Auction Size and Price Dynamics in Sequential Auctions

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Abstract.
This paper uses a unique data-set collected in a consistent way over a number of different auctions to investigate how empirical regularities in sequential auctions depend on the number of lots sold in those auctions. It is shown that prices tend to decline faster in auctions in which a small number of lots were sold. Starting prices tend to be higher in auctions with fewer lots, while average prices are higher in auctions with more lots. Price declines do not appear to be localized at the end of the auctions. Finally, there is no evidence of (i) serial correlation in prices, (ii) changes in price volatility over the course of each auction, and (iii) a systematic relationship between price volatility and number of lots sold.

JEL Classification Codes: D44, Q12.
Keywords: Sequential Auctions, Declining Price Anomaly, Thick Markets, Errors in Variables.

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1. Introduction.

Recent empirical research has documented that prices of similar or identical objects tend to decline over the course of a sequential auction.\(^1\) Most of this research focuses on a single auction at a time and on changes in the expected price. For the most part, it has not investigated whether price dynamics vary systematically across different auctions and how price variability changes within and across auctions. Even studies that include data from multiple auctions make no attempt to estimate the existence of a differential impact of auction size on price behavior.\(^2\)

This paper utilizes data collected in a consistent way across a number of cattle auctions to investigate the impact of auction size on price behavior, within and between auctions. Importantly, and in contrast to most of the existing literature, the data include appraisals of value by an outside expert that were unobserved by the participants. The key findings are: (i) prices, adjusted for appraised value, decline on average over the course of each auction, (ii) this pattern is more pronounced for small auctions, (iii) average adjusted prices are higher in large auctions, (iv) starting prices are somewhat higher in small auctions, (v) there is little evidence for serial correlation or local trends in prices over the course of the auction, (vi) there is no evidence that price volatility varies systematically either across auctions or over the course of an auction, (vii) there is no evidence that price declines are concentrated at the end of an auction (no “sunset” effects), and (viii) appraised value declines over the course of an auction.

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\(^2\) A notable exception is Jones, Menezes, and Vella (1997).
A number of different factors, including bidder risk aversion,³ complementarities⁴, and uncertainty about the value of later lots, combined with a decreasing willingness to pay for additional lots after a bidder has already made some purchases⁵, lead to a declining price pattern over the course of an auction.⁶ The empirical literature simply documents declining prices, the common prediction of all these models and, therefore, can not distinguish among these possibilities.⁷ In this study, a clear pattern is found between the size of the auction and (1) the degree of price decline, (2) the expected starting prices, and (3) the expected average prices. No clear relationship is found between price volatility and auction size. Since theoretical models that predict declining prices may not have the same comparative statics with respect to a change in auction size, these empirical results may shed some light on the underlying causes of declining prices. However, the theoretical literature on sequential auctions considers markets with an exogenous number of items and bidders.⁸ Though it is possible to use some of these models to analyze the effect of exogenous increases in the number of bidders or the number of items, actual markets are characterized by an endogenous number of participants. Therefore, using these empirical results to test the theoretical models requires extending the results of these models.

³ See, for instance, McAfee and Vincent (1993).

⁴ See, for instance, Branco (1997).

⁵ See, for instance, Bernhardt and Scoones (1994) and Engelbrecht-Wiggans (1994).

⁶ Other factors push the equilibrium path of expected prices upwards over the course of the auction. These include increasing stochastic scale effects [Jeitsko and Wolfstetter (1998)], and affiliation of values [Robert, Laffont, and Loisel (1994) among others].

⁷ However, one could attempt to look into bidding environments where all but one of the above factors can be ruled out [Katzman (1998)]. Carefully constructed laboratory experiments can also be used for that purpose.

⁸ Models of single object auctions with endogenous entry have been analyzed by McAfee and McMillan (1987) and Levin and Smith (1994).
A formal analysis of the comparative statics of theoretical models with respect to auction size is beyond the scope of this study. However, we suggest an economic intuition consistent with the empirical results. The main elements of this intuition, which is described in greater detail in the concluding section of the paper, are as follows: (i) With endogenous bidder participation expected bidder surplus is the same across auctions. Larger auctions create greater value per sale by being “thicker” markets: it is more likely that a bidder will find an item that best suits his needs. Bidder indifference to auctions of different size then demands that in equilibrium larger auctions have more bidder participation and, hence, higher prices. Smaller auctions, with lower expected prices, continue to attract sellers because the need to sell is dispersed, both geographically and inter-temporally. (ii) The decline in expected prices can be sustained in equilibrium through any of the theoretical factors discussed above. Prices decline more quickly in smaller auctions because these auctions “thin” more quickly than large auctions. Larger auctions look relatively the same, in terms of bidder participation, from start to finish. (iii) Prices start lower in large auctions because, for any given rate of price decline, the option value of waiting to compete for later lots is higher for a large auction: There are simply more lots to choose from and compete for. This dampens bidder aggressiveness for early lots. (iv) The absence of any trends in price volatility over the course of the auction suggests that two opposing forces essentially cancel each other out: while thinning markets might have more variable prices, price volatility could decline as bidders learn about the other bidders’ willingness to pay in this particular auction. (v) Finally, the absence of local trends and serial correlation indicates that, except for the decline in expected prices, there are no large predictable swings in bidder competition over the course of the auction.

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9 For an option value interpretation of bidding behavior in sequential auctions, see Fehr and Riis (1998).

10 Jeitschko (1998) is a recent analysis of learning in a sequential auction.
This paper is organized as follows. Section 2 describes the data and the collection process. Section 3 develops the empirical methodology while Section 4 includes the results. The paper ends with a brief concluding discussion and an Appendix.

2. Data.

The data consist of prices in 16 public auctions of dairy cattle that took place between October 1987 and April 1988. The data were originally collected to estimate hedonic models of cattle pricing. As part of the data collection process, Calvin Meyer, an experienced appraiser and dairy farmer, was hired to attend all auctions and provide an estimate of the price each animal would fetch if sold by private treaty. The appraisal was made before the auction on the basis of direct inspection of the cattle and information provided by the auction catalogue and was not available to any of the participants. Every effort was made to avoid any influence of the auction itself on the appraiser’s estimates.

Obtaining this measure of value is important because there may be a systematic relationship between the value of cattle and the order in which they are sold. Indeed, it is frequently asserted that auctioneers prefer to place high value lots earlier in the auction and low value lots toward the end. For instance, paintings are often sold in chronological order, which tends to place higher value paintings towards the start of the auction [see Beggs and Graddy (1997)]. In this data-set as well, appraised values for cattle sold in the latter half of an auction are lower than appraised

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11 See Dill (1990) for the results of the hedonic analysis and for additional detailed information on each of these auctions. This data-set has also been used to calibrate the Engelbrecht-Wiggans (1994) model of sequential auctions [see Engelbrecht-Wiggans and Kahn (1998).] The original data includes four additional auctions. One of these was dropped as it only contained six live animals. The other three were auctions which lasted for more than a single day. These auctions were also not incorporated in the present study because the data-set does not distinguish the day in which a lot was sold.
values for cattle sold in the first half of that auction. In the absence of a second estimate of the underlying value of a lot to use as a control, we would observe declining prices over the course of auction even if bidder interest and aggressiveness remained constant.

The number of lots offered for sale varied significantly across auctions. In some auctions fewer than 50 cattle were put up for sale, including an auction with only 21 calves. On the other extreme, in two auctions more than 200 cattle were sold. Given that the data, and in particular the appraised values, were consistently collected across auctions, we have a rare opportunity to test whether the existence and extent of any declining price patterns over the course of a sequential auction depend on the number of lots to be auctioned. We can also compare average (adjusted for value) prices across auctions to determine whether thicker markets yield higher prices.

The key characteristics of the data are summarized in Table 1 below. Average “appraised value” is 15% lower than average price. This however, is not an indication of bias in the appraisal, since the appraiser’s estimate is intended to be the price the cattle would fetch if sold by private treaty and not the price it would fetch in this particular auction. Furthermore, even if there had been some bias in the appraisal, it would only have had an impact on the conclusions of this study if the bias was systematically related to the order in which the cattle was sold and to the size of the auction.

Even though the median and the mean are close, there is some noticeable skewness in the distribution of the sale price and appraised value. This can be inferred from the high, relative to the mean, standard deviation, and is confirmed by looking at the histograms of these two variables. However, the logs of price and appraised value, which are the variables used in the analysis, have a symmetric, bell shaped distribution. There is substantial variation in the average appraised value of cattle sold in the auctions with no particular relationship between auction size

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12 Theoretical analysis suggests that selling higher value lots first increases expected total revenue. See Beggs and Graddy (1997), Benoit and Krishna (1998), and Bernhardt and Scoones (1994).
## TABLE 1. Summary Statistics.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs.</th>
<th>Mean</th>
<th>Median</th>
<th>St. Dev.</th>
<th>Min.</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appraised Value</td>
<td>1503</td>
<td>$1,444</td>
<td>$1,500</td>
<td>$822</td>
<td>$100</td>
<td>$10,000</td>
</tr>
<tr>
<td>Sale Price</td>
<td>1503</td>
<td>$1,647</td>
<td>$1,450</td>
<td>$1,039</td>
<td>$250</td>
<td>$14,000</td>
</tr>
<tr>
<td>Number of Lots&lt;sup&gt;a&lt;/sup&gt;</td>
<td>16</td>
<td>93.9</td>
<td>86.5</td>
<td>55.5</td>
<td>21</td>
<td>207</td>
</tr>
</tbody>
</table>

### Statistics by Auction:

<table>
<thead>
<tr>
<th>Auction Location and Date</th>
<th>Obs.</th>
<th>Mean</th>
<th>Median</th>
<th>St. Dev.</th>
<th>Mean</th>
<th>Median</th>
<th>St. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freeport, IL - 10/24/87</td>
<td>58</td>
<td>$1,645</td>
<td>$1,600</td>
<td>$642</td>
<td>$1,963</td>
<td>$1,875</td>
<td>$869</td>
</tr>
<tr>
<td>Plum City, WI - 10/26/87</td>
<td>207</td>
<td>$889</td>
<td>$800</td>
<td>$367</td>
<td>$1,093</td>
<td>$1,050</td>
<td>$374</td>
</tr>
<tr>
<td>Louisville, KY - 11/07/87</td>
<td>84</td>
<td>$1,547</td>
<td>$1,550</td>
<td>$466</td>
<td>$1,894</td>
<td>$1,750</td>
<td>$895</td>
</tr>
<tr>
<td>Platteville, WI - 02/15/88</td>
<td>207</td>
<td>$1,212</td>
<td>$1,300</td>
<td>$498</td>
<td>$1,078</td>
<td>$1,000</td>
<td>$506</td>
</tr>
<tr>
<td>Baldwin, WI - 02/20/88</td>
<td>49</td>
<td>$2,033</td>
<td>$2,000</td>
<td>$793</td>
<td>$1,467</td>
<td>$1,250</td>
<td>$668</td>
</tr>
<tr>
<td>Lowell, MI - 03/21/88</td>
<td>109</td>
<td>$1,777</td>
<td>$1,800</td>
<td>$631</td>
<td>$2,240</td>
<td>$1,950</td>
<td>$1,416</td>
</tr>
<tr>
<td>Fond du Lac, WI - 03/23/88</td>
<td>116</td>
<td>$1,953</td>
<td>$1,800</td>
<td>$882</td>
<td>$2,085</td>
<td>$1,875</td>
<td>$1,065</td>
</tr>
<tr>
<td>Carlyle, IL - 03/26/88</td>
<td>55</td>
<td>$1,780</td>
<td>$1,600</td>
<td>$699</td>
<td>$2,060</td>
<td>$2,050</td>
<td>$570</td>
</tr>
<tr>
<td>Ft. Atkinson, WI - 03/28/88</td>
<td>103</td>
<td>$2,426</td>
<td>$1,800</td>
<td>$1,547</td>
<td>$2,830</td>
<td>$2,100</td>
<td>$1,850</td>
</tr>
<tr>
<td>Dundee, IL - 03/31/88</td>
<td>50</td>
<td>$1,542</td>
<td>$1,450</td>
<td>$633</td>
<td>$1,694</td>
<td>$1,375</td>
<td>$953</td>
</tr>
<tr>
<td>Fond du Lac, WI - 03/30/88</td>
<td>139</td>
<td>$1,278</td>
<td>$1,200</td>
<td>$538</td>
<td>$1,340</td>
<td>$1,250</td>
<td>$579</td>
</tr>
<tr>
<td>Goshen, IN - 04/07/88</td>
<td>49</td>
<td>$1,451</td>
<td>$1,400</td>
<td>$474</td>
<td>$1,161</td>
<td>$1,025</td>
<td>$500</td>
</tr>
<tr>
<td>Goshen, IN - 04/08/88</td>
<td>54</td>
<td>$1,085</td>
<td>$1,200</td>
<td>$421</td>
<td>$1,131</td>
<td>$975</td>
<td>$725</td>
</tr>
<tr>
<td>Urbana, IL - 04/09/88</td>
<td>21</td>
<td>$1,010</td>
<td>$800</td>
<td>$615</td>
<td>$1,196</td>
<td>$900</td>
<td>$1,100</td>
</tr>
<tr>
<td>Hampshire, IL - 04/14/88</td>
<td>89</td>
<td>$1,671</td>
<td>$1,600</td>
<td>$799</td>
<td>$1,723</td>
<td>$1,525</td>
<td>$942</td>
</tr>
<tr>
<td>Chino, CA - 04/21/88</td>
<td>113</td>
<td>$2,120</td>
<td>$2,000</td>
<td>$412</td>
<td>$1,975</td>
<td>$1,800</td>
<td>$666</td>
</tr>
</tbody>
</table>

Note: (a) An auction is the unit of observation for this variable. See Dill (1990) for additional information on these auctions.
and the value of the cattle offered for sale. On the contrary, there is a strong relationship between appraised value and the order in which a lot is put for sale in an auction. On average, the mean appraised value of the first 20% of the lots offered for sale in any given auction is 35% higher than the mean appraised value of the last 20%. This difference in appraised values raises to 53% when comparing the first with the last 10% of the lots.

Notably absent from the data is any information on the number of active bidders. Indeed, this information is unverifiable because one can not ascertain whether a farmer present in the auction is actively participating, or attending in order to collect information, or for other business and social purposes. It is, however, reasonable to assume, that large auctions attract more bidders. Indeed, as we discuss below, our results are consistent with this hypothesis.


Recall that the appraisal is meant to be equal to the expected price that a lot, \( i \), would fetch if sold in a farm via private treaty. Denote this expected price, which we will also refer to as the value of the lot, by \( E_{FARMPRICE} \). The value of \( E_{FARMPRICE} \) is unobserved to us but is observed, with some error, by both the appraiser, the auctioneer, and the buyers.

Clearly, the sale price in the auction will depend on the value of the lot. Furthermore, when many lots are sold in a single auction, the expected price of a lot may also depend on the order that it is sold. Define

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\(^{13}\) If anything, the relationship is U-shaped. Auctions with approximately 100 lots have the highest average appraised values. Appraised values are somewhat lower for smaller auctions, and substantially lower for the two largest auctions. Overall, the correlation between average appraised value and auction size is -0.15.

\(^{14}\) Simple regression analysis with auction fixed effects demonstrates that the negative relationship between appraised value and the order that a lot is put for sale is statistically significant.
\[ LOCATION_{i,k} = \frac{ORDER_{i,k} - 1}{LOTS_k - 1} \]

to be the relative location of a lot \( i \) in auction \( k \), where \( ORDER_{i,k} \) is the order in which this lot is offered for sale and \( LOTS_k \) is the total number of lots offered for sale in auction \( k \). Observe that \( LOCATION_{i,k} \) ranges from 0 to 1.\(^{15}\)

We assume the price at which lot \( i \) is sold in auction \( k \) is given by

\[
\ln(PRICE_{i,k}) = \alpha_k + \ln(EFARMPRICE_i) + \beta_k \times LOCATION_{i,k} + \tilde{\epsilon}_{i,k} \quad (1)
\]

where \( \tilde{\epsilon}_{i,k} \) is a random variable with \( E[\tilde{\epsilon}_{i,k}] = 0 \) and \( Var[\tilde{\epsilon}_{i,k}] = \sigma^2 \) and the constant \( \alpha_k \) captures the fact that prices in auction \( k \) may differ systematically from the prices would have been obtained by sale under private treaty.\(^{16}\)

The discussion in the introduction about the impact of auction size on average prices and the rate of price decline suggests that

\[
\alpha_k = \alpha_0 + \alpha_1 \times LOTS_k + \tilde{u}_k \quad \text{and} \quad \beta_k = \beta_0 + \beta_1 \times LOTS_k \quad (2)
\]

is a reasonable way to parametrize the coefficients in equation (1). The random effect, \( \tilde{u}_k \), captures the effect of other auction specific factors that affect prices, such as the location of the

\(^{15}\) On average 13.5% of the lots sold in these auctions were not dairy cattle. Except for the order in which they were sold, no information on these lots is available and they are ignored in the analysis. The variable \( ORDER \) does not take into consideration the fact that these other lots were also interspersed among the sale of dairy cattle. Using an alternative definition of \( ORDER \), which takes into consideration the fact that other lots were also sold in the auction, yields essentially identical results.

\(^{16}\) Notice that, for expected prices in an auction to equal the expected price under private treaty, \( \alpha_k \) must be negative. Suppose, for instance, that \( \tilde{\epsilon}_{i,k} \) is normally distributed and that there are no location effects \( (\beta_k = 0) \). Then, the expected price in auction \( k \) is given by \( E[PRICE_{i,k}] = EFARMPRICE_i \cdot e^{\alpha_k + \frac{1}{2} \sigma^2} \).

Therefore, expected auction price equals expected price under sale by private treaty for \( \alpha_k = -\frac{1}{2} \sigma^2 < 0 \).
sale, the number of sales taking place shortly before or shortly after the sale, and perhaps even the weather conditions on the day of sale. The data contain no information on these factors, but we will assume that they are not systematically related to the size of the auction.

Direct estimation of (1) is precluded by the absence of data on $EFARMPRICE_i$. We do observe, however, the estimate of $EFARMPRICE_i$ given by the appraiser. As long as the appraiser does not systematically under-appraise high value lots relative to low value lots or vice-versa, we can write

$$\ln(\text{APPRAISAL}_i) = a + \ln(\text{EFARMPRICE}_i) + \tilde{v}_i$$  \hspace{1cm} (3)$$

where $\text{APPRAISAL}_i$ is the appraised value and $\tilde{v}_i$ is a random variable with mean zero. Combining equations (1) and (3) we have

$$\ln(\text{PRICE}_{i,k}) - \ln(\text{APPRAISAL}_i) = \gamma_k + \beta_k \text{LOCATION}_{i,k} + \tilde{\xi}_{i,k}$$ \hspace{1cm} (4)$$

where $\tilde{\xi}_{i,k} = \bar{u}_k + \bar{\epsilon}_{i,k} - \bar{v}_i$ and $\gamma_k = \alpha_k - a$. Equation (4) can be directly estimated either auction by auction to obtain estimates of $\gamma_k$ and $\beta_k$ for each auction, or by pooling all auctions together and using the parametrization in (2) to obtain estimates of $\alpha_0 - a$, $\alpha_1$, $\beta_0$, and $\beta_1$. In both cases, OLS gives consistent estimates. However, OLS ignores the error components nature of the disturbance term in the “pooled” model. Therefore, this model is also estimated using GLS.

The approach outlined above avoids the errors-in-variables problem that would have arisen had $\text{APPRAISAL}_i$ been included as a regressor. Recall that the value of a lot appears to vary systematically over the auction. Therefore, one must include a measure of the value in the analysis. However, the only measure available, the appraised value, is a noisy signal of the underlying value. Notice that substituting $EFARMPRICE_i$ from equation (3) into equation (1) yields
\[
\ln(PRICE_{i,k}) = \gamma_k + \ln(APPRaisal_{i,k}) + \beta_k \, LOCATION_{i,k} + \tilde{\zeta}_{i,k}
\]

The disturbance term of this regression, \( \tilde{\zeta}_{i,k} = \tilde{u}_{i,k} + \tilde{\epsilon}_{i,k} - \tilde{\nu}_{i,k} \), is correlated with \( APPRAISAL_i \) from equation (3). Therefore, estimating the model

\[
\ln(PRICE_{i,k}) = \gamma_k + \delta \, \ln(APPRaisal_i) + \beta_k \, LOCATION_{i,k} + \tilde{\zeta}_{i,k}
\]  

(5)

would yield biased estimates of all parameters. Using \( \ln(APPRaisal_i) \) as a regressor would not result in substantial bias if the quality of the appraisal was high, i.e., if the appraiser were able to estimate expected price (under sale by private treaty) very precisely. Unfortunately, we do not observe expected price or any variables that are correlated with expected price but uncorrelated with the appraisal error, and hence, we can not directly assess the appraisal accuracy. The approach taken in this paper is to set the coefficient of \( \ln(APPRaisal_i) \) equal to 1, by assuming no systematic bias in the appraisal of high value lots relative to the appraisal of low value lots. Nevertheless, we find that the results are not sensitive to the inclusion of the appraisal value as a regressor.

In the next section we present the estimation results, starting from an auction-by-auction regression analysis, to a pooled auction analysis, and finally to the analysis of pricing patterns within each auction.

\[17\] Using appraisal value as a regressor is also not a problem if one is interested in how sale price and appraised value are correlated in the population.

\[18\] In these results, which are summarized in the next section, the estimate of \( \delta \) is in the 0.70 to 0.75 range. The “bias towards zero” is what one would expect in the presence of this type of measurement error. Unfortunately, Instrumental Variables estimation is not possible as we lack satisfactory instruments.
4. Results.

4.1. Auction by Auction Analysis.

Equation (4) has been estimated for each auction separately. The estimate of $\beta_k$ can be interpreted as indicating the average rate at which prices, adjusted for the value of the lot, decline over the course of the auction. The (unweighted) average coefficient estimate of $LOCATION_{i,k}$ is equal to $-0.280$.\textsuperscript{19} The auction by auction parameter estimates are not of direct interest and are therefore relegated to Table A-1 in the Appendix. Of more interest is the relationship between the speed of price decline and size of the auction. Define by $r_k(\ell_1, \ell_2)$ the expected percentage change in the price of a lot, sold in auction $k$, when this lot is moved from location $\ell_1$ to location $\ell_2$,

$$r_k(\ell_1, \ell_2) = \frac{E[PRICE_{i,k}(\ell_2)] - E[PRICE_{i,k}(\ell_1)]}{E[PRICE_{i,k}(\ell_1)]}$$

Since

$$E[PRICE_{i,k}(\ell)] = e^{\alpha_k + EFARMPRICE_i + \beta_k \ell} E[e^{\tilde{e}_{i,k}}]$$

the above ratio is equal to

$$r_k(\ell_1, \ell_2) = e^{\beta_k (\ell_2 - \ell_1)} - 1$$

Figure 1 below plots $r_k(1/4, 3/4)$ [the expected percentage price decline from the end of the first quartile to the beginning of the fourth quartile] against the number of lots sold in an auction\textsuperscript{20}. Larger auctions appear to have lower average rates of price decline. This slower rate of

\textsuperscript{19} All 16 coefficient estimates are negative, 8 are significant at the 5% level, 10 significant at the 10% level.

\textsuperscript{20} The linear fit and associated $R^2$ in this graph are only meant to facilitate the inspection of the figure and are not intended as a formal fit through the estimated coefficients.
Percent Price Decline:
End of 1rst quantile to end of 3rd quantile.

\[ R^2 = 0.2036 \]

![Figure 1: Average Prices vs. Auction Size](image1)

\[ R^2 = 0.0222 \]

![Figure 2](image2)
price decline results in larger auctions having higher average prices (adjusted for appraised value) than smaller auctions. This can be seen in Figure 2 below, which plots the average value of ln(PRICE/APPRAISAL) against the number of lots sold in the auction.\textsuperscript{21} However, there is no discernible relationship between the residual price variance and auction size: the correlation between the standard deviation of the regression disturbance term and auction size is -0.023, which is statistically indistinguishable from zero by any reasonable measure.

\textbf{4.2. Pooled Analysis.}

A formal test of any relationship between auction size, average price, and the rate of price decline requires estimation using the entire sample of observations from all 16 auctions. The results of the pooled sample estimation are shown in Table 2. The first set of models shows the OLS results. Model (1) assumes the parametrization shown in equation (2). Even though only a small portion of price variability is explained once we control for appraised value, prices are shown to decline over the course of the auctions, with price declines for small auctions exceeding price declines for large auctions.\textsuperscript{22} Also, since $\hat{\alpha}_1 < 0$, the expected price of the first unit sold is higher in smaller auctions. In fact, a simple calculation shows that increasing the number of lots in such a way that the relative position of existing lots does not change decreases the price for the first third of the lots and increases the price on the remaining two-thirds of the lots.\textsuperscript{23} Both the location and lot effects are significant.\textsuperscript{24}

\textsuperscript{21} The correlation between auction size and adjusted prices is 0.149.

\textsuperscript{22} Prices are forecasted to decline for all of the auctions in our sample since $\hat{\beta}_k = 0$ if $LOTS_k = 282$, which exceeds the size of the largest auction in the data.

\textsuperscript{23} This follows by observing that $\hat{\alpha}_1 + \hat{\beta}_1 \text{LOCATION}_{i,k} = 0$ when $\text{LOCATION}_{i,k} = 0.336$. Here the number of lots is treated as a continuous variable.

\textsuperscript{24} The coefficient of $LOTS$ has a t-statistic of -1.557. However, the p-value of the joint effect of auction size on initial prices and rate of price decline is 0.0035. Dropping $LOTS$ from the regression leads to essentially the same results, but it is better to err on the side of caution and allow auction size to affect

\pagebreak
The remaining four OLS models shown in Table 2 explore the robustness of the results to alternative specifications. Model (2) uses the log instead of the level of $LOTS_k$ to parametrize the parameters of equation (4). The results are not only qualitatively identical but also quantitatively similar to those of Model (1). Price declines for small auctions exceed price declines for large auctions. Increasing the number of lots in an auction without affecting the relative position of the existing lots decreases the prices in the first quarter of the lots and increases the prices of the remaining three quarters.

In fact, the declining price effect is remarkably stable across specifications. The estimates of $\hat{\beta}_0$ for the last two models, which are directly comparable, are identical to four significant figures, despite the fact that the model with the auction dummies explains a much greater proportion of the price variation than the models that include only the auction size effect. Model (3), which relaxes the $\delta=1$ assumption, yields broadly similar results with the first two models. In fact, the results are “stronger” with this specification. However, as mentioned above, this specification suffers, at least to some extent, from an errors-in-variables problem. Its inclusion here demonstrates that the results do not rely on the assumption that the appraiser does not systematically over or under-appraise high value lots relative to low value lots. Model (4) which contains the restriction $\beta_1 = 0$, confirms that bigger auctions, on average, have higher prices. This result is not statistically significant: the difference in the average price level is the combination of the difference in starting prices and the difference in the rate of price decline. These two effects push in opposite directions leading to somewhat higher average prices for bigger auctions, but not strikingly so. Finally, Model (5) reveals that other, unobserved, auction

---

25 The estimates of the coefficients of $LOCATION_{i,k}$ for the first two models are not directly comparable with each other and with those of the last two OLS models because of the incorporation of an interaction between $LOCATION_{i,k}$ and $LOTS_k$. 

-14-
TABLE 2. Results of Pooled Analysis.

<table>
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<th>Variable</th>
<th>[parameter]</th>
<th>O.L.S.</th>
<th>Error Components</th>
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<td></td>
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<td>(1) (2)</td>
<td>(3) *</td>
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<tr>
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<td>[α₀ - α]</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
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<td>0.2693</td>
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<tr>
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<td></td>
<td></td>
<td>0.0003</td>
<td>0.0003</td>
</tr>
<tr>
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</tr>
<tr>
<td>LOCATION * LOTS</td>
<td>[β₁]</td>
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<td>0.00172</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.00052</td>
<td>0.00048</td>
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<td></td>
<td></td>
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<td>0.0571</td>
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<td>APPRAISAL</td>
<td>[δ]</td>
<td>0.7402</td>
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</table>

Note: (a) The dependent variable for models (3) and (8) is ln(PRICE). The dependent variable for all remaining models is ln(PRICE/APPRAISAL).

Standard Errors are in italics below the parameter estimates. N = 1503 for all models.
characteristics are important in determining average prices: Using a comprehensive set of auction
dummies increases the $R^2$ from less than 5% to over 22%. Therefore, given that there are only 16
different auctions in the data, it is worthwhile to re-estimate the model taking into consideration
the error components nature of the disturbance term.

The results of the error components analysis confirm that approximately 18% of the
variability in prices comes from auction-specific factors.\textsuperscript{26} As expected, standard errors for
variables that only change across auctions increase, reflecting the smaller effective sample size.
However, standard errors for the remaining variables decrease, reflecting the lower effective
variability of the disturbance term within an auction once the auction specific component is
accounted for. Parameter estimates change very little across the two estimation methods. This is
true regardless of the specification.

Overall, none of the pooled data-set results are surprising given the auction by auction
estimation analysis. However, a lot of information is lost by looking at average rates of price
decline and average prices. To gain insight on the dynamics of price behavior, one needs to focus
on the evolution of prices within each auction. We turn to this next.

\textit{4.3. Within Auction Analysis.}

A cursory look at Table 1 reveals a large variance for both appraised values and sale prices.
The ratio of sale price to appraised value is also highly variable. This makes it difficult to discern
clear patterns in the prices by plotting $\ln(PRICE_{i,k}) - \ln(APPRaisal_i)$ against the relative location
of a lot in the auction in which it was sold. If we are to glean insight on the dynamics of price
behavior within an auction from visual inspection of the data, $\ln(PRICE_{i,k}) - \ln(APPRaisal_i)$ must
be smoothed, and the smoothed series plotted against relative lot location.

\textsuperscript{26} The (random effects) error components analysis follows Fuller and Battesse (1973).
Since an important aspect of this study is determining how the price trends vary across auctions of different size, we grouped auctions according to the number of auctioned lots. This yielded four separate groups, three with three auctions each and one with four auctions. The three auctions that were left out from these groups do not yield any patterns that are observationally different from the auctions that have been grouped.

The series was smoothed non-parametrically in two different ways. In the first one, the bandwidth is equal to the same proportion of sales for all auctions. The smoothed value of \( \ln(PRICE_{i,k}) - \ln(APPRAISAL_{i,t}) \) consists of the average value of the nearest 20% of the observations, 10% on each side. In the beginning of the auction when less than 10% of the sales have taken place prior to the sale for which the smoothed value is calculated, fewer than 20% of the observations are used. Similarly, fewer observations are used for smoothing prices towards the end of the auction when there are less than 10% of the lots remaining to be sold. Therefore, for the observations in the edges there is less smoothing than for the central 80% of the observations. This smoothing is referred to as \textit{Proportional Bandwidth Smoothing} because the same proportion of observations is used for smoothing regardless of the total number of observations in the auction. In the second method of smoothing, referred to as \textit{Fixed Bandwidth Smoothing}, the bandwidth was kept fixed to 25 lots across auctions.\(^{27}\)

The Proportional Bandwidth smoothed values are shown in Figure 3. Comparing the price trends for auctions of different sizes it is apparent that most auctions with a few lots feature a pronounced declining pattern. However, auctions with over 100 lots are equally likely to have broadly constant prices as opposed to declining prices. It has been suggested that the declining price trend in auctions is really a “sunset” effect; it is important only for the last 20 to 40 lots.

\(^{27}\) Both methods use a rectangular kernel which calculates the fitted (smoothed) value by attaching the same weight to all observations within the bandwidth. Alternatively, one could have used a kernel that attaches a higher weight to observations closer to the fitted value. Since the non-parametric analysis is used here for descriptive purposes, the simpler and most transparent kernel was used.
Large auctions may show no clear trends because this “sunset” effect is small relative to the entire auction, while in sufficiently small auctions the entire auction is part of the “sunset”. Figure 3 does not provide support for this hypothesis. Consider, for instance, the three largest auctions. In two of them, prices steadily decrease during the last 50 observations while in the other prices increase during the last 50 observations. In auctions with 100 lots there appear to be no evident differences in price patterns between the first and second halves of the auctions. In fact, prices are somewhat more likely to increase towards the very end of the auction! Finally, in the auctions with only about 50 lots, it is equally likely for the decline to be more pronounced in the first half of the auction than in the second.

This qualitative assessment has been tested by re-estimating the OLS models using linear spline regression, allowing for a “kink” half-way through the auction. When \( \ln(\text{PRICE}/\text{APPRAISAL}) \) is used as the dependent variable, price declines appear only slightly steeper in the 2nd half of the auction: the value of \( \beta_0 \) decreases by about 0.065 after the break-point in all specifications.\(^{28}\) Moreover, one can not reject the null of no structural change: the p-value of the test that the values of the slope and intercept are the same for the 1st and 2nd halves of the auction is around 0.55 for all specifications.\(^{29}\) When \( \ln(\text{PRICE}) \) is used as the dependent variable [Model (3)] the rate of price decline is essentially the same for both halves of the auction: the value of \( \beta_0 \) increases by 0.022 and the p-value for the null of no structural change is 0.84.

A visual examination of Figure 3 also reveals striking variability in the smoothed price series, with noticeable price swings over the course of the auction. Given the number of observations used to compute the smoothed values, one would expect a much lower variability around a

\(^{28}\) The remaining two parameters, \( \alpha_1 \) and \( \beta_1 \), are not allowed to change at the break point. Their estimated values are essentially the same as those reported in Table 2.

\(^{29}\) This test has only one degree of freedom because of the continuity constraint implicit in spline regression.
declining trend. Indeed, simulating pseudo-observations by adding i.i.d. normal disturbances to the predicted prices from the auction-by-auction regressions yields series which, upon smoothing, follow the declining trend very closely. There are some explanations for this “excess” volatility. One possibility is that there is substantial serial correlation or local trends in prices. Another possibility is that the distribution of prices has fatter tails than those of the normal distribution. We first examined the possibility that disturbances follow an AR1 process. Re-estimating the auction-by-auction regressions via the Cochrane-Orcutt method yields positive estimates of serial correlation for 10 of the 16 auctions. However, only in three of these instances was the degree of serial correlation statistically significant at the 10% significance level. This is similar to the results one would expect in the absence of any serial correlation. Therefore, there is little evidence supporting the first hypothesis.

On the contrary, there is very strong evidence that the distribution of prices, conditional on the appraised value and lot location, has fat tails. We computed the kurtosis coefficient of the residuals of the auction-by-auction regressions. Fourteen out of the sixteen auctions in the sample had residuals with a kurtosis coefficient higher than that of the normal distribution. In eight, of these instances the difference was statistically significant at the 10% level. A joint test using the standardized residuals from all 16 auctions yields a coefficient of kurtosis equal to 4.45 which exceeds that of the normal distribution for any reasonable level of significance.

A set of simulations was carried out to demonstrate that fat tails are sufficient to yield price behavior similar to that observed in the data. We obtained for each one of the auctions the predicted price using the estimates of the auction-by-auction regressions. We then added to each

---

30 The variance of the pseudo-disturbances is equal to the predicted variance of the error term from the auction-by-auction regressions.

31 The average estimated value of $\rho$ is equal to 0.056, and the median equal to 0.031.

32 The average coefficient was equal to 4.61, and the median 4.00. The normal distribution has a kurtosis coefficient equal to 3.
one of the predicted prices a disturbance term drawn at random, without replacement, from the actual residuals of the corresponding auction regressions. These simulated price series were then smoothed non-parametrically. If the volatility was due to local trends, then the smoothed simulated series should appear less volatile since any pattern of serial correlation is broken by the re-sampling scheme. If the volatility is due to “fat” tails, the smoothed simulated series would appear as volatile as the original. This simulation exercise was repeated four times. A representative set of results is shown in Figure A-1 in the Appendix. It is apparent that the smoothed simulated series are as volatile and contain as many local “swings” as the original series.

Finally, one might think that price variability changes over the course of the auction. Volatility could be higher towards the end, as the auction “thins out”. Alternatively, it could be higher at the start as participants learn from price levels about the strength of demand by others in this particular auction. However, there is no evidence that price volatility varies over the course of an auction. The regression of the squared residuals from the auction-by-auction analysis on lot location yields negative slope estimates in nine out of the sixteen auctions, 2 of which are significantly different from zero at the 10% level. None of the positive slope estimates are statistically significant. Furthermore, splitting the sample and re-estimating Model (1) using only the lots sold in the first half of the auctions yields an estimate of standard deviation for the disturbance term equal to 0.35. The corresponding estimate using only the lots sold in the second half of the auction is nearly identical and equal to 0.33. Either both of the factors mentioned above are of minor importance or they cancel each other out.

Overall, the evidence indicates high price variability which is constant throughout the auction and to some extent masks the decline in average prices. These price declines are somewhat more pronounced for smaller auctions and they are not concentrated towards the end of the auction.
FIGURE 3
Smoothed Prices - Proportional Bandwidth Smoothing

Auctions with N = 49 to 50

Auctions with N = 54 to 58

Auctions with N = 103 to 116

Auctions with N = 139 to 207
The series obtained via Fixed Bandwidth Smoothing are similar to those obtained under Proportional Bandwidth Smoothing except that there appears to be more variability in larger auctions. This is expected: Since values are smoothed equally for auctions of all sizes but an interval in a large auction includes more sales than an interval in a small auction, there would naturally appear to be more “action” in larger auctions. The results are shown in Figure A-2 in the Appendix.

5. Discussion and Concluding Remarks.

Though it is beyond the scope of this empirical study to explore the comparative statics of formal theoretical models with respect to auction size, an intuition that is consistent with the results of this study and draws from our general theoretical understanding of auctions is easy to outline and could perhaps motivate formal modeling. This intuition must not only explain why prices decline more slowly in large auctions, but also why systematic price differences across auctions are not arbitraged away.33

Consider first the rationale for why prices in large auctions should be higher than equilibrium prices in small auctions. On the demand side, if bidder participation is limited by attendance costs, the number of bidders will be determined by the equality of expected bidder surplus and these costs. If participation costs do not vary systematically with auction size, then bidder surplus will not vary with auction size either. But large auctions, by being thicker markets on the supply side, should be able to create larger total value per sale: Since lots are at best only stochastically identical, for any given number of participants it is more likely that bidders will be able to win an item that closer matches their needs. In addition, larger auctions should have more participants: if

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33 Formal economic modeling (and the intuition described in this paper) assumes that auction participants are aware of these price differences across auctions and of declining prices within an auction. However, Burns (1985) provides evidence that professional bidders are frequently unaware of such a pattern, even though it is clearly discernible to an outside researcher.
not, then prices in larger auctions would be lower\textsuperscript{34}, yielding greater bidder surplus in these auctions. Auctions with more bidders yield higher value than auctions with a few bidders: The expected value of the highest order statistic is increasing in the number of draws. Therefore, large auctions provide higher total value per sale because they are thicker markets both on the supply \textit{and} the demand side.

Notice next that total value is decomposed into price and bidder surplus. Therefore, given that expected bidder surplus does not vary systematically across auctions, expected price must be higher for larger auctions. One must then explain why sellers do not choose to sell their lots on big auctions. Of course, all things being equal, they would prefer to do just that. Lots are sold in separate auctions only because the need to sell is dispersed, both geographically and inter-temporally: Some auctions might be more conveniently located for the seller, and may take place closer to the time that the seller prefers to sell his lots. These trade-offs prevent all sales from being concentrated in a single auction. Instead, a distribution of auction sizes will exist, with larger auctions realizing higher average prices\textsuperscript{35}.

One can speculate about the factors underlying the systematic relationship between the rate of decline and auction size. Consider auctions in which winning a lot reduces the willingness to pay for other lots. An extreme case of this would be when a bidder desires a single lot. One of the driving forces of declining prices is the fact that markets tend to “thin out” towards the end of the auction as winning bidders become satiated\textsuperscript{36}. Anecdotal evidence suggests that most bidders

\textsuperscript{34} It is implicitly assumed that winning a lot reduces the value of additional lots to a bidder. If the value of additional lots is independent of having won previous lots, prices would be the same in both small and large auctions with the same number of bidders. This would still result in higher bidder surplus in large auctions leading to additional bidder entry.

\textsuperscript{35} Clearly, willingness to purchase is also dispersed geographically and inter-temporally. This reduces the positive effect that an increase on the number of lots has on expected prices.

\textsuperscript{36} The theoretical literature mentioned in the introduction shows this pattern of declining prices can be sustained in equilibrium and will not be arbitraged away.
remain in the auctions until their conclusion, but the willingness of past successful bidders to pay for additional lots could be much reduced due to these “stock” effects. The rate of decline is steeper for small auctions because these markets “thin” faster towards the end as bidders who have already purchased are less interested in competing for the remaining lots. Prices may start lower in large auctions because, for any given rate of price decline, the option value of waiting to compete for later lots is higher for a large auction: There are simply more lots to choose from and compete for. This dampens bidder aggressiveness for early lots.

A formal derivation of the effects of auction size, with endogenous bidder participation, on the rate of price decline, price levels, and volatility for different theoretical models might allow us to distinguish amongst the possible underlying causes of declining prices in sequential auctions.
APPENDIX.

This Appendix includes Tables and Figures that, though of potential interest to the reader, are not crucial to the main results of the paper. They are placed here in order to improve the readability and focus of the paper.

**TABLE A-1. Results of Auction by Auction Analysis.**

<table>
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<th>Auction</th>
<th>Constant coefficient</th>
<th>std. error</th>
<th>Relative Location coefficient</th>
<th>std. error</th>
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<th>R²</th>
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FIGURE A-1
Smoothed Simulated Prices - Proportional Bandwidth Smoothing

Auctions with N = 49 to 50

Auctions with N = 54 to 58

Auctions with N = 103 to 116

Auctions with N = 139 to 207
FIGURE A-2
Smoothed Prices - Fixed Bandwidth Smoothing

- **Auctions with N = 49 to 50**
  
  - \( \ln(SalePr) - \ln(AvgVal) \)
  
  - Relative Location

- **Auctions with N = 54 to 58**
  
  - \( \ln(SalePr) - \ln(AvgVal) \)
  
  - Relative Location

- **Auctions with N = 103 to 116**
  
  - \( \ln(SalePr) - \ln(AvgVal) \)
  
  - Relative Location

- **Auctions with N = 139 to 207**
  
  - \( \ln(SalePr) - \ln(AvgVal) \)
  
  - Relative Location

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