- We have:

$$u(c,G) = u(m - \tau p G, G).$$

- The change in utility due to a change in G alone (which is what I derived in class) then is equal to:

$$du = \left[u_c\left(-\tau p\right) + u_G\right]\ dG.$$

- Similarly, the change in utility due to a change in τ alone (which I did *not* derived in class) is equal to:

$$du = u_c\left(-pG\right)\ d\tau.$$

  Observe that while a change in G affects utility on two fronts (because a change in G affects c also), a change in τ affects utility through the change in c only.

- Now, to be on the same indifference curve, the two utility changes should balance one another out so that the final change in utility will be zero. Consequently,

$$\left[u_c\left(-\tau p\right) + u_G\right]\ dG + u_c\left(-pG\right)\ d\tau = 0.$$

- It then follows from the above that

$$\frac{d\tau}{dG} = \frac{u_c\left(-\tau p\right) + u_G}{u_c\left(pG\right)}.$$

  This is the equation for the slope of the indifference curve in (G,τ) space. That is, the above is the equation for the marginal rate of substitution between τ and G.

- Simplifying the right-hand side,

$$\frac{d\tau}{dG} = \frac{1}{pG}\left[-\tau p + \frac{u_G}{u_c}\right].$$

  Observe that the sign of $\frac{d\tau}{dG}$ is the same as the sign of $\frac{u_G}{u_c} - \tau p$.

- My claim (which I tried to show "intuitively" in class on the basis of a change in G alone) was that, for small values of G, $\frac{d\tau}{dG}$ is positive (indifference curve slopes upwards), and for large values of G, $\frac{d\tau}{dG}$ is negative (indifference curve slopes downwards). In other words, for small values of G, $\frac{u_G}{u_c} > \tau p$, and for large values of G, $\frac{u_G}{u_c} < \tau p$.

- This is obvious. For very low values of G (and high values of c), the marginal valuation of G in terms of c (the marginal rate of substitution between G and c) which is given by $\frac{u_G}{u_c}$ is extremely high and greater than τp. As G increases, and c falls, $\frac{u_G}{u_c}$ becomes smaller and smaller until it falls to τp (so that $-\tau p + \frac{u_G}{u_c} = 0$) and the curve attains its maximal value. As G increases further from this point $\frac{u_G}{u_c}$ becomes even smaller, and falls below the value of τp.