Homework #3
(Econ 512)

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1. Rachel has the following utility function for consumption, C, and leisure, l:

$$U(C, l) = C^{0.25} l^{0.75}.$$

There are 50 weeks in the work year and 168 hours per week to be allocated to labor, L, or leisure, l, (so $L + l = 168$). She has no other sources of income.

(i) Given a wage of \$25.00 per hour and a 20% income tax rate, sketch Rachel's weekly budget constraint. How many hours a week will she work? What will her annual income be before paying taxes? What will her annual income be after paying taxes?

(ii) Assume now, that tax code is progressive. On any income less than \$20,000 per year, Rachel must pay 20% in income tax, while on any income over \$20,000 per year, she must pay 40% in income tax. Sketch Rachel's new budget constraint and label the kink-point carefully. How many hours a week will she work? What will her annual income be before and after taxes?

(iii) Assume an additional tax bracket is added so that the marginal tax rate on income over \$30,000 is taxed at the rate of 60%. How many hours of week will Rachel work in this case? What will her annual income be before and after taxes?

2. Kim is a single head of a household with two children and has the following utility function over consumption, C, and leisure, l:

$$U(C, l) = C^{0.2} l^{0.8}.$$

There are 50 weeks in the work year and 168 hours per week. Assume that Kim earns \$10.00 per hour and has no other sources of income.

(i) Suppose initially that the government gives \$4,000 to every family regardless of the family's income. It then taxes all earnings at the rate of 30%. Sketch the family's weekly budget constraint. How many hours a week will Kim work? What will the family's annual consumption be?

(ii) Now assume that instead of the "unearned" credit, there is an earned income tax credit (EITC) for low-income households. The EITC provides a 40% credit if

Kim earns less than $10,000. Continues to pay a $4,000 credit until Kim reaches $15,000 and then is phased out at a 40% rate, so that by $25,000 Kim receives no credit. Any income above $25,000 is taxed at the rate of 30%. Sketch the family's weekly budget constraint with the EITC program in place. How many hours a week will Paul work? What will the family's annual consumption be?

3. Show the superiority of a lump-sum tax to a proportional income tax and a proportional income tax to a progressive tax.

4. Given your answer to 3, why does the government institute a progressive tax?

5. In year 2010, tax rates, standard deduction and exemptions in the U.S. were:

Marginal Tax Rate	Single Returns	Joint Returns
10%	$0–$8,375	$0–$16,750
15%	$8,375–$34,000	$16,750–$68,000
25%	$34,000–$82,400	$68,000–$137,300
28%	$82,400–$171,850	$137,300–$209,250
33%	$171,850–$373,650	$209,250–$373,650
35%	$373,650–	$373,650–
Standard Deduction	$5,700	$11,400
Each personal exemption	$3,650	$3,650

Handsome and Pretty are going to get married. Determine, under each scenario below, whether Handsome and Pretty end up with paying a marriage tax or getting a marriage dowry.

(i) Handsome and Pretty both earn $25,000 each.

(ii) Handsome and Pretty both earn $200,000 each.

(iii) Handsome earns $300,000 and Pretty does not work.

(iv) Handsome earns $50,000 and Pretty $150,000.

6. Suppose married couples and single persons face the same progressive linear income tax schedule: $T = -G + \theta y$.

(i) Suppose the taxpaying unit is a household.

a. Is there a marriage penalty tax?

b. Will all households (regardless of their marital status) pay the same taxes for the same total income?

(ii) Now suppose the taxpaying unit is an individual.

a. Is there a marriage penalty tax?

b. Will all households (regardless of their marital status) pay the same taxes for the same total income?

(iii) How do your answers to (i)–(ii) change if the tax structure is proportional?

7. Prove that given a progressive tax schedule, the two "principles" of "marriage neutrality" and "equal treatment of singles and couples" (with the same income levels) cannot be satisfied simultaneously.

8. Prove that if there are no equity considerations, the optimal linear income tax rate is zero.

9. Prove that if the linear income tax entails no efficiency loss, then the optimal tax rate is 100%.

10. Assume preferences are quasi-linear and represented by

$$u(c, L) = c + f(1 - L),$$

where c is consumption and L is labor supply. Individuals have differing ability and earn wages w distributed with the density function $f(w)$ over the support $[\underline{w}, \overline{w}]$.

(i) Solve the optimal linear income tax problem in this case. What is the expression for γ_w?

(ii) Prove that the optimal tax rate is zero, if the social welfare function is utilitarian.

(iii) Give an intuitive explanation for the result on the basis of γ, the net marginal social utility of income.

11. Consider the optimal linear income tax problem.

(i) Find the tax rate that maximizes the "basic income" (lump-sum rebates).

(ii) Prove that *in general* the "basic-income-maximizing" tax is greater than optimal tax.

12. Consider the optimal linear income tax problem and assume the social welfare function is Rawlsian. Simplify the expression for the optimal tax rate and thus [using 11(i)] prove that the optimal tax rate is that which maximizes the basic income.

13. Consider the optimal linear income tax problem. Assume that a fixed proportion of the population, μ, is "sick" and incapable of work. Everybody else in the society works. The government treats γ_w/λ for all working people equally, represented by a constant ϕ. Similarly, the government treats γ_w/λ for all non-working people equally; this is further set equal to $\rho\phi$ with $\rho > 1$.

(i) Rewrite the optimal tax formula for the characterization of basic income. On the basis of this equation, calculate an expression for ϕ in terms of μ and ρ.

(ii) Simplify the expression for the optimal tax rate and show that the optimal tax moves positively with μ (the proportion of the population who is "sick") and ρ (the weight given to the "sick").

(iii) Prove that in this special case too, the optimal tax rate is less than that which maximizes the "basic income".

14. Show how differentiating tax rates may improve society's welfare.

15. Show why levying differential lump-sum taxes may be incentive incompatible.