Homework #4
(Econ 512)

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1. Consider the two-group version of the optimal general income tax problem. Preferences depend positively on a vector of consumption goods, $\mathbf{x} = (x_1, x_{2,} \ldots, x_n)$, negatively on the labor supply, and represented by the concave utility function $u = u(\mathbf{x}, L)$. High-ability workers have a wage equal to w^h, and low-ability workers a wage equal to w^l. Ability types are publicly unobservable but incomes are.

(i) Describe the set of Pareto-efficient allocations constrained by the resource constraint and incentive compatibility constraints.

(ii) Derive the first-order conditions for this problem.

(iii) Prove that the high-ability workers face no differential commodity taxes.

(iv) Derive an expression for the commodity tax rates faced by the low-ability types.

(v) Prove that the low-ability types face no commodity tax rates either if preferences are weakly separable in labor supply and goods.

(vi) Suppose commodity taxes must always be proportional. That is, all individuals must face the same proportional tax rates (as is commonly the case). Under this circumstance, can you implement the constrained Pareto-efficient allocations you have characterized above?

(vii) Under what informational assumptions commodity taxes must be proportional?

(vii) How do you characterize the constrained Pareto-efficient allocations if commodity taxes have to be proportional?

2. Assume a Mirrleesian economy wherein wage rates, which also represent the skill levels, are distributed according to the distribution function $F(w)$ on the support $[\underline{w}, \overline{w}]$. The corresponding density function, $f(w) = F'(w)$, is assumed to be strictly positive and differentiable for all $w \in [\underline{w}, \overline{w}]$. Individuals have identical quasi-linear preferences that depend on consumption, c, positively, and on labor supply, L, negatively:

$$u = c - h(L). \tag{1}$$

Let $c(w)$, $I(w) = wL(w)$, and $u(w)$ denote consumption, income, and utility of an individual of type w. Let $\underline{u} = u(\underline{w})$ be the utility of the poorest individual.

Assume that the government has no external revenue requirement and it wants to levy an entirely redistributive tax. Thus

$$\int_{\underline{w}}^{\overline{w}} [I(w) - c(w)]\, f(w)dw = 0. \tag{2}$$

(i) Prove

$$\begin{aligned} \underline{u} &= \int_{\underline{w}}^{\overline{w}} \left[I(w) - h\left(\frac{I(w)}{w}\right) \right] f(w)dw - \int_{\underline{w}}^{\overline{w}} \frac{I(w)}{w^2} h'\left(\frac{I(w)}{w}\right) [1 - F(w)]dw, \\ &\equiv \Psi(F), \end{aligned} \tag{3}$$

so that the optimal Rawlsian tax schedule can be found by determining $I(w)$ that maximizes $\Psi(F)$.

(ii) Interpret the two components of $\Psi(F)$.

(iii) Observe that $\Psi(F)$ depends solely on individuals' preferences and the distribution of productivities. Does it make sense for the optimal income tax schedule of the *society* to depend only on the individuals' preferences and the distribution of productivities?

(iv) Characterize optimal tax schedule $T(I(w)) = I(w) - c(w)$.

3. (a) Principle of equal treatment of singles and couples means that if a single-person household and a married-couple household have the same household income, they should pay the same tax. (b) Principle of marriage neutrality states that the tax two persons pay as a couple should be equal to the sum of their individual tax payments before getting married. (c) Principle of spouse anonymity states that married couples with the same *total* income should pay the same tax regardless of which spouse earns how much.

(i) Prove that (a) and (b) together implies a proportional tax system.

(ii) Prove that (b) and (c) together implies that the marginal income tax rate does not change with the level of income.

4. Consider the problem of the voluntary provision of public goods. There are three individuals with preferences over a private good, x, and a public good, G. These preferences are represented by

$$\begin{aligned} u^1 &= \ln\ x^1 + \ln\ G, \\ u^2 &= \ln\ x^2 + 2\ln\ G, \\ u^3 &= \ln\ x^3 + 3\ln\ G. \end{aligned}$$

Individual j (with $j = 1, 2, 3$) has an income level equal to m^j to be spent on the private and the public good. Price of the private good is one and person j who purchases g^j units of the public good will have to pay c for each unit purchased. Everybody gets to consume the total amount purchased: $G = g^1 + g^2 + g^3$.

(i) Using Nash equilibrium as your solution concept, derive the strategy profile $\{\widehat{g}^j\}$ for each individual j (given $\overline{G}^j$).

(ii) Find the values of $\widehat{g}^1, \widehat{g}^2$, and $\widehat{g}^3$ assuming that $m^1 = 20, m^2 = 30, m^3 = 40$, and $c = 10$.