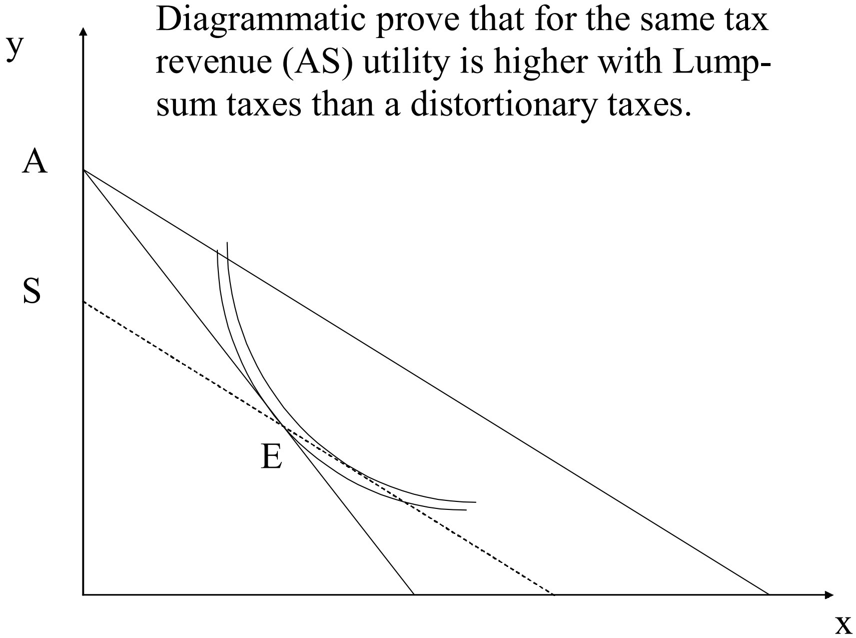
Optimal taxation

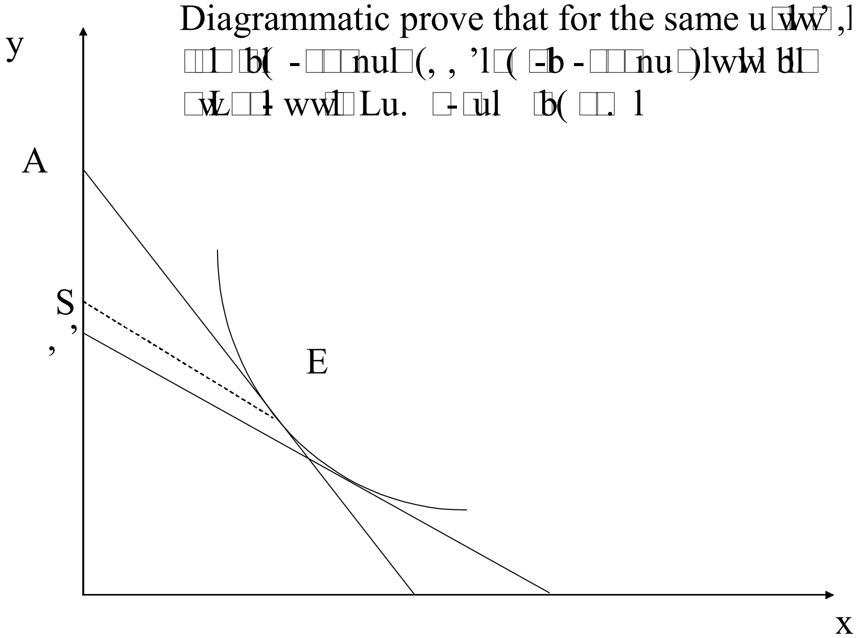
First-Best Taxes

- Lump-sum taxes are taxes that you have to pay them regardless of what you do.
- ⇒ No substitution effects.
- Must depend on *immutable* characteristics.
- Of course, one wants the characteristics to be relevant.
- They are not necessarily fair.
- Substitution and distortion:
- ⇒ It is wasteful to adjust one's behavior to take advantage of the tax system and/or to shift taxes. One's behavior should reflect only one's tastes and *real* opportunities facing him.

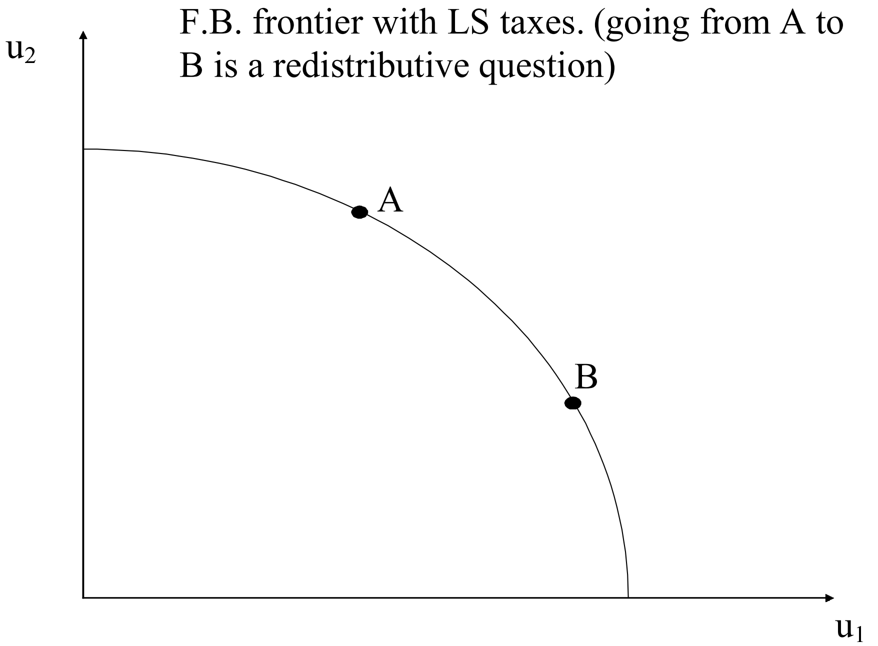
Diagrammatic prove that for the same tax revenue (AS) utility is higher with Lump-sum taxes than a distortionary taxes.

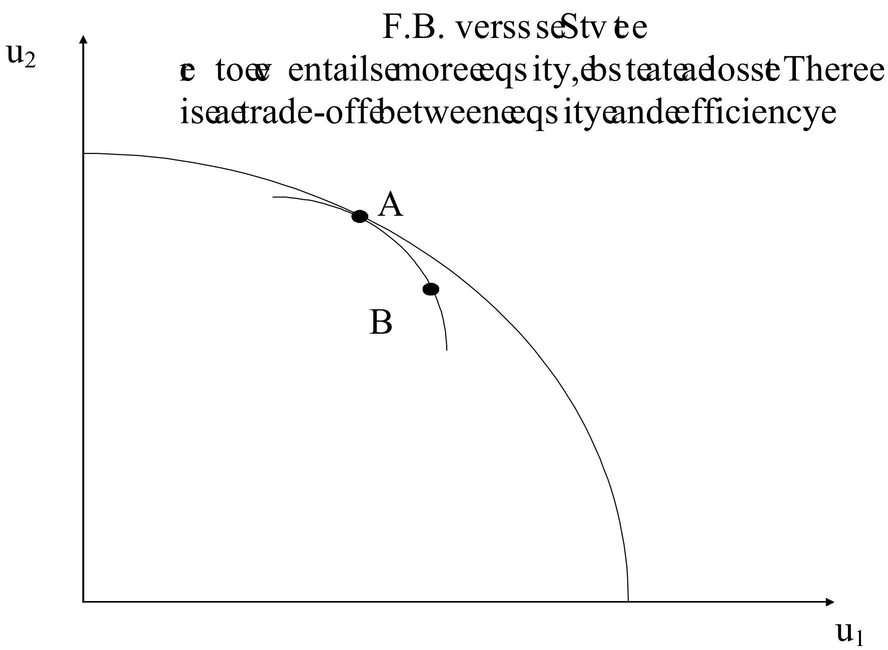


Diagrammatic prove that for the same u [illegible]



- The question of feasibility.
  - Head taxes are feasible but will no do the job.
    (i) Do not raise enough revenue (because of poor people, its level must be low).
    (ii) Equity.
    (iii) Politically; Mrs Thatcher ...
  - Differential lump-sum taxes and the question of information.
- Market outcome is F.B. is the sense of being Pareto-efficient.
- LS taxes does not change F.B. efficiency conditions ⇒ outcome continues to be Pareto-efficient.
- ⇒ The gain in equity does not come at the expense of a loss in efficiency.
- The equity-efficiency trade-off arises with second-best taxes.


F.B. frontier with LS taxes. (going from A to B is a redistributive question)
u2
A
B
u1


F.B. verss seStv te e
r toee entailsemoreeeqs ity,ebsteatcadosst There
isaetrade-offebetweeneeqs ityeandeefficiencye
u2
A
B
u1

Information and incentives

- Informational problems are real problems.
- Ideally, one wants to tax "earning abilities": $w_1, w_2, \ldots, w_n$.
- Lack of public information on w_i's forces the government to use incomes a s a proxy for earning abilities.
- Incomes are: $y_1, y_2, \ldots, y_n$.
- But $y = wL$ is not exogenous.
- Optimal tax theory pinpoints the second-best frontier.
- If we levy sub-optimal taxes, we'll be truly wasteful (inside the second-best frontier).

Introduction to incentives

- Assume preferences are given by:

$$u_i = u(c_i, l_i).$$

- Further assume that the government can levy differential lump-sum taxes.
- Each individual maximizes the above utility function s.t.

$$c_i = w_i(1 - l_i) - T_i.$$

- Where T_i is the lump-sum tax on person i.
- The F.O.C are

$$\frac{\partial u_i / \partial l_i}{\partial u_i / \partial c_i} = w_i.$$

- This determines c_i^* and l_i^* as functions of w_i and T_i.
- $\Rightarrow$

$$u_i^* = u(c_i^*, l_i^*).$$

- The Government's problem.
- Assume a utilitarian framework.
- Maximize $W = \sum u_i^*$ s.t. $\sum T_i \geq \overline{R}$.
- This is represented by the Lagrangian:

$$\pounds = \sum u_i^* + \lambda \left(\sum T_i - \overline{R} \right).$$

- The F.O.C. are

$$\frac{\partial u_j^*}{\partial T_j} = -\lambda, \text{ for all } j.$$

- But,

$$\frac{\partial u_j^*}{\partial T_j} = -\frac{\partial u_j^*}{\partial c_j}.$$

- $\Rightarrow$

$$\frac{\partial u_j^*}{\partial c_j} = \lambda, \text{ for all } j.$$

- In case, it is not obvious to you that $\partial u_j^*/\partial T_j = -\partial u_j^*/\partial c_j$:
- We have:

$$
\begin{aligned}
\frac{\partial u_j^*}{\partial T_j} &= \frac{\partial u_j^*}{\partial c_j}\frac{\partial c_j}{\partial T_j} + \frac{\partial u_j^*}{\partial l_j}\frac{\partial l_j}{\partial T_j} \\
&= \frac{\partial u_j^*}{\partial c_j}\left[\frac{\partial c_j}{\partial T_j} + \frac{\frac{\partial u_j^*}{\partial l_j}}{\frac{\partial u_j^*}{\partial c_j}}\frac{\partial l_j}{\partial T_j}\right] \\
&= \frac{\partial u_j^*}{\partial c_j}\left[\frac{\partial c_j}{\partial T_j} + w_j\frac{\partial l_j}{\partial T_j}\right].
\end{aligned}
$$

- Next, differentiating

$$c_i = w_i(1 - l_i) - T_i$$

* w.r.t. T_i yields

$$\frac{\partial c_i}{\partial T_i} + w_i\frac{\partial l_i}{\partial T_i} = -1.$$

- Which proves the point.

- Next, observe that

$$\frac{\partial u_j^*}{\partial l_j} = \frac{\partial u_j^*}{\partial c_j} w_j = \lambda w_j.$$

- $\Rightarrow$

$$\begin{cases} \frac{\partial u_j^*}{\partial c_j} = \lambda \\ \frac{\partial u_j^*}{\partial l_j} = \lambda w_j. \end{cases}$$

- This, in turn, implies that more able persons are made worse-off!
- Though the claim is general, its proof is made easier if we assume additive preferences.
- Assume:

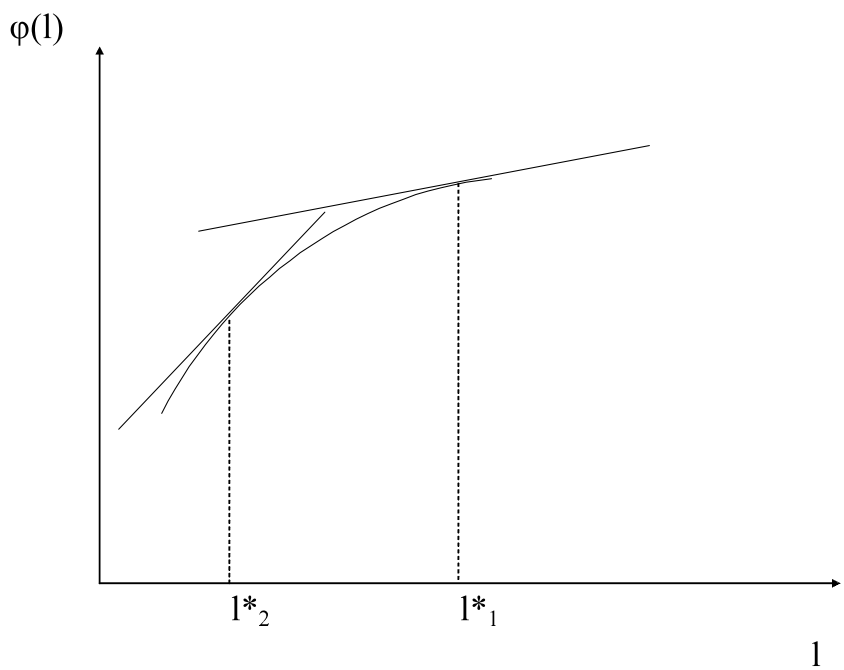
$$u_i = f(c_i) + \varphi(l_i).$$

- Consider two persons 1 and 2 with $w_2 > w_1$.
- From $\partial u_j^* / \partial c_j = \lambda$, $\Rightarrow$
- $f'(c_1^*) = f'(c_2^*)$.
- $\Rightarrow c_1^* = c_2^*$.

- Next, from $\partial u_j^*/\partial l_j = \lambda w_j$, $\Rightarrow$

$$\begin{aligned} \varphi(l_1) &= \lambda w_1, \\ \varphi(l_2) &= \lambda w_2. \end{aligned}$$

- Now that fact that $w_2 > w_1 \Rightarrow \varphi'(l_1^*) < \varphi'(l_2^*)$.
- Given diminishing marginal utility of leisure (i.e. $\varphi''(.) < 0$) $\Rightarrow$ $l_2^* < l_1^*$.



- Individual 2 will thus end up with the same consumption as individual 1, but with less leisure. $\Rightarrow$ He will be worse off.

Second-best tax solution

- In the previous problem, if the government does not know who is endowed with w_2 and who with w_1, it cannot rely on people to reveal their type!
- $\Rightarrow$ Post-tax allocations must satisfy "Incentive compatibility" or "self-selection" constraints.

The optimal linear income tax

- Types: w $F(w)$ over the support $[\underline{w}, \overline{w}]$.
- The government chooses t and G to maximize the SWF

$$\int_{\underline{w}}^{\overline{w}} \phi(u) f(w) dw$$

- s.t.

$$\int_{\underline{w}}^{\overline{w}} (ty - G) f(w) dw \geq \overline{R}.$$

- The solution strikes the "right" balance between efficiency costs and redistributive benefits.

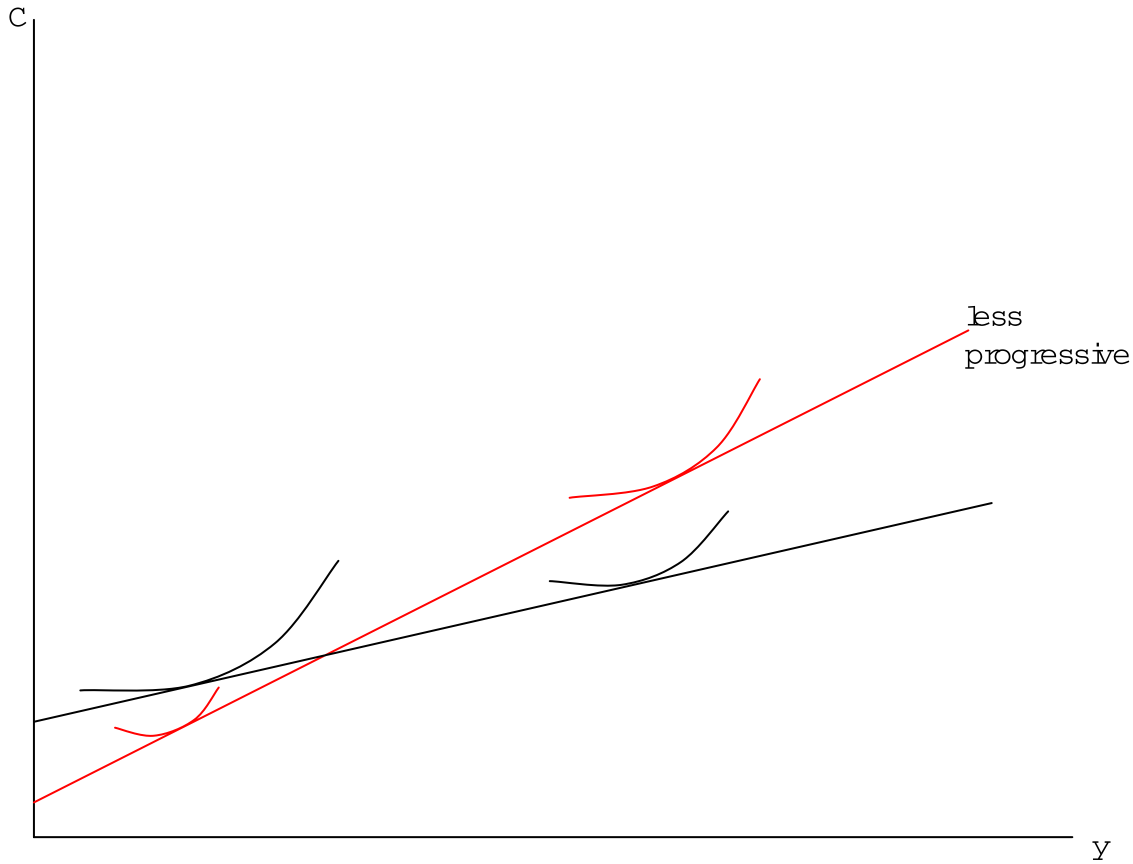
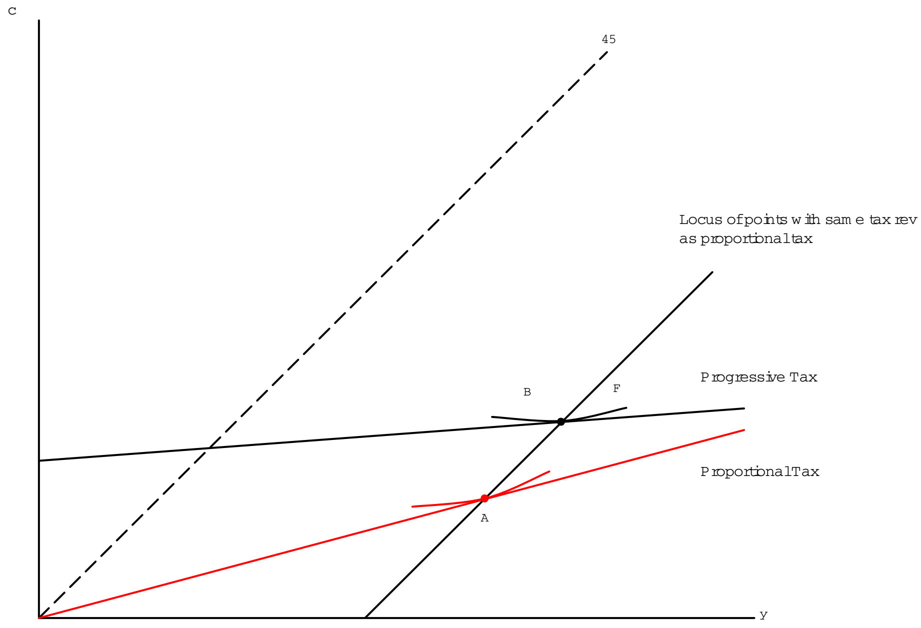


Figure: Equity benefit of progressivity.



- Can the excess burden of a progressive tax be less than the excess burden of a proportional tax?
- Can we have a diagram like above?
- According to such a diagram:

$$L_B > L_A; \quad C_B > C_A; \quad |\text{slope}| \text{ at } B < |\text{slope}| \text{ at } A.$$

- Point F with a slope equal to that of A must be to the *right* of B.
- $\Rightarrow L_F > L_B > L_A$.
- $\Rightarrow$ This is due only to the income effect (slopes at A and F are the same).
- $\Rightarrow$ Normality of leisure rules this out.

Algebraic solution and discussion

- The problem is represented by the Lagrangian:

$$\begin{aligned} \pounds &= \int_{\underline{w}}^{\overline{w}} \phi(u) f(w) dw + \mu \left[\int_{\underline{w}}^{\overline{w}} twLf(w)dw - G - \overline{R} \right] \\ &= \int_{\underline{w}}^{\overline{w}} \left[\phi(u) + \mu(twL - G - \overline{R}) \right] f(w) dw. \end{aligned}$$

- The FOC are

$$\begin{cases} \frac{\partial L}{\partial G} = 0 \\ \frac{\partial L}{\partial t} = 0. \end{cases}$$

- Characterization of the solution:

$$\begin{cases} E(\gamma) = \mu \\ \frac{t}{1-t} = \frac{1}{\mu} \frac{-Cov(y,\gamma)}{\int_{\underline{w}}^{\infty} y \, \varepsilon_{LL} dF} \\ t \int_{\underline{w}}^{\overline{w}} y dF = G + \overline{R}, \end{cases}$$

  * $\gamma \equiv \phi'(u(c,L))\alpha_w + \mu wt \frac{\partial L}{\partial m}$ is the *net* social marginal utility.
  * ε_{LL} is the compensated wage elasticity of labor supply.

- Observe that the solution depends on $\overline{R}$.
- Need to simplify to see the intuition.
- Assume quasi-linear preferences.

  * $\Rightarrow \partial L/\partial m = 0 \Rightarrow \gamma = \phi'(u(c, L))\alpha_w$.
  * $\Rightarrow \alpha_w = 1 \Rightarrow \gamma = \phi'(u(c, L))$

- Dependence on $\overline{R}$ continues.
- Assume, as before,

$$u = c - \frac{\varepsilon}{1+\varepsilon}(L_0)^{-\frac{1}{\varepsilon}} L^{1+\frac{1}{\varepsilon}}$$

- Recall that in this case,

$$\begin{aligned} \varepsilon_{LL} &= \varepsilon = \text{constant} \\ y &= wL = L_0(1-t)^{\varepsilon} w^{1+\varepsilon} \end{aligned}$$

- Substituting in the FOC $\Rightarrow$

$$\begin{cases} E(\phi'(v)) & = \mu \\ \frac{t}{1-t} & = \frac{1}{\mu}\frac{-\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon}\left(\phi'-E(\phi')\right)dF}{\varepsilon\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon}dF} \\ t\left[L_0(1-t)^{\varepsilon}\right]\int_{\underline{w}}^{\overline{w}} w^{1+\varepsilon}dF & = G+\overline{R}. \end{cases}$$

- Observe also that the middle equation can be written as

$$\begin{aligned} \frac{t}{1-t} &= \frac{1}{\mu\,\varepsilon}\frac{-E(w^{1+\varepsilon}\,\phi')+E(\phi')E(w^{1+\varepsilon})}{E(w^{1+\varepsilon})} \\ &= \frac{1}{\varepsilon}\left[1-\frac{E(w^{1+\varepsilon}\,\phi')}{E(\phi')E(w^{1+\varepsilon})}\right]. \end{aligned}$$

- Observe that as long as $\phi'(.)$ depends on $\overline{R}$, so will t.
- Assume $\phi' = \frac{w^{-\gamma}}{E(w^{-\gamma})}$. $\Rightarrow$

  * t is independent of $\overline{R}$.
  * $\overline{R}$ affects the size of G only.

$$G = t_0(1-t)^{\varepsilon}E(w^{1+\varepsilon}) - \overline{R}.$$

  * Sum of the weights are normalized to one:

$$\int \phi' dF = E(\phi') = \frac{E(w^{-\gamma})}{E(w^{-\gamma})} = 1.$$

- Further implications of the weighting scheme:
  - Suppose $w_2 > w_1$: The weight put on the poorer guy (1) relative to the rich (2) is:

$$\frac{w_1^{-\gamma}}{w_2^{-\gamma}} = (\frac{w_2}{w_1})^{\gamma}.$$

  - With $\frac{w_2}{w_1} > 1$, this relative weight increases as γ increases. It is lowest at $\gamma = 0$, where the poor and the rich get the same weight as with utilitarian preferences. It will be highest as $\gamma \to \infty$ as with Rawlsian preferences.
- Assume further that w has a lognormal distribution over the support $(0, \infty)$. $\Rightarrow \ln w$ is normally distributed with mean m and variance σ^2.
- $\Rightarrow$ One can find a closed-form solution for t according to:

$$\frac{t}{1-t} = \frac{1}{\varepsilon}\left[1 - (1+\eta^2)^{-\gamma(1+\varepsilon)}\right].$$

* where $\eta \equiv \frac{\sigma}{m}$ is the “coefficient of variation”.

- Interpreting the optimal tax rule:

$$\frac{t}{1-t} = \frac{1}{\varepsilon}\left[1 - (1+\eta^2)^{-\gamma(1+\varepsilon)}\right].$$

  * ε term represents efficiency.
  * $\left[1 - (1+\eta^2)^{-\gamma(1+\varepsilon)}\right]$ represents equity.

- The higher is ε, the lower is the tax rate (on efficiency grounds).
- The higher is γ, the higher is the tax rate (on equity grounds).
- The higher is η, the higher is the tax rate (on equity grounds).
  - $\Rightarrow$ High degree of inequality calls for a high tax rates.

A numerical study by Nic Stern

- Assume CES preference:

$$u(c,l) = \left[\alpha l^{-\mu} + (1-\alpha)c^{-\mu}\right]^{-1/\mu},$$

* where the elasticity of substitution between l and c, σ, is

$$\sigma = \frac{1}{1+\mu}$$

- The SWF criterion is Atkinson-type,

$$\frac{1}{1-\varepsilon}\int_0^\infty (u(c,l))^{1-\varepsilon} f(w)dw,$$

* with ε denoting the inequality aversion index.

- The income tax is linear so that a person with wage w has a budget constraint

$$c = (1-t)w(1-l) + G$$

- The government's budget constraint is

$$t\int_0^\infty w(1-l)f(w)dw = G + \overline{R}.$$

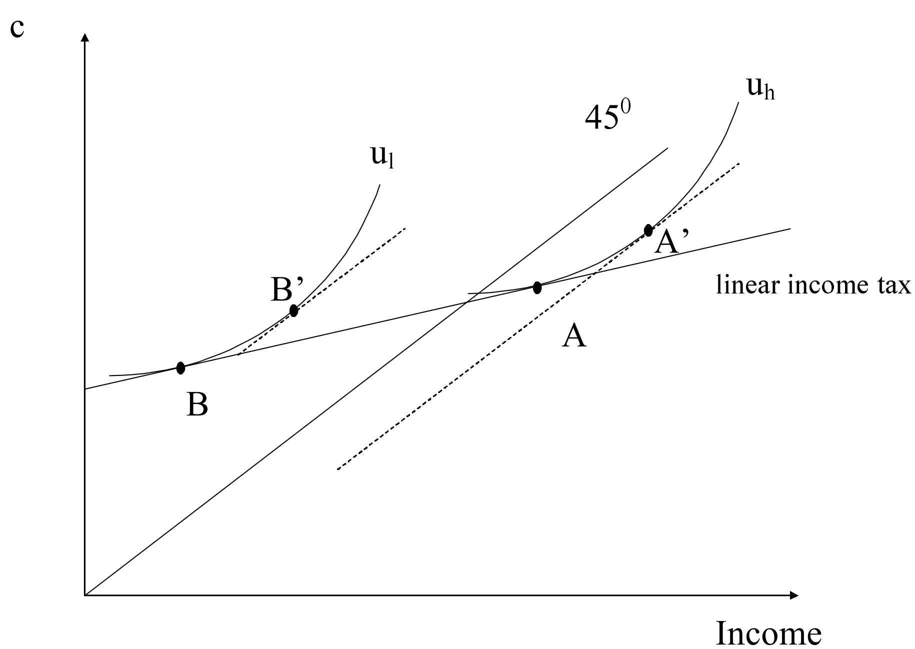
Calculations of Optimal Linear Marginal Tax Rates

(By Nicholas Stern, 1976)

	$\varepsilon = 0$		$\varepsilon = 2$		$\varepsilon = 3$		$\varepsilon = \infty$	
σ	t	G	t	G	t	G	t	G
R=0 (*purely redistributive tax*)								
0.2	36.2	0.096	62.7	0.161	67.0	0.171	92.6	0.212
0.4	22.3	0.057	47.7	0.116	52.7	0.126	83.9	0.167
0.6	17.0	0.042	38.9	0.090	43.8	0.099	75.6	0.135
0.8	14.1	0.034	33.1	0.073	37.6	0.081	68.2	0.111
1.0	12.7	0.029	29.1	0.062	33.4	0.068	62.1	0.094
R=0.05 (*equivalent to about 20 percent of GDP*)								
0.2	40.6	0.063	68.1	0.135	72.0	0.144	93.8	0.182
0.4	25.4	0.019	54.0	0.089	58.8	0.099	86.7	.0139
0.6	18.9	0.000	45.0	0.061	50.1	0.071	79.8	0.107
0.8	19.7	0.000	38.9	0.042	43.8	0.051	73.6	0.082
1.0	20.6	0.000	34.7	0.029	39.5	0.037	68.5	0.064
R=0.10 (*equivalent to about 45 percent of GDP*)								
0.2	45.6	0.034	73.3	0.110	76.7	0.119	95.0+	-
0.4	35.1	0.000	60.5	0.065	65.1	0.076	89.3	0.112
0.6	36.6	0.000	52.0	0.036	57.1	0.047	83.9	0.081
0.8	38.6	0.000	46.0	0.016	51.3	0.026	79.2	0.057
1.0	40.9	0.000	41.7	0.002	47.0	0.011	75.6	0.039

Usefulness of General Income taxes

- Consider an optimal linear income tax schedule, which is optimal for a given SWF *assuming* income taxes have to be linear.
- The question is if we can improve SWF by introducing bracketing.



- Yes! Given two schedules: A linear income tax for all incomes reported bellow I_A, and a lump-sum tax for incomes above I_A (equal to the distance between the 45 degree line and AA'). The poor stays put; the rich goes to A': Individuals have same utility; but tax revenue is higher!
- Note: This does not mean that (B, A') is optimal. We may want to use the extra revenue for further redistribution.
- Question: Why not give them a choice between two Lump-sum taxes?

- With full information, we could. And this would improve things further.
- When incomes are publicly unobservable, we face the IC constraints. $\Rightarrow$ The rich would now want to go to B'.