Homework #7
(Econ 512M)

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I. Mr. Cheat is an expected utility maximizer who values income y according to the utility function

$$u = \ln y.$$

His income is not publicly observable but can be detected through an audit. He faces a tax rate of 20% and maybe audited with a probability of 5%. If he cheats and is caught, he will be fined τ times the taxes he evades. Denote his reported income by αy where $0 < \alpha < 1$. Find Mr Cheat's

(i) Net of tax income if he is not audited.

(ii) Net income if he is audited.

(iii) Expected net income.

(iv) Expected tax payment.

(v) Expected utility.

(vi) Discuss what Mr. Cheat will do if $\tau = 1$. Why?

(vii) Discuss what Mr. Cheat will do if τ is very high. Why?

II. There are four income groups of equal size in Tooliville with incomes of 94, 98, 102 and 106 thousand of rupees.

(i) Calculate the following inequality measures for this society: Range, variance, coefficient of variation, logarithmic variance, relative mean deviation and GINI coefficient.

(ii) Draw Pen's parade of dwarfs, cumulative distribution function and the Lorenz curve for Tooliville.

(iii) Suppose we redistribute 2 thousand rupees from the second quartile and give it to the third. What are the implications of this for each of the above inequality measures?

(iv) Can you think of an inequality index for which the above change does not show a deterioration in income distribution?

III. Suppose annual consumption, c, is related to annual income, y, according to

$$c = \alpha + \beta y,$$

where $\alpha > 0$ and $0 < \beta < 1$. How do the inequality measures based on c compare with those based on y? Do the comparisons for range, variance, coefficient of variation, logarithmic variance, relative mean deviation and GINI coefficient.

IV. Suppose we are measuring wealth inequalities with some people being in debt (having a negative net worth). How do you draw (i) Pen's parade of dwarfs and (ii) the Lorenz curve?

V. Consider an economy with the following levels of income ranked from the lowest to the highest:

$$y_1, y_2, \ldots, y_{i-1}, y_i, y_{i+1}, \ldots, y_{j-1}, y_j, y_{j+1}, \ldots, y_H.$$

Transfer \$1 from the j th person to i th person. Prove that the change in the value of the GINI coefficient depends on $j - i$.