Optimal and Private Provision of Public Goods
(identical persons)

- **Assumptions**:

  - There are N identical individuals who consume a private good x and a public good G with preferences
  $$u = x + \ln G.$$
  - Marginal cost of producing one unit of public good is p.
  - An individual's budget constraint is
  $$x + pg = m,$$
  where the price of one unit of private good is \$1 and g denotes the number of units of the public good purchased by one person.
  - The economy's resource cost is thus
  $$Nm \geq Nx + pG.$$
  This means that the total incomes of all individuals must be sufficient to pay for their total consumption. Note that there is no N in front of G because G is consumed by everybody.

- **Optimal provision: Direct method**

  - Maximize the sum of utilities, $N(x + \ln G)$, subject to the economy's resource constraint.
  - We can incorporate the resource constraint in the maximand, $N(x+\ln G)$, by substituting for Nx from the economy's resource constraint.
  - This yields $Nm - pG + N\ln G$, which we want to maximize with respect to G.
  - The first-order condition for this maximization is
  $$-p + N\frac{1}{G} = 0.$$
  - This equation gives us the optimal amount of G which is
  $$G = N/p.$$

- **Optimal provision: using Samuelson's rule**
  - Set the sum of all marginal rates of substitution equal to marginal cost of production for the public good:
$$\sum \frac{1}{G} = p.$$
  - Since we have N persons,
$$N\frac{1}{G} = p.$$
  - Or $G = N/p$ which is what we had before.

- **Private provision**
  - Each individual maximizes utility, $x + \ln G$, subject to his/her budget constraint $x + pg = m$.
  - Substituting for x from the budget constraint into the utility function, we want to maximize $m - pg + \ln G$.
  - Clearly, the individual cannot determine G which is the total purchased by everyone. One can only determine his/her own purchases, g.
  - Let $\bar{G}$ denote the number of units of the public good purchased by all "other" individuals. By "other" we mean everyone else except the one who is deciding how many units of the public good he/she should buy.
  - The problem of the individual is then to choose his/her purchase, g, by maximizing
$$m - pg + \ln(g + \bar{G}).$$
  - The first-order condition for this maximization is
$$-p + \frac{1}{g + \bar{G}} = 0.$$
  - Substituting for $\bar{G} = (N-1)g$, then gives us,
$$g = \frac{1}{pN}, \qquad \text{and} \qquad G = Ng = \frac{1}{p}.$$
  - Comparing this solution with the optimum G which was N/p tells us that private provision falls short of the optimum.

- **What is the problem of first substituting Ng for G?**

- Another way of saying this is to ask why can't we substitute $(N-1)g$ for $\bar{G}$.

- Observe that if we do this, the maximization problem becomes one of choosing g to maximize
$$m - pg + \ln Ng.$$

- Now the first-order condition for maximization becomes
$$-p + \frac{N}{Ng} = 0.$$

- This yields $g = 1/p$ or $G = Ng = N/p$ which is the optimal solution. Recall that in the method I employed above for private provision, the answer I got was $G = 1/p$.

- What gives?

- First, observe that the solution I am deriving here *is* correct based on the assumption. In other words, this is not a mathematical problem.

- We thus have to think what it means conceptually if (i) We choose g to maximize $m - pg + \ln(g + \bar{G})$, versus (ii) first substitute $\bar{G} = (N-1)g$ in $m - pg + \ln(g + \bar{G})$ and then maximize $m - pg + \ln Ng$.

- If we follow (i), we are assuming that when one person varies his/her consumption of g, he/she *assumes* his action is not matched by others. That is, we keep $\bar{G}$ to be fixed in taking the derivative of the expression with respect to g.

- If we follow (ii), we are assuming that when one person varies his/her consumption of g, he/she *assumes* his action is matched exactly by others. That is, when the person changes g, say increases it by one unit, then $\bar{G}$ changes by $(N-1)$ units.

- Neither assumption is completely correct or completely wrong. The point is which one is a more "realistic" description of how individuals behave.

- The first assumption is the so-called Nash assumption. (Yes, Nash as the man in the movie "A Beautiful Mind" who is a Nobel laureate in economics). The second assumption is referred to as "Lindahl assumption". No wonder then that under it we get the optimal solution from individuals' behavior.

- That you get the optimal outcome under (ii) should not surprise you. While the free rider problem arises because of non-excludability, it is also based on the belief that the others do not do as you do and that is why you can "free ride" on them. If they were to do exactly as you do, then there will be nothing for you to free ride on. If one thinks this way, then one is bound to do the "right" thing.

- To see which assumption is more realistic, ask yourself questions like: Do you think when a person decides to cheat on his/her taxes, he/she thinks that if he/she cheats everybody else cheats as well, or whether others cheat or not does not depend on what he does? Do you think if you speed everybody else will also speed? Do you think if you decide to contribute to WILL, everyone else does too? Do you think if you go to vote, everybody else will also do the same?
- You may also want to think how your decision change with the assumption you adopt.