire population. The result of statistical uncertainty quantification is used to examine if a likely change in the ranking of the poverty measure due to a change in the values of $(\mathbf{T}_1, T_2)$ is statistically significant.

The formulation in Ref. 21 has the following strengths with respect to other formulations of poverty measurement: (1) it measures poverty in a multidimensional way by identifying the poor first and then aggregating the deprivation intensity of the deprived functionings among those identified as poor; (2) it satisfies the *population decomposability property* (i.e., the overall poverty of a community can be computed either by considering the entire population or as a weighted average of subgroups of the entire population where the subgroup size is used as a weight) and the *dimensional breakdown property* (i.e., the same formulation can be used considering any subset of dimensions to investigate their contribution to poverty); and (3) it is applicable both to cardinal variables (e.g., years of schooling) as well as ordinal (e.g., self-reported health) and categorical variables (e.g., modes of access to drinking water). In the case of ordinal and categorical variables, since there is no unique way to measure the deprivation intensity, it only measures the headcount of the poor. However, there are three limitations: (1) it does not account for and evaluate the role of all of the relevant sources of uncertainty in quantifying poverty; (2) it does not consider the deprivation of the nonpoor individuals; and (3) it makes comparison of poverty measurements of different societies difficult by allowing for the use of different weighting vectors.

Regarding the first limitation, Alkire *et al.*[38] acknowledge the necessity of addressing the uncertainty but their scope of uncertainty treatment is limited to some sensitivity analyses. In particular, they do not discuss the role of the various sources of uncertainty in statistical inference about the poverty measure of a given populations, how additional information might help to improve the inference, and how this relevant uncertainty should be propagated through the models. The significance of accounting for different sources of uncertainty, in addition to statistical uncertainty, was elaborated earlier in this article.

The second limitation is that it does not consider the deprivations of the nonpoor individuals. The nonpoor individuals might still suffer from a number of deprived functionings (i.e., $d_{ij} \neq 0$). If the different capabilities are incommensurable in moral value (as is widely recognized), a measure of deprivations should also include the extent of deprivations of the nonpoor. Therefore, there is a need to develop a formulation that also captures the variation of well-being among the nonpoor.

The third limitation is about the subjective weights of indicators, $\mathbf{w}$, in Equation (2) that show their relative importance. Using different $\mathbf{w}$s does not allow us to compare the multidimensional poverty measurement in different societies. In practice, it is possible to justify the use of different $\mathbf{w}$s in measuring poverty in different societies, and, in fact, different studies use different $\mathbf{w}$s. However, different $\mathbf{w}$s change the contribution of d_{ij}s in the value of D_is and subsequently in the value of D. This difference makes comparing poverty measurement in two different societies difficult.

4. RELIABILITY-BASED CAPABILITY APPROACH

This section presents the proposed RCA and shows how this approach addresses the limitations of the current formulations, as discussed in the previous section. A reliability analysis is generally concerned with determining the probability that a component or system performs a specified function under certain conditions.[15] A system is an interconnected assembly of components where its state depends on the states of its components and their roles in the system (i.e., the definition of the system in terms of its components). For example, the state of a transportation system depends on the states of the bridges, roads, etc., which constitute it. A system may fail if certain subsets of its components fail. A component fails when its performance is no longer satisfactory. For example, we can define the failure of the transportation system when the connectivity between any two nodes in the network is lost, which occurs if selected sets of components fail.

In the proposed RCA, the well-being of each individual is treated as a system of indicators such as life expectancy, number of schooling years, and income, which are the components of the system. In order to determine the state of well-being, we need to know the value/state of each indicator and how the indicators are collectively related to well-being.

We define three states for each indicator index as follows:

$$\mathcal{S}_j := \begin{cases} \text{Acceptable}, & I_j > T_{j,\text{acc}}, \\ \text{Tolerable}, & T_{j,\text{tol}} < I_j \leq T_{j,\text{acc}}, \\ \text{Intolerable}, & I_j \leq T_{j,\text{tol}}, \end{cases} \quad (4)$$

where $\mathcal{S}_j$ is an auxiliary variable that describes the state of the indicator index j; $I_j \in [0, 1]$ is the value of the indicator index j; and $T_{j,\text{acc}} \in [0, 1]$ and $T_{j,\text{tol}} \in [0, T_{j,\text{acc}})$ are the corresponding acceptable and tolerable thresholds of the indicator index j.

As mentioned earlier, the values of functionings and their indicators are influenced by different factors such as wealth, income, socioeconomic status of the society, and infrastructure status. To quantify the influence of such factors, we propose probabilistic predictive models that define each I_j as functions of regressors that represent the different influencing factors. The proposed models also account for the effect of the various sources of uncertainty in predicting I_js. Because of the uncertainty in predicating the value of I_j, $\mathcal{S}_j$ is a random variable, where its three states are the possible outcomes. To determine the probability of each state of $\mathcal{S}_j$, we formulate a component reliability problem, described next. Subsequently, we formulate a system reliability problem to determine the probability of each state of well-being.

4.1. Mathematical Formulation of Component Reliability Problem

To formulate the component reliability problem, we first develop a probabilistic predictive model for each I_j. Following the formulation proposed in Ref. 17, we write the general form of the probabilistic predictive models as:

$$C_j(\mathbf{x}, \mathbf{\Theta}_j) = \hat{c}_j(\mathbf{x}) + \gamma_j(\mathbf{x}, \mathbf{\Theta}_j) + \sigma_j \varepsilon_j, \quad j \in \{1, \ldots, J\}, \tag{5}$$

where $C_j(\mathbf{x}, \mathbf{\Theta}_j)$ is the predicted value of the jth indicator index or a suitable transformation thereof; $\mathbf{x}$ is the set of input variables (regressors that capture the socioeconomic conditions and the characteristics of the built environment); $\mathbf{\Theta}_j := (\boldsymbol{\theta}_j, \sigma_j)$ is a set of unknown model parameters, corresponding to the jth indicator index, which need to be estimated; $\hat{c}_j(\mathbf{x})$ is an existing deterministic model for predicting the value of the jth indicator index (e.g., a predefined function of the average of the measured values over the population); $\gamma_j(\mathbf{x}, \mathbf{\Theta}_j)$ is a correction term for $\hat{c}_j(\mathbf{x})$ that captures some of the dependencies of C_j on $\mathbf{x}$; and $\sigma_j \varepsilon_j$ is an additive model error (additivity assumption), in which σ_j is the standard deviation of the model error that is assumed to be independent of $\mathbf{x}$ (homoskedaticity assumption) and ε_j is a standard normal random variable (normality assumption). The model error captures the variability in predicting C_j using $\hat{c}_j(\mathbf{x}) + \gamma_j(\mathbf{x}, \mathbf{\Theta}_j)$ due to, for example, inaccuracy of the model form, missing variables, and statistical uncertainties. Measurement error can be included in the model calibration as discussed later.

In general, the ε_j's in Equation (5) are correlated. Thus, letting $\mathbf{\Sigma}$ denote the covariance matrix of $\sigma_j \varepsilon_j$'s, the set of all unknown model parameters is $\mathbf{\Theta} := (\boldsymbol{\theta}, \mathbf{\Sigma})$, where $\boldsymbol{\theta} := (\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_J)$.

In order to satisfy the additivity, homoskedasticity, and normality assumptions, we may use a transformation to define $C_j(\mathbf{x}, \mathbf{\Theta}_j) := \mathcal{T}_j[I_j(\mathbf{x}, \mathbf{\Theta}_j)]$ and $\hat{c}_j(\mathbf{x}) := \mathcal{T}_j[\hat{I}_j(\mathbf{x})]$, where $\mathcal{T}_j(\cdot)$ is the transformation function for the jth indicator index; $I_j(\mathbf{x}, \mathbf{\Theta}_j)$ is the predicted value of the jth indicator index; and $\hat{I}_j(\mathbf{x})$ is the deterministic prediction of the jth indicator index.

The suitability of a specific choice of $\mathcal{T}_j(\cdot)$ (e.g., a logit model) can be examined by means of diagnostic plots.[39]

The correction term in Equation (5), $\gamma_j(\mathbf{x}, \mathbf{\Theta}_j)$, can be written in the simplest form as:

$$\gamma_j(\mathbf{x}, \mathbf{\Theta}_j) = \sum_{q=1}^{Q} \theta_{jq} h_{jq}(\mathbf{x}), \tag{6}$$

where θ_{jq}'s are the elements of $\boldsymbol{\theta}_j$ and h_{jq}'s are a set of explanatory functions defined in terms of the elements of $\mathbf{x}$. The explanatory functions are defined in terms of the influencing factors that are believed to be important in predicting C_j. Examples of h_{jq}s include individuals' age, gender, ethnicity, income, language, socioeconomic status of the society, and the infrastructure status. To develop an empirical model that is both parsimonious (with as few θ_{jq}s as possible) and accurate (with small $\sigma_j \varepsilon_j$), one can use a model selection process[17,40] to eliminate unimportant terms in $\gamma_j(\mathbf{x}, \mathbf{\Theta}_j)$ that do not significantly contribute to predicting C_j.

The unknown model parameters, $\mathbf{\Theta}$, can be estimated based on the observed data using, for example, a Bayesian approach.[41] The observed data are the values of I_js, or the corresponding values of C_js, along with the values of the $\mathbf{x}$s, for a group of individuals. Using the Bayesian approach, we can combine previous information about $\mathbf{\Theta}$ (which could possibly also be no information) with information obtained from the observed data to arrive at an updated PDF of $\mathbf{\Theta}$. Such updating can be carried out using the Bayesian updating rule.[41] The Bayesian updating rule can be written as:

$$f(\mathbf{\Theta}) = \kappa L(\mathbf{\Theta}) p(\mathbf{\Theta}), \tag{7}$$

where $f(\mathbf{\Theta})$ is the posterior PDF, containing the updated information about $\mathbf{\Theta}$; $L(\mathbf{\Theta})$ is the likelihood function, representing the objective information about $\mathbf{\Theta}$ obtained from the observed data; $p(\mathbf{\Theta})$ is the prior PDF, reflecting our state of knowledge about $\mathbf{\Theta}$ before obtaining the observations; and $\kappa := [\int L(\mathbf{\Theta}) p(\mathbf{\Theta}) d\mathbf{\Theta}]^{-1}$ is a normalizing constant.

Assuming there is no previous information about $\mathbf{\Theta}$, we use a noninformative $p(\mathbf{\Theta})$ to reflect that no or minimal information about $\mathbf{\Theta}$ is available before the observed data.[41] Thus, the inferences are unaffected by information external to the observations. Further discussion on noninformative priors can be found in Refs. 42 and 43. For the set of unknown model parameters $\boldsymbol{\Theta} := (\boldsymbol{\theta}, \boldsymbol{\Sigma})$, it can generally be assumed that $\boldsymbol{\theta}$ and $\boldsymbol{\Sigma}$ are approximately independent;[17,41] hence, $p(\boldsymbol{\Theta}) \approx p(\boldsymbol{\Theta})p(\boldsymbol{\Sigma})$. Then, we use a locally uniform noninformative prior on $\boldsymbol{\theta}$ such that $p(\boldsymbol{\Theta}) \cong p(\boldsymbol{\Sigma})$. Furthermore, following Gardoni *et al.*,[17] we can write the noninformative prior of $\boldsymbol{\Sigma}$ as:

$$p(\boldsymbol{\Sigma}) \propto |\mathbf{R}|^{-(J+1)/2} \prod_{j=1}^{J} \frac{1}{\sigma_j}, \tag{8}$$

where $|\mathbf{R}|$ is the determinant of $\mathbf{R}$, which is the correlation matrix of the ε_js. Following Gardoni *et al.*,[17] $L(\boldsymbol{\Theta})$ can be written by dividing the observed data into three groups: (1) equality data, when the measured values are the values of I_js; (2) lower bound data, when the measured values are less than the actual values of I_j; and (3) upper bound data, when the measured values are greater than the actual values. For example, when we do not know the exact income of an individual but we know it is greater than a certain amount, that amount is a lower bound datum. Similarly, when we know that the income is not greater than a certain amount, that amount is an upper bound datum. Lower and upper bound data are also called censored data.

In a general setting, we can write $L(\boldsymbol{\Theta})$ as:

$$L(\boldsymbol{\Theta}) \propto \mathbf{P}\left\{ \bigcap_{\text{observation } i} \left\{ \bigcap_{\text{equality data}_j} [\sigma_j \varepsilon_j = r_i(\boldsymbol{\theta}_j)] \bigcap_{\text{lower bound data}_j} [\sigma_j \varepsilon_j > r_i(\boldsymbol{\theta}_j)] \bigcap_{\text{upper bound data}_j} [\sigma_j \varepsilon_j < r_i(\boldsymbol{\theta}_j)] \right\}\right\}, \tag{9}$$

where $\mathbf{P}(A)$ is the probability of the event A; $\bigcap_j B_j$ is the intersection of B_js; and $r_i(\boldsymbol{\theta}_j) := C_{ji} - \hat{c}_j(\mathbf{x}_i) - \gamma_j(\mathbf{x}_i, \boldsymbol{\theta}_j)$ is the prediction residual of C_j for the ith individual.

In the specific case that observations (i.e., $r_i(\boldsymbol{\theta}_j)$'s for different is) are statistically independent, we can write $L(\boldsymbol{\Theta})$ as:

$$L(\boldsymbol{\Theta}) \propto \prod_{\text{observation } i} \mathbf{P}\left\{ \bigcap_{\text{equality data}_j} [\sigma_j \varepsilon_j = r_i(\boldsymbol{\theta}_j)] \bigcap_{\text{lower bound data}_j} [\sigma_j \varepsilon_j > r_i(\boldsymbol{\theta}_j)] \bigcap_{\text{upper bound data}_j} [\sigma_j \varepsilon_j < r_i(\boldsymbol{\theta}_j)] \right\}. \tag{10}$$

Note that Equation (10) still accounts for the statistical dependence of indicator indices (i.e., for each observation i, $\{r_i(\boldsymbol{\theta}_j)\}_{j=1}^{J}$ are statistically dependent).

To model the measurement errors, we use the formulation proposed by Gardoni *et al.*[17] We can write the actual values of C_{ji} and $\mathbf{x}_i$ as $C_{ji} = \hat{C}_{ji} + e_{C_{ji}}$ and $\mathbf{x}_i = \hat{\mathbf{x}}_i + e_{\mathbf{x}_i}$, where $\hat{C}_{ji}$ and $\hat{\mathbf{x}}_i$ are the measured values for the ith individual and $e_{C_{ji}}$ and $e_{\mathbf{x}_i}$ are the corresponding measurement errors. It is assumed that the measurements are corrected for any systematic errors. As a result, the mean values of $e_{C_{ji}}$ and $e_{\mathbf{x}_i}$ are zero. Let s_{ji}^2 and $\boldsymbol{\Sigma}_i$ denote the variance of $e_{C_{ji}}$ and the covariance matrix of $e_{\mathbf{x}_i}$. We allow the statistical dependence between the measurement errors for different elements of the vector $\mathbf{x}_i$ but assume that the measurement errors for different individuals are statistically independent. To write $L(\boldsymbol{\Theta})$ in Equation (10), accounting for the measurement errors, we replace $r_i(\boldsymbol{\theta}_j)$ with $r_i(\boldsymbol{\theta}_j, e_{\mathbf{x}_i}) := \hat{r}_i(\boldsymbol{\theta}_j) + \nabla_{\hat{\mathbf{x}}_i}\hat{r}_i(\boldsymbol{\theta}_j)e_{\mathbf{x}_i}$ and σ_j with $\hat{\sigma}_j := \sqrt{\sigma_j^2 + s_{ji}^2 + \nabla_{\hat{\mathbf{x}}_i}\hat{r}_i(\boldsymbol{\theta}_j)\boldsymbol{\Sigma}_i\nabla_{\hat{\mathbf{x}}_i}\hat{r}_i(\boldsymbol{\theta}_j)^T}$, where $\hat{r}_i(\boldsymbol{\theta}_j) := \hat{C}_{ji} - \hat{c}_j(\hat{\mathbf{x}}_i) - \gamma_j(\hat{\mathbf{x}}_i, \boldsymbol{\theta}_j)$ and $\nabla_{\hat{\mathbf{x}}_i}$ is the gradient row vector with respect to $\hat{\mathbf{x}}_i$.

To obtain $f(\boldsymbol{\Theta})$, we have to calculate the normalizing constant, κ, in Equation (7), which requires evaluating a complex, multifold integral. In general, this integral is not analytically tractable. However, we can use simulation methods to estimate the posterior statistics of $\boldsymbol{\Theta}$. The details of various simulation methods are discussed, for example, in Ref. 44.

While other regression techniques are generally available to estimate the unknown model parameters in probabilistic models, we presented a Bayesian approach because it is ideally suited to consider different types of data (equality or censored data) and possible information about the model parameters that might be available before considering the observed data.

Now, we can formulate a component reliability problem for each of the J components to obtain the corresponding probabilities of the acceptable, tolerable, and intolerable states. To formulate the component reliability problem, we define the limit-state function $g[I_j(\mathbf{x}, \mathbf{\Theta}_j), T_j] := I_j(\mathbf{x}, \mathbf{\Theta}_j) - T_j$, for $j = 1, \ldots, J$. Accordingly, we can write the probabilities associated with each state in terms of $g[I_j(\mathbf{x}, \mathbf{\Theta}_j), T_j]$. For instance, the probability associated with the intolerable state can be written as $\mathbf{P}\{g[I_j(\mathbf{x}, \mathbf{\Theta}_j), T_{j,\text{tol}}] \leq 0\}$. We can solve this problem using the reliability methods, including the first-order reliability method (FORM) and the second-order reliability method (SORM).[15] In explaining the proposed formulation, we assume that the value of each T_j is defined *a priori*. For example, one can use the values of T_js described in Ref. 32 as possible deterministic values. However, one could also develop separate probabilistic models for the T_js, as we did for the $C_j(\mathbf{x}, \mathbf{\Theta}_j)$'s in Equation (5).

4.2. Mathematical Formulation of System Reliability Problem

To determine the state of well-being, we can treat the well-being of each individual as a series system in which failure of any component (e.g., intolerability of the state of any indicator indices) results in the failure of the system (e.g., intolerability of the state of well-being). This is because the value of each indicator index is incommensurable and no amount of gain in the value of any indicator indices can offset the reduction in the value of the others. Note that the proposed formulation is not restricted to the series system (defined next) and it can simply be extended to other systems as well. To clarify this point, Fig. 2 shows two possible configurations of a transportation system consisting of four bridges, connecting cities A and B. Fig. 2(a) shows a series system, while Fig. 2(b) shows an example of a general system. In terms of failure (cities A and B are disconnected), the series system fails if any bridges in the system fail. On the other hand, the general system fails if either set of bridges {1} or {2, 3, 4} fails. Analogously, it is possible to formulate the well-being of individuals as a general system of indicator indices.

Following Murphy and Gardoni,[32] and similarly to what was presented for the individual indicators,

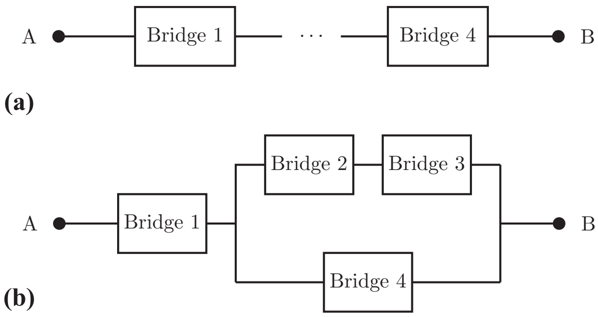

Fig. 2. Illustration of two different configurations of a transportation system: (a) series system and (b) general system.

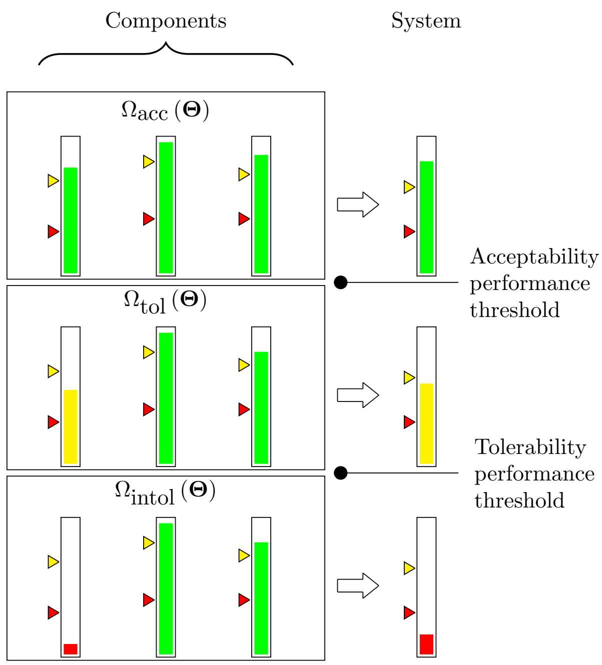

Fig. 3. Illustration of the system states and their relations with the indicator indices.

we define three states of well-being that are delimited by two performance thresholds: an *acceptability performance threshold* that delimits the acceptable and tolerable states and a *tolerability performance threshold* that delimits the tolerable and intolerable states. Specifically, we define the three states of the system as follows: (1) the state of well-being is acceptable if all the indicator indices are in their acceptable states; (2) the state of well-being is not acceptable but is still tolerable if at least one indicator index is in its tolerable state and the other indicator indices are in the acceptable state; and (3) the state of well-being is intolerable if at least one indicator index is in its intolerable state. Fig. 3 schematically explains different states of well-being and the relation with the values/states of the indicator indices in the case of $J = 3$. Note that if one requires to obtain a more refined information on the state of well-being (i.e., beyond the three considered states), the proposed formulation can be extended to model multistate systems, composed of multistate components.[45]

Mathematically, we can write the following expressions for the states of well-being:

$$\begin{aligned}
\Omega_{\mathrm{acc}}(\boldsymbol{\Theta}) &:= \left\{ \mathbf{z} : \bigcap_{j=1}^{J} g[I_j(\mathbf{x}, \boldsymbol{\Theta}_j), T_{j,\mathrm{acc}}] > 0 \right\}, \\
\Omega_{\mathrm{tol}}(\boldsymbol{\Theta}) &:= \left\{ \mathbf{z} : \left[\bigcup_{j=1}^{J} g[I_j(\mathbf{x}, \boldsymbol{\Theta}_j), T_{j,\mathrm{acc}}] \le 0 \right] \right. \\
&\qquad \left. \bigcap \left[\bigcap_{j=1}^{J} g[I_j(\mathbf{x}, \boldsymbol{\Theta}_j), T_{j,\mathrm{tol}}] > 0 \right] \right\}, \\
\Omega_{\mathrm{intol}}(\boldsymbol{\Theta}) &:= \left\{ \mathbf{z} : \bigcup_{j=1}^{J} g[I_j(\mathbf{x}, \boldsymbol{\Theta}_j), T_{j,\mathrm{tol}}] \le 0 \right\},
\end{aligned} \tag{11}$$

where $\Omega_{\mathrm{acc}}(\boldsymbol{\Theta})$ is a set containing all the vectors $\mathbf{z}$ that lead to the acceptable state of system, in which $\mathbf{z} := (\mathbf{x}, \varepsilon_1, \ldots, \varepsilon_J)$; $\bigcup_{j=1}^{J} A_j$ and $\bigcap_{j=1}^{J} A_j$ are the union and the intersection of the events A_js. Similarly, $\Omega_{\mathrm{tol}}(\boldsymbol{\Theta})$ and $\Omega_{\mathrm{intol}}(\boldsymbol{\Theta})$ are the sets containing all the vectors $\mathbf{z}$ that lead to the tolerable and intolerable states of the system.

If we call $\Omega_{\mathcal{S}}(\boldsymbol{\Theta})$ the domain of the system state, as a function of $\boldsymbol{\Theta}$ (i.e., $\Omega_{\mathrm{acc}}(\boldsymbol{\Theta})$, $\Omega_{\mathrm{tol}}(\boldsymbol{\Theta})$, or $\Omega_{\mathrm{intol}}(\boldsymbol{\Theta})$), we can write the probability associated with the system state as:

$$\mathbf{P}_{\mathcal{S}}(\boldsymbol{\Theta}) = \int_{\Omega_{\mathcal{S}}(\boldsymbol{\Theta})} f_{\mathbf{Z}}(\mathbf{z}) d\mathbf{z}, \tag{12}$$

where $\mathbf{P}_{\mathcal{S}}(\boldsymbol{\Theta})$ is the probability associated with the system state $\mathcal{S} \in \{$Acceptable, Tolerable, Intolerable$\}$ as a function of $\boldsymbol{\Theta}$ and $f_{\mathbf{Z}}(\mathbf{z})$ is the joint PDF of $\mathbf{Z}$.

In addition, we can determine the contribution of each component to the state $\mathcal{S}$. For this purpose, we define the importance measure of the jth component as follows:[46]

$$IM_{j,\mathcal{S}}(\boldsymbol{\Theta}) := \frac{1}{\mathbf{P}_{\mathcal{S}}(\boldsymbol{\Theta})} \int_{\{\Omega_{\mathcal{S}}(\boldsymbol{\theta}_j) \cap \Omega_{\mathcal{S}}(\boldsymbol{\Theta})\}} f_{\mathbf{Z}}(\mathbf{z}) d\mathbf{z}, \tag{13}$$

where $IM_{j,\mathcal{S}}(\boldsymbol{\Theta})$ is the importance measure of the jth component as a function of $\boldsymbol{\Theta}$ and $\{\Omega_{\mathcal{S}}(\boldsymbol{\theta}_j) \cap \Omega_{\mathcal{S}}(\boldsymbol{\Theta})\}$ is the domain containing all $\mathbf{z}$s such that, for the given $\boldsymbol{\Theta}$, both the jth component and the system are in the same state $\mathcal{S}$ (e.g., intolerable state).

To solve Equations (12) and (13), we can use the simulation methods. Specifically, we can first obtain $f_{\mathbf{Z}}(\mathbf{z})$ using a Nataf model.[47] The Nataf model requires as inputs the individual PDFs of each element of $\mathbf{Z}$ (i.e., their marginal PDFs) and their correlation matrix. Then, we can compute $\mathbf{P}_{\mathcal{S}}(\boldsymbol{\Theta})$ in Equation (13) numerically as follows:

$$\mathbf{P}_{\mathcal{S}}(\boldsymbol{\Theta}) \approx \frac{1}{K} \sum_{k=1}^{K} \mathbf{1}_{\{\mathbf{z}_k \in \Omega_{\mathcal{S}}(\boldsymbol{\Theta})\}}, \tag{14}$$

where K samples of $\mathbf{z}_k$s are drawn from $f_{\mathbf{Z}}(\mathbf{z})$; and $\mathbf{1}_{\{\mathbf{z}_k \in \Omega_{\mathcal{S}}(\boldsymbol{\Theta})\}} = 1$ if $\{\mathbf{z}_k \in \Omega_{\mathcal{S}}(\boldsymbol{\Theta})\}$ is a true statement and $\mathbf{1}_{\{\mathbf{z}_k \in \Omega_{\mathcal{S}}(\boldsymbol{\Theta})\}} = 0$, otherwise. Similarly, we can compute $IM_{j,\mathcal{S}}(\boldsymbol{\Theta})$ in Equation (13) as:

$$IM_{j,\mathcal{S}}(\boldsymbol{\Theta}) \approx \frac{1}{\mathbf{P}_{\mathcal{S}}(\boldsymbol{\Theta})} \frac{1}{K} \sum_{k=1}^{K} \mathbf{1}_{\{\mathbf{z}_k \in [\Omega_{\mathcal{S}}(\boldsymbol{\theta}_j) \cap \Omega_{\mathcal{S}}(\boldsymbol{\Theta})]\}}, \tag{15}$$

where $\mathbf{P}_{\mathcal{S}}(\boldsymbol{\Theta})$ is obtained from Equation (14).

There are two possible ways of incorporating the uncertainty in $\boldsymbol{\Theta}$ in computing $\mathbf{P}_{\mathcal{S}}(\boldsymbol{\Theta})$ and $IM_{j,\mathcal{S}}(\boldsymbol{\Theta})$.[17] First, we may ignore the uncertainty in $\boldsymbol{\Theta}$ and obtain a point estimate of the state probability, $\hat{\mathbf{P}}_{\mathcal{S}}$, by replacing $\boldsymbol{\Theta}$ in Equation (14) with a fixed set of values, $\hat{\boldsymbol{\Theta}}$ (e.g., the posterior mode of $\boldsymbol{\Theta}$). Alternatively, to incorporate the uncertainty in $\boldsymbol{\Theta}$ in Equation (14), we can estimate the predictive state probability, $\tilde{\mathbf{P}}_{\mathcal{S}}$, as:

$$\tilde{\mathbf{P}}_{\mathcal{S}} := \int \mathbf{P}_{\mathcal{S}}(\boldsymbol{\Theta}) f(\boldsymbol{\Theta}) d\boldsymbol{\Theta}. \tag{16}$$

Intuitively, $\tilde{\mathbf{P}}_{\mathcal{S}}$ is a weighted average of $\mathbf{P}_{\mathcal{S}}(\boldsymbol{\Theta})$ for different values of $\boldsymbol{\Theta}$ where the weights are proportional to $f(\boldsymbol{\Theta})$. Similarly, we can define a point estimate $\widehat{IM}_{j,\mathcal{S}}$ and a predictive estimate $\widetilde{IM}_{j,\mathcal{S}}$ of $IM_{j,\mathcal{S}}$.

We can perform the system reliability analysis for all individuals in a given region and develop a map that visualizes the spatial distribution of the states of well-being over the region. For example, a map that shows the spatial distribution of the intolerable state can give insights on which subpopulations in the study region are suffering more in the aftermath of a disruptive event. Furthermore, by calculating the importance measures corresponding to the intolerable state for all individuals, we can determine which indicator indices are the main causes of being in an intolerable state. This information can inform the decision making and resource allocation both for predisaster mitigation and for postdisaster recovery.

5. NUMERICAL EXAMPLE

In this section, we illustrate, through a hypothetical example, how the proposed RCA can be used. The example provided here is given in the context of risk analysis; however, the proposed formulation can also be used for other applications of the capability approach, including the multidimensional poverty assessment. In this example, we consider the well-being in terms of the functioning of meeting physiological needs that includes three indicators: (1) the source of drinking water; (2) problems with having access to drinking water; and (3) problems satisfying food needs.

The selected indicators are modeled as random variables with Beta probability distributions. Table I summarizes the parameters of the assigned Beta distributions, before and after a disruption, together with the corresponding acceptability and tolerability thresholds. Considering actual data, one can develop probabilistic predictive models, similar to Equation (5), for each indicator index, instead of assuming probability distributions. Assuming that the indicator indices are statistically independent random variables, we calculate the probability that the state of well-being is acceptable, tolerable, or intolerable, before and after the disruption, using Equations (11) and (12), as follows.

Before the disruption, the probability of each state of well-being is:

$$\mathbf{P}_{\text{acc}} = \mathbf{P}\left\{\bigcap_{j=1}^{3} I_j \in [0.6, 1]\right\} = \prod_{j=1}^{3} \mathbf{P}\{I_j \in [0.6, 1]\}$$
$$= (0.3520)(0.5248)(0.8154) = 0.1506, \quad (17)$$

$$\mathbf{P}_{\text{intol}} = \mathbf{P}\left\{\bigcup_{j=1}^{3} I_j \in [0, 0.4]\right\} = 1 - \mathbf{P}\left\{\bigcap_{j=1}^{3} I_j \in [0.4, 1]\right\}$$
$$= 1 - \prod_{j=1}^{3} \mathbf{P}\{I_j \in [0.4, 1]\}$$
$$= 1 - (0.6480)(0.8208)(0.9657) = 0.4864. \quad (18)$$

Because the states are pairwise disjoint and collectively exhaustive, we can write $\mathbf{P}_{\text{tol}}$ as:

$$\mathbf{P}_{\text{tol}} = 1 - \mathbf{P}_{\text{acc}} - \mathbf{P}_{\text{intol}} = 0.3630. \quad (19)$$

The current approaches using capabilities, like the ones discussed earlier in this article, ignore the uncertainty in the values of indicator indices and use deterministic values (e.g., the means) to represent the achieved functionings. In this example, using only the mean values of the indicator indices, the state of well-being would be "tolerable" because the mean value of the indicator index 1 is below the corresponding acceptability threshold and the mean values of all indicator indices are above their tolerability thresholds. However, accounting for the uncertainty in the values of the indicator indices, using the proposed RCA, the most likely state of well-being is "intolerable." Moreover, using Equation (13), the importance measures of the indicator indices for the tolerable state of well-being are:

$$IM_{1,\text{tol}} = \frac{1}{\mathbf{P}_{\text{tol}}}\mathbf{P}\left\{I_1 \in [0.4, 0.6], \bigcap_{j=2}^{3} I_j \in [0.4, 1]\right\}$$
$$= \frac{1}{\mathbf{P}_{\text{tol}}}\mathbf{P}\{I_1 \in [0.4, 0.6]\} \prod_{j=2}^{3} \mathbf{P}\{I_j \in [0.4, 1]\}$$
$$= \frac{1}{0.3630}(0.2960)(0.8208)(0.9657) = 0.6463. \quad (20)$$

Similarly, the importance measures of the other two indicator indices are:

$$IM_{2,\text{tol}} = 0.5103, \quad IM_{3,\text{tol}} = 0.2202. \quad (21)$$

The obtained result shows that when considering the tolerable state of well-being, the contribution of I_1 is more significant than I_2 and I_3. To explain this observation, we note that the assumed probability distributions are such that I_1 has a higher probability of being in the tolerable state (i.e., in the range [0.4, 0.6]) with respect to I_2 and I_3. As a result, it becomes more likely that tolerable state of I_1 is the main cause of the tolerable state of well-being.

The importance measures of the indicator indices for the intolerable state of well-being are:

$$IM_{1,\text{intol}} = \frac{1}{\mathbf{P}_{\text{intol}}}\mathbf{P}\{I_1 \in [0, 0.4)\}$$
$$= \frac{1}{0.4864}(0.3520) = 0.7237. \quad (22)$$

Similarly, the importance measures of the other two indicator indices are obtained as:

$$IM_{2,\text{intol}} = 0.3684, \quad IM_{3,\text{intol}} = 0.0706. \quad (23)$$

Similar to the observations for the tolerable state, we observe that the contribution of I_1 to the intolerable state of well-being is more significant than

Table I. Probability Distributions of Indicator Indices and Their Thresholds

Variable	Before Disruption			After Disruption				
	Distribution	Mean	Standard Deviation	Distribution	Mean	Standard Deviation	T_{acc}	T_{tol}
I_1	Beta (2.0,2.0)	0.50	0.22	Beta (1.10,1.25)	0.47	0.27	0.6	0.4
I_2	Beta (3.0,2.0)	0.60	0.20	Beta (1.55,1.45)	0.53	0.25	0.6	0.4
I_3	Beta (4.5,1.5)	0.75	0.16	Beta (1.75,1.25)	0.58	0.25	0.6	0.4

Table II. Probabilities of the State of Well-Being along with the Importance Measures, before and after Disruption (Independent Indicator Indices)

Variable	Before Disruption					After Disruption				
	IM_{tol}	IM_{intol}	$\mathbf{P}_{acc}$	$\mathbf{P}_{tol}$	$\mathbf{P}_{intol}$	IM_{tol}	IM_{intol}	$\mathbf{P}_{acc}$	$\mathbf{P}_{tol}$	$\mathbf{P}_{intol}$
I_1	0.6463	0.7237	0.1506	0.3630	0.4864	0.5195	0.5996	0.0697	0.2038	0.7265
I_2	0.5103	0.3684				0.5200	0.4799			
I_3	0.2202	0.0706				0.4309	0.3524			

I_2 and I_3. We also observe that the contribution of I_1 to the intolerable state becomes even more significant with respect to the tolerable state. To explain these observations, we note that, in this example, the mean value of I_1 is less than those of I_2 and I_3. Also, the probability distribution of I_1 is symmetric but those of I_2 and I_3 are left skewed. As a result, when considering the intolerable range of indicator indices (i.e., the range $[0, 0.4)$), the focus is on the left tail of the probability distributions, where the contribution of I_1 becomes more significant than those of I_2 and I_3 and also with respect to the tolerable state.

After the disruption, the values of the indicator indices and the probability of each state of well-being might change. The changes in the values of the indicator indices are represented by updating the parameters of their distributions, as shown in Table I. The assumed changes in the distribution parameters of the indicator indices lead to smaller mean values and larger standard deviations, representing more uncertainties. Table II summarizes the calculated probabilities of the three states of well-being.

The deterministic estimation of the state of well-being, based on the new mean values of the indicator indices, remains "tolerable." The most likely state of well-being is again "intolerable" ; however, now the probability of the intolerable state is increased with respect to the predisruption one. Table II also summarizes the calculated importance measures of the indicator indices after the disruption.

The proposed formulation can also account for the likely correlation between the values of the indicator indices. To study the effects of the correlation, we assume that the three indicator indices have the same distributions as in the previous case (and summarized in Table I) but they are statistically dependent with correlation coefficients $\rho_{ij} = 0.3$ for $i, j \in \{1, 2, 3\}$, and $i \neq j$.

Table III summarizes the new results that reflect the ability of the proposed method to account for the correlation between indicator indices. We observe that the introduction of the positive correlation between the indicator indices increases the probability of the acceptable state. To explain this observation, we note that the positive correlation enforces similar behavior of the three indicator indices (i.e., all three take large values or all three take small values). Because the probability distributions of I_2 and I_3 are such that they are more likely to be in the acceptable range (i.e., take large values), the positive correlation, which favors similar behavior, helps to increase the probability of the acceptable state of well-being. The contribution of I_1 to the tolerable state of well-being becomes more significant when introducing positive correlation with respect to the independent case. Because the probability distribution of I_1 is symmetric, the probability of I_1 being in a tolerable state is less affected by introducing a positive correlation in comparison to I_2 and I_3, which have skewed probability distributions. As a result, its

Table III. Probabilities of the State of Well-Being along with the Importance Measures, before and after Disruption (Correlated Indicator Indices)

Variable	Before Disruption					After Disruption				
	IM_{tol}	IM_{intol}	$\mathbf{P}_{acc}$	$\mathbf{P}_{tol}$	$\mathbf{P}_{intol}$	IM_{tol}	IM_{intol}	$\mathbf{P}_{acc}$	$\mathbf{P}_{tol}$	$\mathbf{P}_{intol}$
I_1	0.7059	0.7898	0.2129	0.3411	0.4460	0.5358	0.6718	0.1302	0.2210	0.6487
I_2	0.5058	0.4021				0.5135	0.5376			
I_3	0.1691	0.0772				0.3922	0.3946			

contribution to the tolerable state of well-being becomes even more significant with respect to the independent case.

6. CONCLUSIONS

This article proposed a general-purpose mathematical approach, called a Reliability-based Capability Approach (RCA) to evaluate the well-being of individuals. Though the specific contexts of application discussed here are risk analysis and poverty assessment, the RCA can be used for different applications as well. In the proposed RCA, the well-being of each individual is treated as a system that is composed of interconnected indicator indices that define the components of the system. The values or states of the indicator indices collectively determine the state of well-being. To predict the value of each indicator index, probabilistic predictive models are proposed. A Bayesian approach is presented to estimate the unknown parameters of the predictive models. The proposed RCA integrates the predictive models into a system reliability problem to determine the probability that the state of well-being is acceptable, tolerable, or intolerable. Such calculations can be performed for each individual in a study region, and a map could be developed to visualize the spatial distribution of each state of well-being over the entire region. Such maps can help decisionmakers visualize which subpopulations suffer more in the aftermath of a disruptive event. In addition, an importance measure is developed that determines the contribution of each indicator index to an unfavorable state of well-being (i.e., tolerable or intolerable). Such information is particularly important in optimal allocation of limited resources to mitigate a hazard or expedite the recovery process. The proposed formulation is general and applicable to several different fields and therefore makes a step forward toward the development of a uniform approach to the societal risk assessment and decision making across all fields that use a capability approach.

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