# Economics of Taxation 

(Econ 512M)
Midterm
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Name: $\qquad$
Spring 2011

Instructions: Answer all questions. Credit will be given only to well-argued answers. Write legibly.

1. (40 points) Bai-Belle is the beautiful home to a thriving population of hard workers with different productivity or wage rates $w$ distributed over the support $[\underline{w}, \bar{w}]$ with a density $f(w)$. All Bai-Bellians have the same preferences over consumption $c$ and labor supply $L$ represented by the utility function

$$
U(c, L)=c-\frac{1}{2} L^{2} .
$$

They face a linear income tax schedule given by

$$
T=-a+t y
$$

where $a>0$ is the "basic income" everybody receives from the government regardless of his/her income, $t$ is the income tax rate, and $y=w L$ denotes the (gross of tax) income. The tax is purely redistributive in that the net positive taxes some people pay just offsets the net subsidies others receive. Put differently, sum of taxes paid on incomes (sum of $t y$ 's) i equal to sum of all the basic incomes everyone receives (some of $a$ 's).
(a) Derive an equation for the average tax rate faced by a Bai-Bellian and draw its graph.
(b) Define a progressive tax system and show that Bai-Bellians face such a progressive tax system.
(c) What is the threshold level of income, in terms of basic income and the tax rate, above which a Bai-Bellian consumes less than his/her income (positive net tax payment) and below which he/she consumes more than his/her income (negative net tax payment).
(d) Amelia and Chenia are two Bai-Bellians who earn $x$ and $z$ yuan respectively. Assume the tax system is based on households. That is, $T=-a+t y$ is the tax schedule a household faces both when the household consists of a single person as well as when it consists of a married couple.
i. What marriage tax will they pay if they form a union?
ii. Can they change their tax liability by transferring one party's income to the other?
(e) Derive the equation for a Bai-Bellian's budget constraint, relating $c$ to $y$. Draw this budget constraint.
(f) Derive a Bai-Bellian's labor supply function and its elasticity.
(g) Derive a Bai-Bellian's compensated labor supply function.
(h) What is the effect of non-labor income $a$ on a Bai-Bellian's labor supply?
(i) Show the excess burden imposed on a Bai-Bellian by the income tax in the diagram depicting his/her labor supply
(j) Derive an expression for the excess burden imposed on a Bai-Bellian (in terms of $t$ and his/her wage $w$ ).
(k) Derive an expression for Bai-Belle's total national income (equal to its $G D P$ ).
(l) How does Bai-Belle's $G D P$ vary with the tax rate?
(m) Derive an expression for the sum of utilities in Bai-Belle.
(n) How does the sum of utilities change with the tax rate?
(o) Derive an expression for the sum of excess burdens.
(p) How does the sum of excess burdens change with the tax rate.
(q) Which expression do you think is a better measure of welfare loss in BaiBelle: the loss in GDP as the tax rate changes or the loss in the sum of utilities? Why?
2. (12 points)
(a) Define the "equivalent variation" $(E V)$ measure for a change in welfare.
(b) Illustrate $E V$ via drawing a diagram.
(c) What is the advantage of "equivalent variation" over the "compensating variation" measure for a change in welfare.
3. (12 points) Consider a two-sector general equilibrium model using capital and labor to produce $X$ and $Y$.
(a) Write down the equations that determine the equilibrium allocations in this economy.
(b) Assume there is a tax on workers producing $y$.
i. Under what conditions, if any, does the factor intensity effect work in favor of consumers?
ii. Under what conditions, if any, does the factor substitution effect work in favor of consumers?
4. (12 points) Consider the following demand and supply equations:

$$
\begin{aligned}
& P^{c}=50-5 X \\
& P^{s}=4 X
\end{aligned}
$$

(a) Draw these curves and find the equilibrium quantity, consumer and producer price.
(b) Consider a per unit subsidy of $\$ 4$ on consumers. Find the new equilibrium quantity, consumer price and producer price for both cases. Who benefits more from the subsidy?
(c) Calculate the demand and supply elasticities at the first equilibrium point(s) for both cases. Which curve is more elastic? Does this reinforce or contradict your answer in (b) about the incidence of the subsidy? Explain.
(d) Find the government's expenditures and the excess burden using the elasticity formula.
5. (12 points) You want to divide $\$ 240$ between Wendy and Jade. Find the optimal distribution of the following cases (when $x_{i}, i=R, T$, denotes $i$ 's share of the money):
(a) Utility of Wendy is $U_{W}=50 x_{W}-x_{W}^{2}$, utility of Jade is $U_{J}=200 x_{J}-x_{J}^{2} / 2$, and the Social Welfare Function $(S W F)$ is utilitarian.
(b) Utility of Wendy is $U_{W}=10 x_{W}-3 x_{W}^{2}$, utility of Jade is $U_{J}=10 x_{J}-x_{J}^{2}$, and $S W F$ is utilitarian.
(c) Utility of Wendy is $U_{W}=10 x_{W}-3 x_{W}^{2}$, utility of Jade is $U_{J}=10 x_{J}-x_{J}^{2} / 2$, and the society cares only about Jade. (Sorry Wendy!)
(d) Utility of Wendy is $U_{W}=10 x_{W}-3 x_{W}^{2}$, utility of Jade is $U_{J}=10 x_{J}-3 x_{J}^{2}$, and $S W F$ is Rawlsian.
6. (12 points) State True or False and explain:
(a) The higher the elasticity of demand, the lower would be the incidence of a tax on consumers.
(b) The higher the elasticity of compensated demand, the lower would be the excess burden.
(c) The "principle of fairness" requires that one divides the tax rate on employment equally on employers and employees.
(d) Tax expenditures are a harmless feature of the tax system.
7. (6 points) Extra credit questions. The following three questions are taken from Jim Harrick Jr.'s Coaching Principles and Strategies of Basketball course at the University of Georgia in the fall of 2001. Even though we did not cover them in class and the homeworks, you should see if you can answer them! No explanation is needed.
(a) How many halves are in a college basketball game? $1,2,3$, or 4 ?
(b) How many quarters are in a high school basketball game? $1,2,3$, or 4 ?
(c) How many points does a 3-point field goal account for in a basketball game? $1,2,3$, or 4 ?

