Homework #2
(Econ 512)

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1. Assume that the markets for beer (b) and cigarettes (c) in Illinois are linked because people like to smoke when drinking. Their weekly demand functions are given by

$$\begin{aligned} X_c &= 10 - p_c - 0.5p_b \\ X_b &= 20 - 2p_b - p_c, \end{aligned}$$

where X_c is the quantity of cigarettes (in millions of packs), X_b is the quantity of beer (in millions of pints), p_c is the price of a pack of cigarettes and p_b is the price of a pint of beer. Assume that both industries are perfectly competitive. The marginal cost of producing one pack of cigarettes is fixed and equal to one dollar ($MC_c = 1$). The marginal cost of producing a pint of beer is also fixed and equal to two dollars ($MC_b = 2$).

(i) Find the initial no-tax competitive equilibrium.

(ii) The state of Illinois taxes both alcohol and cigarettes. Assume the tax on cigarettes is 50 cents per pack and on beer is $1 per pint. Find the equilibrium with the tax as well as the tax revenues.

(iii) What is the total excess burden of these taxes if we calculate them on the basis of the above ordinary demand functions?

(iv) Comment on your answer to the above question. (Micah thinks this is too vague a question. Prove him wrong by thinking carefully if there might be anything special about your answer to the above question that you may want to mention).

2. Consider a one-sector general equilibrium model in which labor supply is upward-sloping.

(i) Determine the impact of a sales tax (at the rate of t) on net factor returns if capital is supplied perfectly inelastically. It may be easier if you calculate the "percentage changes"; that is, for every variable x, calculate $(dx/dt)/x$.

(ii) How does your answer to above changes if the elasticity of substitution between capital and labor is zero?

(iii) Determine the impact of a sales tax on net factor returns if capital is supplied perfectly elastically.

3. Consider taxing labor in a labor intensive industry. Use a traditional two sector general equilibrium model to answer the following questions.

(i) Can labor ever bear more than 100% percent of the incidence of the tax?

(ii) Can the incidence on labor ever be less than that on capital?

4. In the Country of Wineapples wine and apple pies are produced using capital and labor. Capital in each industry is fixed in place, while labor is freely mobile.

(i) What is the incidence of a proportional labor tax in the country of Wineapples?

(ii) What is the incidence of a tax levied on labor used in the wine industry?

(iii) Does your answer to above question depend on whether wine is more or less labor intensive?

(iv) What is the incidence of a tax on capital used in the wine industry?

(v) How would your answer to (iv) differ if capital as well as labor was mobile?

5. Assume there are m workers, $n \leq m$ of whom are employed. Workers derive utility only from the net income they consume, supplying one unit of labor regardless of the wage. Labor markets are non-competitive. The government, the firms and the "union" behave as follows. In stage 1, the government sets a payroll tax at the rate of τ on the wage (w) paid out by firms, an income tax rate of t on the employed workers, and an untaxed benefit level of $\bar{w} < w(1-t)$ to be paid to the unemployed workers. In stage 2, the "union" sets the wage rate with a view to maximize the sum of the employed workers' earnings and the unemployed workers' benefits. In stage 3, the profit maximizing firms (represented here by one "firm") decide on the number of workers to hire. The production function is $f(n)$ where $f(.)$ is increasing in n and concave.

(i) Determine n and show that it is a decreasing function of $w(1+\tau)$.

(ii) Determine w as a function of n, t, τ and $\bar{w}$.

(iii) Derive a sufficient condition for the second order condition of the union to be satisfied.

Next suppose the government decides to change its financing of $\bar{w}$. It keeps $\bar{w}$ fixed while increasing t and reducing τ such that the net tax rate $(1+\tau)/(1-t)$ remains constant.

(iv) What is the impact of this tax adjustment on employment?

(v) What is the impact of this tax adjustment on government's tax revenues?

(vi) Who bears the incidence of this tax adjustment?

6. In my class discussion on Ramsey tax problem, and the note I distributed, I made explicit use of the individual's budget constraint and the government's budget constraint, but not of the economy's resource constraint. Why? Specifically, consider the case when producer prices are not constant. In considering the individual's optimization problem (and thus deriving his indirect utility function), I made use of his budget constraint

$$\sum_{i=0}^{n} q_i x_i^d = q_0 + \pi - \theta,$$

where π denotes profit, θ is the profit tax, $q_i = p_i + t_i$, $i = 0, 1, \ldots, n$, withp_0 denoting the gross-of-tax wage, and $t_0 = -\tau$ where τ is the wage tax. Then, in setting up the government's problem, I considered the government's budget constraint

$$\sum_{i=1}^{n} t_i x_i^d + \tau_0(L^p + L^g) + \theta = \sum_{i=1}^{n} p_i x_i^g + p_0 L^g.$$

The economy's resource constraint,

$$\sum_{i=1}^{n} p_i x_i^s + p_g G = p_0(L^p + L^g) + \pi$$

was never explicitly utilized (G is the government good produced at the per unit cost of p_g). How can we characterize the society's optimum without considering its resource constraint? What is going on?

7. Generalize the "inverse elasticity" rule, derived when cross-price derivatives are zero, to a setup with many consumers.

8. We define the net marginal social utility of income to individual h to be

$$\gamma^h \equiv \frac{\partial W}{\partial v^h}\alpha^h + \mu \sum_{i=1}^{n} t_i \frac{\partial x_i^h}{\partial m^h}.$$

Clearly, if there are no commodity taxes, the above will simplify to

$$\gamma^h \equiv \frac{\partial W}{\partial v^h}\alpha^h.$$

It thus appears that the net marginal social utility of income is higher when goods are taxed (and assuming that these goods are normal). Does this make sense? What is going on?