Homework #1
(Econ 512)

F. Gahvari — Fall 2011

1. Consider the following quasi-linear utility function:

$$u = y + \ln x.$$

(i) Show that the corresponding indifference curves are vertically parallel.

(ii) Given a budget constraint

$$p_x x + p_y y = m,$$

determine the demand functions for x and y.

(iii) Derive the compensated demand functions for x and y.

(iv) Derive the indirect utility function.

(v) Derive the expenditure function.

(vi) Find the consumer's surplus in going from p_x^0 to p_x^1.

(vii) Find the compensating variation in going from p_x^0 to p_x^1.

(vii) Find the equivalent variation in going from p_x^0 to p_x^1.

2. Suppose prices of goods x and y both change from (p_x^0, p_y^0) to (p_x^1, p_y^1).

(i) Show that the compensating variation (CV) measure of a welfare change satisfies path independence.

(ii) Depict the above change diagrammatically in terms of areas under compensated demand curves.

(iii) Prove that the Marshallian consumer surplus measure for the change in consumer's welfare is path dependent (unless the cross-price derivatives of ordinary demand functions are equal).

3. Recall that we used

$$EB(t_1) = E(p_1, u_a) - E(p_0, u_a) - t_1 x^c(p_1, u_a)$$

as a general measure for excess burden of a tax equal to t_1 levied on x. We also used

$$MEB = E(p_2, u_a) - E(p_1, u_a) - [t_2 x^c(p_2, u_a) - t_1 x^c(p_1, u_a)]$$

as a measure of additional excess burden associated with increasing the tax on x from t_1 to t_2.

(i) Prove that MEB satisfies the additive property; namely, $MEB = EB(t_2) - EB(t_1)$.

(ii) Prove that if one were to work with the CV or EV measures when imposing these taxes (i.e. t_1 first followed by t_2, the additive property will be lost.

4. Assume preferences for a composite consumption good, c, and housing, x, are represented by the following Cobb-Douglas utility function,

$$u = c^{.75} x^{.25}.$$

Further assume that the consumer has an annual income of \$50,000, the producer prices are constant, the (annual) price of housing per square meter is \$10, and c is the numeraire. Assume that initially there is no tax on x.

(i) Determine the equilibrium quantities of c and x.

Next assume that a 25% tax is levied on housing.

(ii) Determine the new equilibrium quantities of c and x.

(iii) Using equivalent variation, determine how much worse off the consumer is as a result of the tax.

(iv) Calculate the excess burden of the tax.

(v) Calculate excess burden per dollar of tax revenue raised.

(v) Approximate the excess burden by using the ordinary demand curve for x and assuming that it is locally linear.

5. Assume the representative consumer's preferences for a composite consumption good, c, and leisure, l, are represented by the following Cobb-Douglas utility function,

$$u = \ln c + \ln l.$$

The consumer's budget constraint is

$$c = w(1 - \theta)L + m,$$

where θ is the tax rate on labor income and $m \geq 0$ is exogenous income.

(i) Calculate the indirect utility function.

(ii) Calculate marginal utility of income.

(iii) Calculate the marginal tax revenue function.

(iv) Show that if $m = 0$, labor supply is perfectly inelastic.

(v) Does your findings for $m = 0$ mean the tax is not distortionary? Discuss.

6. Prove that quasi-linear preferences are of Gorman-polar form.