Homework #6
(Econ 512M)

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I. Show the superiority of a lump-sum tax to a proportional income tax and a proportional income tax and a proportional income tax to a progressive tax.

II. In the SteveLee Land people like to consume goods, c, and leisure, l, according to the utility function

$$u(c, 1) = l + \ln c.$$

The people of SteveLee are strong and do not need to sleep; they can spend all their time working so that $l + L = 1$. They are of differing ability however earning different wages distributed with the density function $f(w)$ over the support $[\underline{w}, \overline{w}]$. The population size is normalized at one and there are no external tax revenue requirements. The rulers of SteveLee like "flat" taxes and subject their citizens to a linear income tax schedule of the type

$$T = ty - a$$

where T is an individuals's net tax payments, $y = wL$ is the income (=output) of SteveLee person with wage w, and a is her "basic income".

1. Derive and draw a SteveLee person's budget constraint.

2. Do the people of SteveLee Land face a progressive or regressive tax. Why?

3. Draw a SteveLee person's indifference map. What do you notice about the shapes of these curves?

4. Derive a SteveLee person's demand for c and supply of L.

5. Derive a SteveLee person's compensated demand for c and supply of L.

6. What is a SteveLee person's elasticity of demand for c and compensated demand for c?

7. What are a SteveLee person's uncompensated and compensated labor supply elasticities?

8. Determine the size of the basic income as a function of the tax rate.

9. What tax rate maximizes the basic income level guaranteed each person?

10. Find how is SteveLee's GNP affected by the size of its tax rate? Would you interpret this as an efficiency loss or an equity item?

11. Write a SteveLee person's indirect utility function, v.

12. Write v as a function of the tax rate only; then identify equity and efficiency terms.

13. Suppose initially there is no income tax. Show that an imposition of a small tax would benefit the poor people in SteveLee at the cost of the rich ones.

14. Determine an expression for the average utility in SteveLee Land as a function of the tax rate only.

15. How does the average utility change as the tax rate changes.

16. Cast your answer to above in terms of efficiency and equity ideas.

17. There is a consensus in SteveLee Land, that when a person with wage w has a utility of $v(w)$ the society gains, $w^{-\gamma}v(w)$ where $\gamma \geq 0$. Interpret γ.

18. How does social welfare change as the tax rate changes.

19. Do you think the optimal tax should maximize the basic benefit G? Why or why not.

20. How does γ affect the size of the optimal tax rate?

III. Show how differentiating tax rates may improve society's welfare.

IV. If lump-sum taxes are best, why does the government institute a progressive tax?

V. Show why levying differential lump-sum taxes may be "incentive incompatible".