Public Economics
(Econ 512)

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Answer all questions. I wish everyone a wonderful holiday break.

1. (9 points.) Assume that the representative consumer has a utility function $U = \sqrt{x_1} + \ln x_2 - L$, where x_1 and x_2 are the consumption levels of good 1 and good 2 respectively, while L is the labor supply. Producer prices, p_i, and the wage, w, are constant. Consumer prices are denoted by $q_i = p_i + t_i$, where t_i is a per unit tax. Taxes are set according to a Ramsey problem with revenue requirement $R_0 > 0$.

(i) Solve the individual's optimization problem.

(ii) Give the expressions for the optimal tax *rates* $\tau_i = t_i/(p_i + t_i) = t_i/q_i$.

(iii) Find the value of τ_1/τ_2.

(iv) Determine the value of the labor supply.

(v) Determine the value of the individual's marginal utility of income.

2. (19 points.) Consider an economy consisting of two types of consumers, $i = 1, 2$ with of equal size with wage levels $w_2 > w_1$. The two types are of equal size and have identical preferences given by

$$U_i = u\left(x_i - \frac{L_i^2}{2}\right),$$

where $u' > 0$ and $u'' < 0$. (This is a concave transformation of quasi-linear preferences). Production technology is linear and an individual's pre-tax income is denoted by $I_i = w_i L_i$. There is no exogenous revenue requirement.

(i) Determine the utilitarian first-best (full information) allocation.

(ii) Write the incentive constraints and show that this allocation is not incentive compatible.

(iii) State the problem determining the constrained Pareto-efficient allocations that can be achieved with an optimal nonlinear income tax $T(I)$.

(iv) Write the Lagrangian expression that is associated with this problem and derive the first-order conditions.

(v) Prove that an i-type individual, with the above preferences facing a nonlinear income tax schedule $T(I)$, chooses his optimal allocation such that

$$T'(I_i) = 1 - \frac{I_i}{w_i^2}.$$

(vi) Assuming that the downward incentive compatibility constraint is binding, prove that

(a) Individuals of type 2 face a zero marginal income tax rate and choose the same level of labor supply as in the first best.

(b) Individuals of type 1 face a positive marginal income tax rate.

3. (12 points.) Consider an economy consisting of H individuals, indexed $h = 1, \ldots, H$, who have the same preferences but differ in their income m_h with $m_1 \leq m_2 \ldots \leq m_h \ldots \leq m_H$. Individuals have identical preferences over a private good x and a public good G.

(i) Does the Samuelson's rule for optimal provision of the public good determine the level of public good uniquely? Why or why not? Under what circumstances G is determined uniquely?

(ii) What is a Lindahl equilibrium? Is it efficient? Is it implementable?

(iii) Assume the public good is produced from the numeraire good x with a constant marginal cost of 1. It is provided through voluntary contribution $g_h \geq 0$, with

$$G = \sum_{h=1}^{H} g_h.$$

Preferences are represented by

$$U(x_h, G) = \ln(x_h) + \ln(G).$$

(a) Is the Nash equilibrium of the contribution game efficient? Why or why not?

(b) Prove that all *contributors* consume the same amount of the private good.

(c) Prove that the higher is the income of a contributor, the more he contributes (to the provision of the public good).

4. Why would the government want to redistribute through in-kind as opposed to cash transfers? (A good answer covers at least five distinct reasons.)