Demand Revelation Mechanism

• Homeowners in a home association are deciding on 3 options (mowing the neighborhood lawns once a week, twice a week, or once a month; installing 20 lights in the neighborhood, 15 lights, or 10 lights; etc.)

• The neighborhood association manager asks homeowners how much they are prepared to pay for each of the three options. Each homeowner declares a value which may be He adds up the declared values and the option that has the highest declared value is chosen.

• What one is actually prepared to pay for something measures one's valuation for something. However, one may think it is better to lie and misrepresent one's valuation by reporting a number which is not one's true valuation. He may want to do this to bring about a better outcome for himself or pay less taxes.

• Information structure:

- Each homeowner knows his own valuation of the 3 options.
- He does not know the other neighbors' true valuations.
- He does not know the other neighbors' declared values.
- He does not know who tells the truth and who does not.
- He does not know what plan will be chosen without his participation.
- He does not know what plan will be chosen with his participation.
- The only other piece of information he has is the structure of the tax.

• Tax structure:

- Each homeowner will pay the cost that he imposes on other neighbors, whatever that will be.
- He knows only the tax *rule*; he does not know the amounts involved.

• Homeowner's decision process:

- Denote the homeowner's most-preferred option by N, his true valuation of it by n, and the *sum* of other neighbors' declared valuations for it by $\overline{N}$.

- Denote the homeowner's second favorite option by M, his true valuation of it by m, and the *sum* of other neighbors' declared valuation of others for it by $\overline{M}$.

- Denote the homeowner's least favorite option by C, his true valuation of it by c, and the *sum* of other neighbors' declared valuations for it by $\overline{C}$.

• Given the naming of the options, we have

$$n > m > c. \tag{1}$$

• The key to understanding why the solution to this problem entails truth telling is that *for any given outcome*, one's tax payment does not depend on one's reported valuation, but on the sum of the other neighbors' reported valuations. Thus suppose the total *reported* valuations by all other neighbors are \$900 for project N, \$1,000 for project M, and \$1,100 for project C. Without our homeowner's participation, then, project C will be chosen.

If our homeowner reports a set of three numbers such that C remains the choice of the home association, he will pay a zero tax. This tax does not change with his declaration as long as the choice remains to be C. On the other hand, if the homeowner's declaration changes the choice of the options to N, our homeowner will be asked to pay \$200 in taxes. Again, as long as the chosen option remains N, the tax payment does not change. It depends on the sum of what the others have reported (\$1,100 and \$900).

• If the outcome changes as a result of one's declarations, then the person's declaration has had a crucial effect on the choice and he is called "pivotal". If the person's declaration does not change things, his declarations does not matter, and he is called "not pivotal".

• Since the homeowner does not know the declared valuations, he considers all possibilities that may happen without his participation and all that can happen with his participation. First, *if he does not participate*, we can have either of the following possibilities

$$\text{I } M \text{ is chosen} \begin{cases} (a) & \overline{M} > \overline{C} > \overline{N} \\ (b) & \overline{M} > \overline{N} > \overline{C} \end{cases}$$

$$\text{II } N \text{ is chosen} \begin{cases} (a) & \overline{N} > \overline{M} > \overline{C} \\ (b) & \overline{N} > \overline{C} > \overline{M} \end{cases}$$

III C is chosen $\begin{cases} (a) & \overline{C} > \overline{N} > \overline{M} \\ (b) & \overline{C} > \overline{M} > \overline{N} \end{cases}$

• We have to consider cases $I(a) - I(b), II(a) - II(b)$, and $III(a) - III(b)$ separately.

• Given any starting position, the homeowner compares the *net gain* he achieves from truth telling and lying, choosing the one with the higher net gain.

$I(a): \ \overline{M} > \overline{C} > \overline{N}$

• Truth-telling $\begin{cases} (i) & \text{He is pivotal, changing } M \text{ to } N. \\ (ii) & \text{He is not pivotal, } M \text{ stays.} \end{cases}$

• Observe that we cannot have a situation in which the homeowner is pivotal, changing M to C through truth telling. Why? Because we have in this case $\overline{M} > \overline{C}$. He himself prefers M to C so that $m > c$. Consequently, if he tells the truth, $m + \overline{M} > c + \overline{C}$ so that M will stay.

• Observe that under (i), truth telling changes M to N. We can thus deduce that *if (i) holds,*

$$n + \overline{N} > m + \overline{M}. \tag{2}$$

• Similarly, under (ii), the true reporting does not changes M. We can thus deduce that *if (ii) holds,*

$$\begin{aligned} m + \overline{M} &> n + \overline{N} && (3) \\ m + \overline{M} &> c + \overline{C}. && (4) \end{aligned}$$

• Note carefully that the sign of equation (3) is opposite of equation (2). This is not a contradiction. The point is that these two equations do not hold simultaneously. Equation (2) holds under (i), and equation (3) holds under (ii).

$$\bullet \text{ Lying} \begin{cases} (e) & \text{He is pivotal, changing } M \text{ to } N. \\ (f) & \text{He is pivotal, changing } M \text{ to } C. \\ (g) & \text{He is not pivotal, } M \text{ stays.} \end{cases}$$

- **Calculations:**

1. *(i) versus (e), (f), and (g).*

- Net gain under (i): In changing the outcome from M to N, the home-owner gains $n - m$ but has to pay a tax equal to $\overline{M} - \overline{N}$. His net gain is thus equal to

$$(n - m) - (\overline{M} - \overline{N}).$$

Now, given that equation (2) holds under (i), we will have,

$$(n - m) - (\overline{M} - \overline{N}) > 0.$$

- Net gain under (e): This is the same as (i) so that net gain is

$$(n - m) - (\overline{M} - \overline{N}) > 0.$$

- Net gain under (f): In changing the outcome from M to C, he gains $c - m$ but has to pay a tax equal to $\overline{M} - \overline{C}$. His net gain is thus equal to

$$(c - m) - (\overline{M} - \overline{C}) = (c - m) + (\overline{C} - \overline{M}).$$

Observe, however, that $(c - m) < 0$ and $(\overline{C} - \overline{M}) < 0$ so that the net gain is negative.

- Net gain under (g): This does not change the outcome; he neither gains in terms of valuation nor pays any taxes. The net gain is zero.

- **Conclusion:** Lying is never better than truth-telling in this case.

2. *(ii) versus (e), (f), and (g).*

- Net gain under (ii): The outcome does not change so that the net gain is zero.

- Net gain under (e): In changing the outcome from M to N, he gains $n - m$ but has to pay a tax equal to $\overline{M} - \overline{N}$. His net gain is thus equal to

$$(n - m) - (\overline{M} - \overline{N}) = (n + \overline{N}) - (m + \overline{M}).$$

However, now we are operating under (ii). In this case, from equation (3) we have $m + \overline{M} > n + \overline{N}$. It then follows that the net gain is negative.

- Net gain under (f): In changing the outcome from M to C, he gains $c - m$ but has to pay a tax equal to $\overline{M} - \overline{C}$. His net gain is thus equal to

  $$(c - m) - (\overline{M} - \overline{C}) = (c - m) + (\overline{C} - \overline{M}).$$

  Recall, however, that $(c - m) < 0$ and $(\overline{C} - \overline{M}) < 0$ so that the net gain is negative.

- Net gain under (g): This does not change the outcome; he neither gains in terms of valuation nor pays any taxes. The net gain is zero.

• **Conclusion:** Lying is never better than truth-telling in this case either.

• We now have to repeat the same steps for cases I(b), II(a)–II(b), and III(a)–III(b) - (VI). Try it, it is fun!

• **An example:** There are four homeowners deciding on the choice of a project that will be useful to all of them. There are three options A, B, and C. Suppose the decision is to be made on the basis of the demand revelation mechanism I have described. Their reported valuations appear in the Table below. Determine the tax that each homeowner will be asked to pay.

	Plan			
Homeowner	**A**	**B**	**C**	**Tax**
1	\$60	\$50	\$40	
2	30	70	50	
3	20	80	25	
4	40	20	90	
Total Willingness to pay	\$150	\$220	\$205	