

**Public Economics**  
(Econ 512)

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Answer all questions. I wish everyone a wonderful holiday break.

1. In Karma-Land live many Ms who like to consume M&Ms, denoted by  $c$ , but hate to work, denoted by  $L$ . They have identical preferences represented by the utility function

$$u(c, L) = c - \frac{\varepsilon}{1 + \varepsilon} (L_0)^{-\frac{1}{\varepsilon}} (L)^{1 + \frac{1}{\varepsilon}},$$

where  $\varepsilon \geq 0$ . Although Ms have identical preferences, they have different earning abilities or wage rates,  $w$ . Earning abilities are distributed over the support  $[\underline{w}, \bar{w}]$  with a density function  $f(w)$ . The economy is closed and population size is normalized at one. Ms face a linear income tax schedule of the type

$$T = ty - G$$

where  $T$  is an individual's net tax payments,  $y(w) = wL(w)$  is the income of an M with wage  $w$ , and  $G$  is his "basic income" (what he receives from the government) which is the same for everyone. The government of Karma Land has no external revenue requirement.

1. Derive an expression for national income in Karma-Land and show that it is decreasing in the tax rate.
2. Derive an expression for loss of output as a function of the tax rate and show that the loss is increasing in the elasticity of labor supply.
3. Can we think of the output loss as the "efficiency loss" of taxation? Why or why not?
4. Derive an expression for the indirect utility function of a  $w$ -type M (as a function of his wage and tax rate).
5. Which  $w$ -types see their utility increase and which ones see their utility shrink as a result of levying  $t$ ?
6. Derive an expression for average utility in Karma-Land.
7. Show that the ratio of average utility at a tax rate of  $t$  to average utility at a tax rate of 0 is decreasing in  $t$ .
8. Show that although average utility shrinks as a result of levying a tax rate of  $t$ , it shrinks by less than the factor than national income shrinks. Why do you think this is?

9. Suppose the social welfare function is utilitarian. What is the optimal tax rate?
  10. Suppose the social welfare function is Rawlsian. What is the optimal tax rate?
2. In the Country of Wineapples wine and apple pies are produced using capital and labor. Capital in each industry is fixed in place, while labor is freely mobile.
- (i) What is the incidence of a proportional labor tax in the country of Wineapples?
  - (ii) What is the incidence of a tax levied on labor used in the wine industry?
  - (iii) Does your answer to above question depend on whether wine is more or less labor intensive?
  - (iv) What is the incidence of a tax on capital used in the wine industry?
  - (v) How would your answer to (iv) differ if capital as well as labor was mobile?

3. Steven Orental Billiams, aka SOB, is a pretentious man claiming to be for any grand-sounding principle. He does not know much about economics of taxation, but when he heard of the “principle of equal treatment of singles and couples,” he proclaimed his support for it. Similarly, he proclaimed his support for the “principle of marriage neutrality,” and the “principle of spouse anonymity.” Much to his disappointment, his learned public economics colleague, Holder of the Throne (HOT) told SOB that his views on taxation, like all other facets of his knowledge, are FLAT. What is this all about?

Hint: (i) Principle of equal treatment of singles and couples means that if a single-person household and a married-couple household have the same household income, they should pay the same tax.

(ii) Principle of marriage neutrality states that the tax two persons pay as a couple should be equal to the sum of their individual tax payments before getting married.

(iii) Principle of spouse anonymity states that married couples with the same *total* income should pay the same tax regardless of which spouse earns how much.

You should prove that (i) and (ii) together implies a proportional tax system, and (ii) and (iii) together implies same marginal income tax rate.

4. As it happens, the SOB is also a cheater. He spends  $L$  hours at the office and receives a salary of  $wL$ . His utility function  $v(c, L)$  depends positively on his consumption,  $c$ , and negatively on  $L$ . His income is not publicly observable but can be detected through an audit. He faces a tax rate of  $t$  and may be audited with probability of  $\beta$ . If caught, the SOB will have to pay the taxes he evaded plus a fine equal to  $(\tau - 1)$  times the taxes he evaded.

- (i) Derive a set of equations that determine SOB's hour at work and undeclared income,  $z$ . Interpret these equations.
- (ii) Is there a policy that may prevent the SOB from cheating on his taxes? If there is, should the government adopt it?
- (iii) Assume  $v(c, L) = u(c) - f(L)$ . Further assume that  $u(c)$  has the property that the mean of marginal utilities of consumption is equal to the marginal utility of mean consumption. That is, given two values for  $c$ ,  $c_1$  and  $c_2$ :  $\alpha u'(c_1) + (1 - \alpha) u'(c_2) = u'[\alpha c_1 + (1 - \alpha) c_2]$ , where  $0 \leq \alpha \leq 1$ . Prove that tax evasion causes the SOB to work less.
- (iv) What kind of utility function exhibits the above-mentioned property:  $\alpha u'(c_1) + (1 - \alpha) u'(c_2) = u'[\alpha c_1 + (1 - \alpha) c_2]$ ?

5. Assume a Mirrlesian economy wherein wage rates, which also represent the skill levels, are distributed according to the distribution function  $F(w)$  on the support  $[\underline{w}, \bar{w}]$ . The corresponding density function,  $f(w) = F'(w)$ , is assumed to be strictly positive and differentiable for all  $w \in [\underline{w}, \bar{w}]$ . Individuals have identical quasi-linear preferences that depend on consumption,  $c$ , positively, and on labor supply,  $L$ , negatively:

$$u = c - h(L). \tag{1}$$

Let  $c(w)$ ,  $I(w) = wL(w)$ , and  $u(w)$  denote consumption, income, and utility of an individual of type  $w$ . Let  $\underline{u} = u(\underline{w})$  be the utility of the poorest individual.

Assume that the government has no external revenue requirement and it wants to levy an entirely redistributive tax. Thus

$$\int_{\underline{w}}^{\bar{w}} [I(w) - c(w)] f(w) dw = 0. \tag{2}$$

- (i) Prove

$$\begin{aligned} \underline{u} &= \int_{\underline{w}}^{\bar{w}} \left[ I(w) - h\left(\frac{I(w)}{w}\right) \right] f(w) dw - \int_{\underline{w}}^{\bar{w}} \frac{I(w)}{w^2} h'\left(\frac{I(w)}{w}\right) [1 - F(w)] dw, \\ &\equiv \Psi(F), \end{aligned} \tag{3}$$

so that the optimal Rawlsian tax schedule can be found by determining  $I(w)$  that maximizes  $\Psi(F)$ .

- (ii) Interpret the two components of  $\Psi(F)$ .
- (iii) Observe that  $\Psi(F)$  depends solely on individuals' preferences and the distribution of productivities. Does it make sense for the optimal income tax schedule of the *society* to depend only on the individuals' preferences and the distribution of productivities?
- (iv) Characterize optimal tax schedule  $T(I(w)) = I(w) - c(w)$ .