Conditional cash transfers, public provision of private goods, and income redistribution

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Abstract

This paper examines the role of cash transfers as a screening device when combined with in-kind transfers. It shows that linking in-kind to cash transfers makes first-best redistribution possible despite the government's inability to tell the rich and the poor individuals apart. Second, the maximal attainable welfare for the poor can be pushed beyond its first-best level by distorting downwards the quality of the indivisible good the poor receive (relative to the cash value of their net transfers). Third, the first- and second-best frontiers each will have at most on point in common with the feasible utility frontier under the Besley and Coate (1991) scheme so that their solution is third best. Finally, the extension to an economy with many income types is discussed.

JEL classification: H42, H31, H21, H23.

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1 Introduction

In designing redistributive policies, the public sector invariably finds it too difficult or too costly to tell the intended welfare recipients apart from the rest of the population. The recent literature on the public provision of private goods incorporates this limitation on information gathering into account when determining the efficiency properties of such schemes. The literature points out that in the absence of the required information for targeting benefits, one should devise "self-targeting" mechanisms that induce only the intended recipients to participate with the others opting out. One way to achieve this is by imposing certain costs on the participants (e.g., a low quality product, workfare, time-consuming application procedures etc) that only the targeted population is prepared to endure. The potential pretenders will find, for a variety of reasons, the costs to be prohibitively high; see, among others, Nichols and Zeckhauser (1982), Moffit (1983), Blackorby and Donaldson (1988), and Besley and Coate (1991).[1]

Besley and Coate (1991) has a particularly simple structure. The authors consider the public provision of an indivisible good which is produced in different variants each embodying a particular quality level. Every person may consume only one variant of this good; they cannot be combined. An example is education, when a person can go to only one type of school.[2] The quality is normal in the sense that people with higher income levels would opt for higher quality variants of the good. They further assume that the publicly-provided good will be financed through a head tax. Redistribution is then achieved as long as only the poor households consume the good. In such a

[1]Blomquist and Christiansen (1995) also have a self-targeting model where people either participate in the public provision program or opt out. By contrast, Boadway and Marchand (1995), and Cremer and Gahvari (1997) consider models of in-kind transfers with uniform provision to everyone without achieving self-targeting. These papers, however, are concerned mainly with the question of the usefulness of in-kind transfers in the presence of a general income tax in which the government observes income but not earning abilities. The relevance of the impact of in-kind transfers on labor supply (and tax revenues) had previously been emphasized by Gahvari (1994, 1995) on the basis of a linear income tax structure.

[2]This assumes, as Besley and Coate (1991) state, that hiring private tutors to educate one's children during after-school hours is not the same as going to a school of higher quality.

scheme, all transfers, bar one, involve a deadweight loss. The exception arises when the publicly-provided quality level is precisely what the poor would choose for themselves if they received its value in cash. The authors point out that, because of the welfare loss associated with an inefficient quality (or level) of public provision, their scheme will not necessarily be "part of a properly designed redistributional package". They argue that whether this would be the case or not depends on "the cost to the government of observing its citizens' incomes".[3] Their point is that while their scheme has a deadweight loss, it does not rely on the costly activity of information-gathering regarding the characteristics of different households: There exists a trade-off between the cost of acquiring information on the part of the government and the deadweight loss inherent in not providing the poor their desired level of the publicly-provided good.

The aim of this paper is to show that one can alter the Besley-Coate transfer package in such a way as to result in no deadweight loss associated with an inefficient quality (or level) of public provision, and yet demand no extra informational requirement on the part of the government. Any trade-offs that may exist will thus solely be due to the costs of administrating the proposed transfer programs versus the costs in observing the individuals' incomes. The "trick" is to link the acceptance of the public assistance to a lump-sum tax or rebate which we call a "conditional cash transfer". Under this scheme, incomes remain unobservable and there will be self-selection on the part of taxpayers. Yet, all transfers are carried out on the first-best frontier of the economy.

The cash-cum-in-kind-transfer scheme we are suggesting is also a mechanism that achieves self-targeting. However, the introduction of conditional cash transfers (in addition to public-provision part that forms the basis of the Besley and Coate's scheme) allows for a policy design that punishes only (at least for a certain range of transfers) the potential pretenders (the so-called "mimickers") while sparing the truly poor participants. Consequently, when the net transfers are not high enough to entice the rich to participate in the program, the conditional cash transfers, in conjunction with the head

[3]The quotations are from Besley and Coate's Concluding Remarks, p. 983.

tax, forms a system of differential lump-sum taxation. It should not then be surprising that the redistribution it achieves is first-best.

We also show that while the first-best conditional cash-cum-in-kind transfer scheme dominates the Besley and Coate's mechanism, it *may* result in less redistribution in that the maximum attainable welfare for the poor will be lower under it (as compared to the poor's maximal welfare under the Besley and Coate's mechanism). Specifically, we prove that if the maximum incentive compatible quality level under the Besley and Coate's scheme exceeds the efficient quality level (i.e. the level that the poor would choose for themselves if they received the cost of the publicly-provided good in cash), the maximum attainable utility by the poor will necessarily be higher when conditional cash transfers accompany public provision than when they do not. Otherwise, conditional cash transfers may increase or decrease the extent of redistribution towards the poor.

Secondly, we show that the maximal attainable welfare for the poor can be pushed beyond its level under the first-best cash-cum-in-kind transfer scheme. This possibility arises when the redistribution is high enough to encourage the rich persons to participate in the program. Under this circumstance, the downward incentive compatibility constraint for the rich becomes binding, and we have a second-best solution. The maximum attainable welfare in this case will as a rule be higher than its corresponding level under the Besley and Coate mechanism.[4]

2 The Besley and Coate model

Consider an economy with two goods: a numeraire consumption good c and a second indivisible good which one consumes in whole or not at all. This latter good may

[4]Observe that, unlike the first-best version, the poor participants will now also be punished as the quality of the indivisible good they receive will be distorted downwards (relative to the cash value of their net transfers). That is, the quality level of the publicly-provided good will be *less* than the level the poor would purchase for themselves if they were to receive the value of their net transfers in cash. The way that conditional cash transfers help is by slackening the otherwise (i.e. in the absence of cash transfers as in the Besley and Coate's scheme) downward incentive compatibility of the rich, thus allowing further redistribution.

be packaged in different variants, each embodying a different level of quality q. The consumer can buy only one variant; different variants cannot be combined. Examples include education or housing. The economy is inhabited by two types of individuals: The "rich" who have y^h endowment of the numeraire good and the "poor" who have an endowment of $y^l < y^h$. The two types have identical preferences over the goods represented by a smooth and strongly quasi-concave utility function $u(c, q)$ which is increasing in both of its arguments. The numeraire good is normal and so is the quality embedded in the indivisible, and publicly-provided, good. The production technology is linear, converting pq units of the numeraire into one unit of the indivisible good with quality q. The economy is perfectly competitive so that p is the price of quality at the margin.

Let $c(p, y^j)$ and $q(p, y^j)$ denote the j-type's $(j = h, l)$ demand functions for the numeraire good and the quality level of the indivisible good, if he were to purchase them from the market. These correspond to type j's maximizing $u(c, q)$ subject to $c + pq = y^j$, and yield the indirect utility function $v(p, y^j) \equiv u\left(c(p, y^j), q(p, y^j)\right)$. Now assume that the government is to provide the second good at the quality level of $\overline{q}$ for free to whoever wants it. The good will be financed by a lump-sum tax T levied on everyone. Besley and Coate point out that it is never efficient to provide a quality level such that both types want to consume it. Efficiency requires a separating equilibrium: The poor prefer $\overline{q}$ to the alternative of buying their most-preferred quality level from the market, while the rich prefer to buy from the market. The following incentive compatibility constraints must be satisfied.

$$u(y^h - T, \overline{q}) \leq v(p, y^h - T), \tag{1}$$

$$u(y^l - T, \overline{q}) \geq v(p, y^l - T). \tag{2}$$

Observe also that as long as only the poor participates in the public provision scheme, the government's budget constraint is given by

$$T = \pi^l p\overline{q}, \tag{3}$$

where π^l denotes the proportion of the poor in the population. Naturally, $\pi^h = 1 - \pi^l$ denotes the proportion of rich individuals in the total population.

Let Q denote the value of q under the Besley and Coate scheme. Assuming the rich do not participate in the transfer program, the quality of the publicly-provided good must be less than their desired level: $\overline{Q} < q(p, y^h - T)$.[5] The discussion below will be simplified by introducing a definition and distinguishing between certain values that $\overline{Q}$ may take.

Definition 1 *The quality level of the publicly-provided good is said to be efficient/less than efficient/more than efficient (from the perspective of the poor), if in comparison to his current position, the transfer recipient would be indifferent between/strictly better off by/strictly worse off by an offer of spending one dollar more to enhance the quality level of the good, financed through a one dollar increase in his lump-sum tax.*

It is plain, on the basis of this definition, that the efficient level of $\overline{Q}$ for the poor is characterized by $Q^* = q\left(p, y^l + \pi^h p Q^*\right)$. The poor receive $p\overline{Q}$ in-kind and pay $T = \pi^l p\overline{Q}$ in taxes; the monetary value of the net transfer to a poor individual is thus $p\overline{Q} - \pi^l p\overline{Q} = \pi^h p\overline{Q}$. At $\overline{Q} = Q^*$, the poor are indifferent between receiving one extra dollar in cash and one extra dollar worth of the publicly-provided good. At this point, it is also the case that $u_{\overline{q}}/u_c = p$. Moreover, if $\overline{Q} < Q^*$, $u_{\overline{q}}/u_c > p$ and $\overline{Q}$ is less than efficient; while if $\overline{Q} > Q^*$, $u_{\overline{q}}/u_c < p$ and $\overline{Q}$ is more than efficient. Finally, given that the rich always choose their most desired bundle (not participating in the transfer program), Q^* must be first best. Point E in Figure 1 shows the distribution of the utilities between the poor and the rich when $\overline{Q} = Q^*$.

Next, observe that while Q^* is efficient from the perspective of the poor, it does not result in the maximal utility for them. The point is that Q^* would maximize their utility (they would choose it voluntarily) *provided that* the size of their net transfer is constant. This is not the case here. The poor receives a net transfer of $\pi^h p\overline{Q}$ which directly

[5] Assume the contrary: $\overline{Q} \geq q(p, y^h - T)$. To have the inequality (1) satisfied, it must be the case that $y^h - T \leq c(p, y^h - T) = y^h - T - pq(p, y^h - T)$, or $pq(p, y^h - T) \leq 0$.

Table 1. Different conceptual values for $\overline{Q}$

Q^*	The efficient value of $\overline{Q}$ for the poor: $q\left(p, y^l + \pi^h p Q^*\right) = Q^*$.
$\widehat{Q}$	The value of $\overline{Q}$ that maximizes the utility of the poor: $u_q/u_c = \pi^l p$.
Q^{min}	The *minimum* value of $\overline{Q}$ that satisfies the incentive compatibility constraint (2): $u(y^l - T, Q^{min}) = v(p, y^l - T)$.
Q^{max}	The *maximum* value of $\overline{Q}$ that satisfies the incentive compatibility constraint (1): $u(y^h - T, Q^{max}) = v(p, y^h - T)$.
$\widetilde{Q}$	The (minimum) value of $\overline{Q}$ that makes the poor as happy as they would be without the government transfer policy: $u(y^l - T, \widetilde{Q}) = v(p, y^l)$.

increases with $\overline{Q}$. In effect, a one unit increase in $\overline{Q}$ will cost the poor $p - \pi^h p = \pi^l p$ instead of p (which would be the case if net transfers were constant). It should not then be surprising to find that their utility would increase if $\overline{Q}$ exceeds Q^*. Lemma 1 summarizes the relationships between these and other conceptually interesting values of $\overline{Q}$ as introduced and characterized in Table 1 (all proofs are given in the Appendix which is posted on the *AER* Web site.)

Lemma 1 *We have:*

(i) $Q^{min} < \widetilde{Q} < Q^* < \widehat{Q}$.

(ii) Q^{max} *can take values from below* $\widetilde{Q}$ *to above* $\widehat{Q}$.

Figures 1 and 2 reproduce Besley and Coate's Figures with some additions. In both Figures, AC shows the first-best frontier (the possible distribution of utilities if the types were publicly observable). Figures 1 and 2 depict the feasible utility frontiers under the Besley and Coate scheme if $Q^{max} > Q^*$ ($DB'EF$), and if $Q^{max} < Q^*$ ($DB'E'$). Both Figures are drawn under the assumption that $\widetilde{Q} < Q^{max} < \widehat{Q}$.[6] Observe that the feasible utility frontiers do not cross the 45 degree line: As pointed out by Besley and Coate, the after-transfer utility of the poor can never exceed the after-tax utility of the rich.

[6] Thus, if the downward incentive compatibility of the rich were not binding at F and E', $u^l(\overline{Q})$ would continue to increase with $\overline{Q}$. This is why the $DB'EF$ and $DB'E'$ loci are drawn with negative slopes at F and E'.

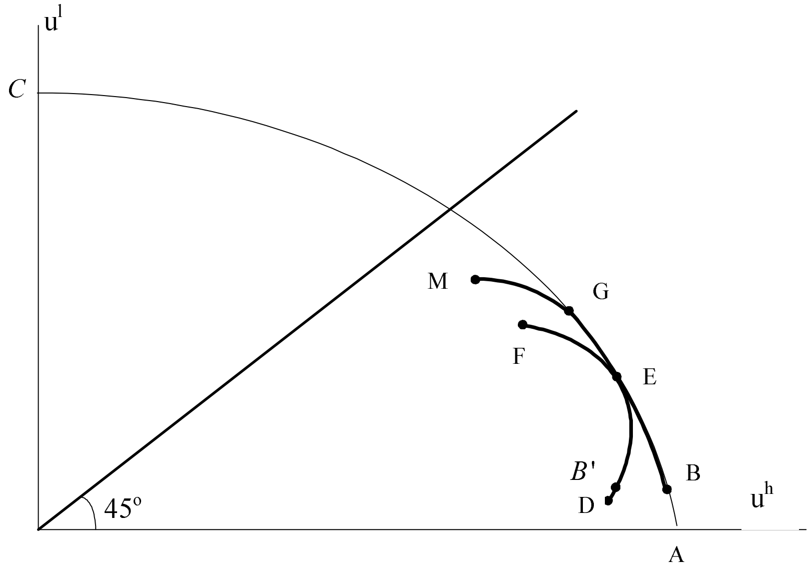

Figure 1: Economy's Pareto efficient frontier (AC), utility feasibility frontier under Besley and Coate (DEF), the first-best frontier under conditional cash transfers (BEG), and the second-best frontier with conditional cash transfers (GM).

We have:

$$u(y^l - T, \overline{Q}) < u(y^h - T, \overline{Q}) \leq v(p, y^h - T).$$

The first inequality follows from $y^l < y^h$, and the second from (1).

Observe also that $DB'EF$ and $DB'E'$ are inside the first-best frontier AC. Moreover, $DB'EF$ in Figure 1 is just tangent to the first-best frontier point E; this corresponds to Q^*. Other relevant points, in the context of Besley and Coate scheme, include point B (in both Figures) which corresponds to a situation with no transfer policies. It is plain that this point must also be on the first-best frontier. Point D (in both Figures) corresponds to Q^{min}; point B' (in both Figures) corresponds to $\widetilde{Q}$; point F (in Figure 1) and point E' (in Figure 2) correspond to Q^{max}.

3 First-best cash-cum-in-kind transfers

Consider now enriching the Besley and Coate setup by offering the participants not just $\overline{q}$, but a bundle consisting of $\overline{q}$ plus a cash transfer, t, where t can take both positive

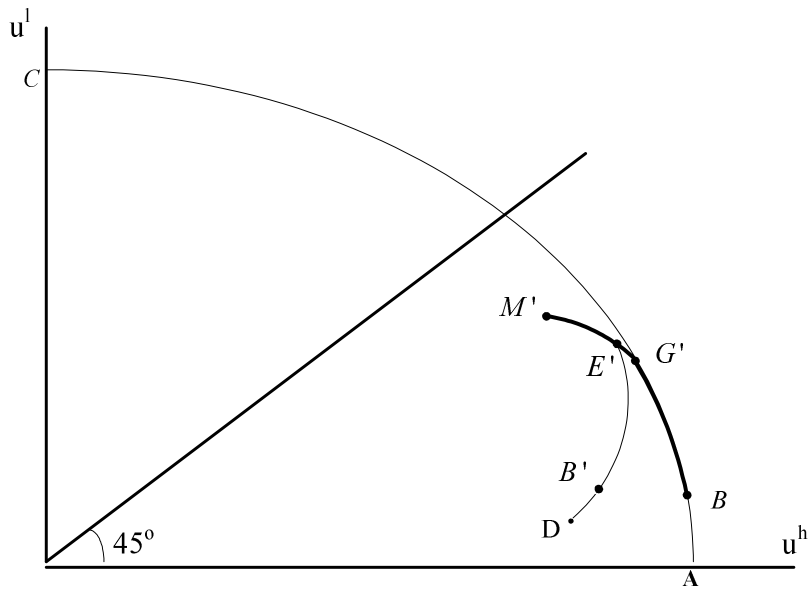

Figure 2: Economy's Pareto efficient frontier (AC), utility feasibility frontier under Besley and Coate (DE'), the first-best frontier under conditional cash transfers (BG') and the second-best frontier with conditional cash transfers ($G'E'M'$).

(cash transfer) and negative (lump-sum tax) values. The introduction of t changes the government's budget constraint to

$$T = \pi^l(p\overline{q} + t), \tag{4}$$

as long as only the poor participate in the public provision scheme. More importantly, it gives the government an additional degree of freedom to provide the poor with their efficient choice of quality. Specifically, introduce N to denote the "net transfer" to a poor individual. Then, for any given choice of N, set $\overline{q}(N)$ according to the poor's demand for q at an income level of $y^l + N$; that is, set

$$\overline{q}(N) \equiv q(p, y^l + N). \tag{5}$$

Additionally, set the accompanying cash transfer at

$$t(N) = \frac{N}{\pi^h} - p\overline{q}(N). \tag{6}$$

Observe also that to satisfy the government's budget constraint (4), T must be equal to[7]

$$T(N) = \frac{\pi^l}{\pi^h} N. \tag{7}$$

This procedure is depicted in Figure 3 (when $t(N) < 0$) and Figure 4 (when $t(N) > 0$). In both Figures, line IK indicates the poor's budget constraint in the absence of any transfers. The Figures also show the poor's optimal choice of q for a given value of N (point P), as well as the implementing values for $t(N)$ and $T(N)$.

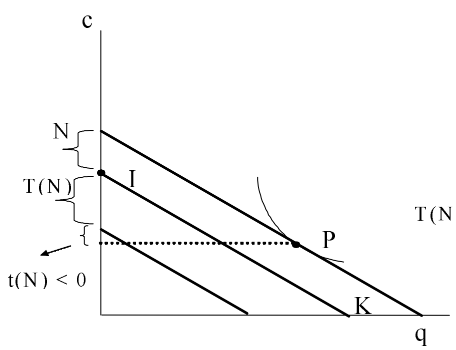

Figure 3: The efficient provision of q with a small level of net transfers N requiring t(N) < 0.

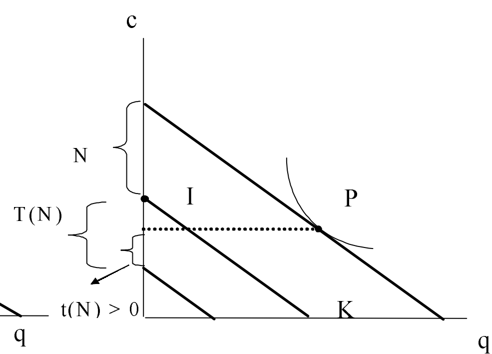

Figure 4: The efficient provision of q with a large level of net transfers N requiring t(N) > 0.

3.1 The limit of redistribution

At $N = 0$, a poor individual is offered his optimal choice of q given y^l and is taxed its full cost: $t(0) = -p\,\overline{q}(0)$ with $T(0) = 0$. There is thus no redistribution towards the poor and they remain at point B on Figure 1 and Figure 2. A positive N results in a

[7]With these values for $\overline{q}(N), t(N)$ and $T(N)$, we have

$$N = p\overline{q}(N) + t(N) - T(N),$$

which shows why we have termed N as "net transfer".

positive redistribution towards the poor which increases in size with N. The resulting utility of the poor is given by

$$u^l(N) = u\left(y^l - T(N) + t(N), \overline{q}(N)\right).$$

However, with $\overline{q}(N) = q\left(p, y^l + N\right)$ being the poor's optimal choice, unlike the Besley and Coate solution, the redistribution takes place on the first-best frontier. Specifically, we will have the loci BG in Figure 1 and BG' in Figure 2. Finally, observe that $\overline{q}(N), t(N)$ and $T(N)$ increase with the extent of redistribution, N.[8]

It is plain that there will be a redistribution from the rich to the poor only if the rich prefer not to participate in the public provision scheme. Thus the redistribution is limited by the incentive compatibility constraint of the rich:

$$u\left(y^h - T(N) + t(N), \overline{q}(N)\right) \leq v\left(p, y^h - T(N)\right) = u^h(N). \quad (8)$$

As with the Besley and Coate solution, it is the case here that the after-transfer utility of the rich always exceeds the after-transfer utility of the poor: The rich enjoy a utility level equal to or above $u\left(y^h - T(N) + t(N), \overline{q}(N)\right)$, while the utility of the poor is $u^l(N) = u\left(y^l - T(N) + t(N), \overline{q}(N)\right)$. With $y^h > y^l$, it then follows that $u^h(N) > u^l(N)$.

It is not just that the rich will always have a higher utility than the poor. Inequality (8) also sets a limit on the extent of net transfers to the poor. This is attained when (8) is satisfied as an equality. This situation is depicted in Figure 5 where the resulting maximal value of N is denoted by N_{FB}^{max}. The budget line for the rich is drawn *net* of $T(N_{FB}^{max})$, the lump-sum tax to be paid by everyone when $N = N_{FB}^{max}$. Point M shows the (c, q) bundle the rich will consume if they participate in the cash-cum-transfer program. Point R, on the other hand, shows the rich's optimal bundle if they do not participate. The two points lie on the same indifference curve for the rich, when $N = N_{FB}^{max}$. The

[8] This is obvious for $\overline{q}(N)$, from (5), and for $T(N)$, from (7). In the case of $t(N)$, the result follows from differentiating $c(p, y^l + N) = c^l(N) = y^l - T(N) + t(N) = y^l - (\pi^l/\pi^h)N + t(N)$ with respect to N.

corresponding maximal utility level of the poor, $u^l(N_{FB}^{max})$, is shown by point G in Figure 1 and point G' in Figure 2. Observe that, with this amount of net transfers, the conditional cash transfers, $t(N_{FB}^{max})$, as drawn in Figure 5, is negative; but it can also be positive.

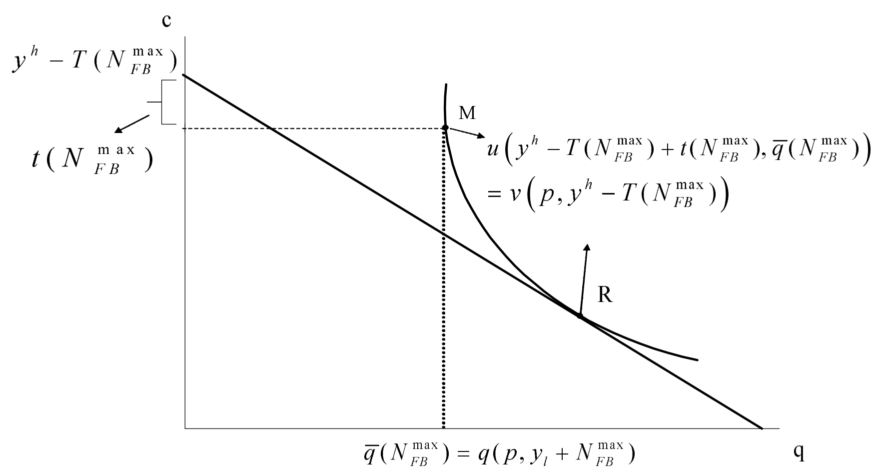

Figure 5: The rich is just indifferent between public provision and buying from the market (N= N_{FB}^{max}).

3.2 Comparison with the Besley and Coate solution

At $t = 0$, the solution to our system is identical to Q^* in Besley and Coate. Specifically, from (5)–(6), it follows that at $t(N) = 0$, $\overline{q}(N) = q\left(p, y^l + \pi^h p\overline{q}(N)\right)$, which is the characterization of Q^*. The two programs will then entail identical costs as well. To compare the two schemes when $t(N) \neq 0$, consider

Definition 2 *Define the "Besley-and-Coate-equivalent" cash-cum-in-kind transfer policy as the* $(t(N), \overline{q}(N))$ *bundle which satisfies equations (5)–(7), with* N *being set equal to the net expenditures on* $\overline{q}$ *under the Besley and Coate scheme.*

Let T_{BC} denote the solution for T, and N_{BC} the imputed value of N, under Besley and Coate. Thus, given any $\overline{Q}$, from (3), $T_{BC} = \pi^l p\overline{Q}$; and based on the definition of net transfers, $N_{BC} = p\overline{Q} - T_{BC}$. To construct the Besley-and-Coate-equivalent cash-cum-in-kind transfer policy, N must then be set equal to

$$N = N_{BC} = p\overline{Q} - T_{BC} = p\overline{Q} - \pi^l p\overline{Q} = \pi^h p\overline{Q}.$$

Note that we will then also have, from (7), $T(N_{BC}) = \pi^l N_{BC}/\pi^h = \pi^l p\overline{Q} = T_{BC}$. The corresponding values for $\overline{q}$ and t are found from (5)–(6) as $\overline{q}(N_{BC}) = q\left(p, y^l + \pi^h p\overline{Q}\right)$ and $t(N_{BC}) = p\overline{Q} - p\overline{q}(N_{BC})$. More interestingly, we have

Lemma 2 *Set $N = N_{BC} = \pi^h p\overline{Q}$ in the cash-cum-in-kind-transfer scheme to attain the Besley and Coate's equivalent transfer policy. It must then be the case that*

$$\overline{Q} \lesseqqgtr Q^* \;\;\Leftrightarrow\;\; \overline{q}(N_{BC}) \gtreqqless \overline{Q} \;\;\Leftrightarrow\;\; t(N_{BC}) \lesseqqgtr 0.$$

Intuitively, Lemma 2 tells us that, when a poor individual receives $p\overline{Q}$ in cash instead of $\overline{Q}$ in kind while continuing to pay the same lump-sum tax as before, his demand for q will be higher (lower) than $\overline{Q}$ if $\overline{Q}$ is less (more) than efficient. Moreover, if we were to provide him with his demanded value of q, we should levy a further tax on him (give him a cash rebate) to equalize the cost of the program to that under the Besley and Coate scheme.

We are now in a position to compare the extent of redistribution under conditional cash transfers with that under the Besley and Coate's model. The following proposition states our result in this regard. Recall that, by definition, the redistribution towards the poor is maximal under the Besley and Coate's scheme if $\overline{Q} = Q^{max}$.

Proposition 1 *Combining public provision with conditional cash transfers allows redistribution to take place on the first-best frontier. Moreover, if $Q^{max} > Q^*$, the maximum utility that the poor can attain will necessarily be higher when conditional cash transfers accompany public provision than when they do not. If $Q^{max} < Q^*$, conditional cash transfers may increase as well as decrease the extent of redistribution towards the poor.*

To gain an intuition for this result, first observe that one can always improve *the poor's* utility over its maximum level under the Besley and Coate scheme (i.e. when they are given Q^{max}) by switching to the Besley-and-Coate-equivalent conditional cash transfer policy (i.e. by setting $N = N_{BC}^{max} = \pi^h p Q^{max}$).[9] The key question then is when such a policy switch is feasible in the sense of being incentive compatible for the rich. This will be the case if $Q^{max} > Q^*$ because then $\overline{q}(N_{BC}^{max}) = q(p, y^l + N_{BC}^{max}) < Q^{max}$ and Q^{max} is incentive compatible. Observe that in this case one can increase N over N_{BC}^{max} (up to N_{FB}^{max}) and make the poor even more better-off. Figure 1 depicts this situation where point G shows $u^l(N_{FB}^{max})$ and point F shows $u^l(Q^{max})$; with $u^l(N_{FB}^{max}) > u^l(Q^{max})$. On the other hand, if $Q^{max} < Q^*$, $\overline{q}(N_{BC}^{max}) = q(p, y^l + N_{BC}^{max}) > Q^{max}$ and the maximum N that one can achieve under the stipulated conditional cash transfer policy, N_{FB}^{max}, must be lower than N_{BC}^{max} in order to be incentive compatible. Now, with $N_{FB}^{max} < N_{BC}^{max}$, either policy can yield a higher utility level for the poor. The point is that while transferring less resources to the poor would make them worse-off, allowing them to purchase their most-preferred quality level from the market would make them better-off.[10] Figure 2 depicts one of the two possible outcomes in this case, namely, $u^l(N^{max}) < u^l(Q^{max})$: Point G' shows $u^l(N_{FB}^{max})$ and point E' shows $u^l(Q^{max})$.

4 Second-best cash-cum-in-kind-transfers

It is plain that, *given* any desired level of redistribution, it is preferable to carry it out through the first-best mechanism of the previous section than the Besley and Coate's second-best scheme. However, there is a limit to this type of redistribution (N_{FB}^{max} in our first-best setup). In order to push the redistribution beyond N_{FB}^{max}, however, one must resort to a second-best mechanism. One such scheme is Besley and Coate's which under certain conditions, as specified in Proposition 1, can bring about a higher degree of redistribution than N_{FB}^{max}. This raises the question of specifying the optimal second-best

[9] Unless $Q^{max} = Q^*$ in which case the two policies will be identical.

[10] One can easily construct examples to show both cases are possible.

scheme. Interestingly, it will be the case that even when a high degree of desired redistribution calls for using a second-best scheme, conditional cash-cum-in-kind-transfers lead to Pareto superior outcomes (over the Besley and Coate's procedure).[11]

To characterize the optimal cash-cum-in-kind-transfers scheme, denote the social weight assigned to the utility of the poor by $0 \leq \gamma^l \leq 1$ and to the utility of the rich by $0 \leq \gamma^h \leq 1$, where $\gamma^l + \gamma^h = 1$. Then maximize $\gamma^l u^l + \gamma^h u^h$, with respect to $T, \overline{q}, t$, subject to the government's budget constraint,

$$T - \pi^l(\overline{q} + t) \geq 0, \tag{9}$$

and the incentive compatibility constraints of the rich (the "downward" constraint) and the poor (the "upward" constraint),

$$\begin{aligned} v(p, y^h - T) &\geq u(y^h - T + t, \overline{q}), & (10)\\ u(y^l - T + t, \overline{q}) &\geq v(p, y^l - T). & (11) \end{aligned}$$

Specifically, let μ, λ^h, and λ^l denote the Lagrangian multipliers associated with the government's budget constraint (9) and the incentive compatibility constraints (10)–(11). Define, for $j = h, l$, $u^j \equiv u(y^j - T + t, \overline{q}), v^j \equiv v(p, y^j - T)$ and $q^j \equiv q(p, y^j - T)$. Denote the partial derivatives of u^j, v^j and q^j with respect to any of their arguments by a subscript indicating that argument. One can then easily show, from the first-order

[11] One may alternatively characterize the first- and second-best policies using a mechanism design approach. Consider a direct revelation mechanism in which one offers two "bundles" to the consumers: T^h, intended for the rich, and (T^l, q^l), intended for the poor, with the incentive compatibility constraints,

$$\begin{aligned} v(p, y^h - T^h) &\geq u(y^h - T^l, q^l),\\ u(y^l - T^l, q^l) &\geq v(p, y^l - T^h). \end{aligned}$$

This is identical to the problem posed below where $T^h = T$, and $(T^l, q^l) = (T - t, \overline{q})$. Observe that this mechanism assumes that q^h is not publicly observable. If market purchases of q are observable at a personal level, the direct revelation mechanism will instead consist of the bundles (T^l, q^l) and (T^h, q^h). While the two direct mechanisms yield identical allocations, under the latter assumption, one is not restricted to rely on in-kind transfers for the purpose of implementation. A combination of differential consumptions taxes and differential income taxes will also suffice. (When allocation is first-best and there are no consumption taxes, the h-type cannot claim to be l-type, pay T^l, and then purchase q^h. The observability of personal purchases rules this out, as one can punish the cheaters severely.)

conditions of this problem, that[12]

$$\frac{u^l_{\overline{q}}}{u^l_c} = \frac{\mu\pi^l p + \lambda^h u^h_{\overline{q}}}{\mu\pi^l + \lambda^h u^h_c} = p + \frac{\lambda^h u^h_c (u^h_{\overline{q}}/u^h_c - p)}{\mu\pi^l + \lambda^h u^h_c}. \tag{12}$$

It follows from (12) that if $\lambda^h = 0$, $u^l_{\overline{q}}/u^l_c = p$ regardless of the value of λ^l. Thus optimal redistribution does not distort the consumption of the poor for a range of values of $\overline{q}$ that starts from a minimum value at which the poor decide to participate in the program (when $\lambda^l > 0$ and the incentive compatibility constraint of the poor just binds.)[13] Now, as the value of $\overline{q}$ increases from its minimum value, the upward incentive constraint slackens and $\lambda^l = 0$. Moreover, as long as the redistribution to the poor is not high enough to make the incentive compatibility of the rich binding, λ^h will also be equal to zero and again $u^l_{\overline{q}}/u^l_c = p$. This range of values of $\overline{q}$ thus coincides with the first-best outcomes of the previous section. The initial no policy solution (point B in Figures 1 and 2) corresponds to some value of γ^l when one maximizes $\gamma^l u^l + \gamma^h u^h$. As γ^l increases from this value, the poor's utility level increases along the first-best utility frontier attaining its highest value at the point where the incentive compatibility constraint of the rich starts to bind (point G in Figure 1 and point G' in Figure 2).

When the redistribution is high and the downward constraint of the rich is binding, the nature of the solution changes. This is characterized by setting $\lambda^h > 0$ in equation (12). Recall that the rich who do not participate in the cash-cum-in-kind-transfer

[12] The Lagrangian expression associated with this problem is written as

$$\mathcal{L} = \gamma^l u^l + \gamma^h v^h + \mu \left[T - \pi^l (p\overline{q} + t) \right] + \lambda^h (v^h - u^h) + \lambda^l (u^l - v^l).$$

The first-order conditions are,

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial t} &= \gamma^l u^l_c - \mu\pi^l - \lambda^h u^h_c + \lambda^l u^l_c = 0, \\
\frac{\partial \mathcal{L}}{\partial \overline{q}} &= \gamma^l u^l_{\overline{q}} - \mu\pi^l p - \lambda^h u^h_{\overline{q}} + \lambda^l u^l_{\overline{q}} = 0, \\
\frac{\partial \mathcal{L}}{\partial T} &= -\gamma^l u^l_c - \gamma^h v^h_y + \mu + \lambda^h(-v^h_y + u^h_c) + \lambda^l(-u^l_c + v^l_y) = 0.
\end{aligned}$$

[13] One can easily show that the two incentive compatibility constraints (10)–(11) cannot bind simultaneously so that $\lambda^l > 0 \Rightarrow \lambda^h = 0$.

scheme find the quality level of $\overline{q}$ to be less than efficient so that $u^h_{\overline{q}}/u^h_c > p$. It then follows from equation (12) that $u^l_{\overline{q}}/u^l_c > p$. This suggests a less than efficient provision of $\overline{q}$ for the poor in the second best.

The intuition for a downward distortion in quality comes from its impact on the rich's incentive compatibility constraint. The lower the quality level, the less inclined the rich will be to participate in the cash-cum-in-kind-transfer scheme and the higher will be the feasible degree of redistribution to the poor (the higher will be the quality level $\overline{q}$ at which their incentive compatibility constraint becomes binding).[14]

Figure 1 and Figure 2 depict the second-best utility feasibility frontier under cash-cum-in-kind-transfers. The boundary starts from point G in Figure 1, and point G' in Figure 2, where redistribution has reached its limit under the first-best cash-cum-in-kind-transfer scheme. Recall that these points are attained when γ^l reaches its maximum value consistent with a first-best solution. As γ^l increases from this point, we will have different second-best solution values for $\overline{q}, t, T, u^l$ and u^h (where $\gamma^l u^l + \gamma^h v^h$ is being maximized and the downward incentive constraint for the rich remains binding). Clearly, u^l attains its highest possible value when $\gamma^l = 1$. This is depicted by point M in Figure 1 and point M' in Figure 2, with GM and $G'E'M'$ each depicting the second-best frontier.

Observe also that because point G in Figure 1 entails a higher value for u^l as compared to point F (the maximum of u^l under the Besley and Coate's scheme), the second-best cash-cum-in-kind-transfer feasibility frontier (GM) has no point in common with the utility feasibility frontier under Besley and Coate's scheme (DEF). However, following Proposition 1, if maximal redistribution to the poor happens to be higher under Besley and Coate's scheme, as depicted by point E' versus point G' in Figure 2, then the second-best cash-cum-in-kind-transfer frontier $G'E'M'$ must pass through point E' on Besley and Coate's frontier (DE'). This follows because when the optimal value of t

[14]Observe, however, that the downward distortion is with respect to the poor's "implicit income" upon the transfer. That is, they would wish to purchase a good higher in quality than $\overline{q}$ *if* they were to receive the income equivalent of their net transfers in cash. As compared to what they may purchase for themselves in the absence of any transfers, the poor may very well be consuming a higher quality level of q.

(in the second-best cash-cum-in-kind-transfer scheme) happens to be $t = 0$, its solution coincides with that of Besley and Coate's.[15]

Proposition 2 summarizes the main points of this section.

Proposition 2 *The second-best cash-cum-in-kind transfer scheme requires the quality level of the publicly-provided good to be less than efficient for the poor. This scheme results in a maximum attainable welfare for the poor which will always be higher than its corresponding level under the Besley and Coate mechanism.*[16]

5 Discussion

While our model postulates only two groups of people, the possibility of effecting first-best redistribution through cash-cum-in-kind-transfers carries over to an economy with many poor and many rich groups of people. To achieve this, the indivisible good must be provided in as many variants as there are poor groups with each variant being combined with a different level of cash transfers. As long as none of the variants is sufficiently high in quality to attract the rich people, each variant can be offered at one of the poor groups' most-preferred quality level (i.e. at the level they would buy for themselves if they were to receive the value of the transfers in cash). On the other hand, if the bundles offered the poor groups are less in variety than the number of the poor groups, one cannot effect first-best redistribution through cash-cum-in-kind-transfers. Only, second-best redistribution will be possible. With n bundles and m groups of poor people, if $n < m$, *at least* $m - n$ poor groups cannot get their first-best allocations.

It is also interesting to point out that many developing countries have in recent years instituted in-kind transfer programs that award the recipients with some cash; see September 17, 2005 issue of the *Economist*. Two prominent examples are *Bolsa-Escola* in Brazil and PROGRESA in Mexico. Under *Bolsa-Escola*, families receive a monthly

[15] This common point cannot be any other point on DE' other than point E' because $G'M'$, being the second-best frontier when t is unconstrained, must (at least weakly) dominate Besley and Coate's frontier which restricts t to be equal to zero.

[16] Unless, by chance, the $G'E'M'$ frontier ends at point E'.

stipend for each child enrolled in public schools. Mexico's PROGRESA distributes nutritional supplements in addition to cash and is conditioned on school attendance as well as regular health checkups. Similar such programs exist in Bangladesh, Colombia, Honduras, Jamaica and Nicaragua. Education and health services are not the only examples of "low quality" goods that developing countries provide publicly for the benefit of their poor. Low quality foodstuff is another example. The government of Tunisia subsidizes the provision of such goods in a way that only the poor households will want to consume them. Another variant of these schemes links public provision of one good to another (rather than linking cash to goods). In Bangladesh and Philippines, for example, school children receive free food if they attend school. Mexico's PROGRESA also provides nutritional supplements to people who visit health centers. In these schemes, food works as a substitute for cash inducing the recipients to go to school and/or to have health checkups (who otherwise may not want to do so).

We do not mean to imply that these programs are organized to achieve efficient redistribution. Indeed, most of them cite not one (i.e. redistribution) but many social objectives. They include, in addition to redistribution, promotion of human capital accumulation among the poor, eliminating child labor practices, and providing a social safety net for the poor; see Rawlings and Rubio (2004). Some of these programs use means testing to screen the poor; some are offered non-universally, and the self-selection criterion is not the only mechanism used to separate targeted groups from the rest of the population. In certain instances, for example, location is used to choose the targeted municipality. Then a means test (Brazil and Mexico), or a proxy-means test (Nicaragua, Colombia), is used to identify the targeted group within that municipality.[17] Many such

[17] A number of recent empirical papers examine the efficacy of different mechanisms for achieving targeting [e.g., means testing, geographic targeting, demographic targeting, self-selection based on work requirement (public work at low wages), and self-selection based on consumption (food subsidies and cash as in our model)]. See, among others, Bourguignon *et al.* (2002), Das *et al.* (2004), and Galasso and Ravallion (2004). The results seem to suggest that the programs have generally been effective in increasing school attendance and health checkups but less effective as a means of redistribution. They also suggest, unsurprisingly, that means testing, when possible, and geographic targeting might be superior to targeting via self selection.

program also appear to operate on a belief that the recipients do not "value" the publicly provided goods to the extent that "they should." They offer cash (rather than spend more on the goods in question) to induce the recipients to consume those goods.

6 Concluding remarks

The redistributive capability of the public sector is determined by the nature of the information that it has. When the government does not know who is poor and who is rich, and has to rely on the information revealed by the people themselves, its ability to redistribute is severely limited. Besley and Coate (1991) have shown that when the rich consume a higher quality variant of an indivisible good and the poor a lower quality variant, a certain degree of redistribution will be possible if the government provides the low quality product to whoever wants it at no charge and finance it with a head tax. The method they propose entail a welfare loss because in general the low quality product is provided at a quality different from the poor's desired level (if they were to get the cash equivalent of the transfer).

This paper has argued that the welfare loss inherent in Besley and Coate's procedure can be avoided, if the desired degree of redistribution is not very "high," by linking the provision of the indivisible good to the payment of a lump-sum tax or the receipt of a rebate. Intuitively, the double provision of cash and in-kind transfers will act as a system of differential lump-sum taxes for the poor and the rich, thus achieving first-best redistributions. The paper has further shown that even when the desired degree of redistribution is high, the introduction of cash into the Besley and Coate's scheme increases the extent of feasible redistribution within the economy. The second-best version of our procedure entails, of course, a welfare loss. However, this will be less than the welfare loss incurred in the Besley and Coate's scheme for the same amount of redistribution. Moreover, it will achieve a degree of redistribution that exceeds what is possible under the first-best version of our model.

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Appendix

A1 Proofs

Proof of Lemma 1: To prove part (i), observe that $\widehat{Q}$ maximizes $u\left(y^l - T, \overline{Q}\right) = u\left(y^l - \pi^l p\overline{Q}, \overline{Q}\right) = u\left(y^l - p\overline{Q} + \pi^h p\overline{Q}, \overline{Q}\right)$, while Q^* maximizes $u\left(y^l - p\overline{Q} + \pi^h pQ^*, \overline{Q}\right)$. Now comparison of these two expressions reveals

$$u\left(y^l - p\overline{Q} + \pi^h p\overline{Q}, \overline{Q}\right) \lesseqqgtr u\left(y^l - p\overline{Q} + \pi^h pQ^*, \overline{Q}\right) \Leftrightarrow \overline{Q} \lesseqqgtr Q^*.$$

Consequently, $u\left(y^l - p\overline{Q} + \pi^h p\overline{Q}, \overline{Q}\right)$ crosses $u\left(y^l - p\overline{Q} + \pi^h pQ^*, \overline{Q}\right)$ at $\overline{Q} = Q^*$ with a positive slope, and $\widehat{Q} > Q^*$. Second, observe that $\widetilde{Q}$ is defined as the (minimum) value of $\overline{Q}$ at which, $u(y^l - T, \overline{Q}) = v(p, y^l)$. This implies that $\widetilde{Q} < q(p, y^l)$ so that at $\widetilde{Q}$, $u_q/u_c > p$. With u_q/u_c being decreasing in q, it follows that $\widetilde{Q} < Q^*$. Finally, we have at $\overline{Q} = Q^{min}$,

$$u(y^l - T, Q^{min}) = v(p, y^l - T), \tag{A1}$$

and at $\overline{Q} = \widetilde{Q}$,

$$u(y^l - T, \widetilde{Q}) = v(p, y^l). \tag{A2}$$

Clearly, the value of the right-hand side of (A1) is less than the value of the right-hand side of (A2). Consequently, the left-hand side of (A1) will also be smaller than the left-hand side of (A2), implying that $Q^{min} < \widetilde{Q}$.

To prove part (ii), first observe that if y^h is "very close" to y^l, Q^{max} will be "very close" to Q^{min} and one may not even be able to separate the types. Under this circumstance, and given that $Q^{min} < \widetilde{Q}$, we have $Q^{max} < \widetilde{Q}$. Secondly, while Q^{max} increases with $y^h - y^l$, $\widehat{Q}$ is independent of y^h (although it does depend on the relative size of the rich to the poor in the total population). This means that even if $\overline{Q}$ is set above $\widehat{Q}$, it may still be not high enough for the rich to participate in the public provision program. Then, $Q^{max} > \widehat{Q}$.

Proof of Lemma 2: When a poor individual receives, under the Besley and Coate scheme, $\overline{Q} \lesseqqgtr Q^*$ (see Figure 2 for $\overline{Q} < Q^*$ and Figure 1 for $\overline{Q} > Q^*$), his marginal rate of substitution between quality and the numeraire at $\overline{Q}$ will be equal to $u_q/u_c \gtreqqless p$. On the other hand, if instead he were to receive $p\overline{Q}$ in cash (while continuing to pay $\pi^l p\overline{Q}$ in taxes so that his net transfer is $\pi^h p\overline{Q}$), he would demand $\overline{q}(N_{BC}) = q(p, y^l + \pi^h p\overline{Q})$ such that $u_q/u_c = p$. Consequently, in going from receiving $\overline{Q}$ in kind to receiving $p\overline{Q}$ in cash and choosing his own q, the poor individual's marginal rate of substitution between quality and the numeraire decreases/remains the same/increases (i.e. his demand for q increases/remains the same/decreases) depending on $\overline{Q} \lesseqqgtr Q^*$. That is, $\overline{q}(N_{BC}) = q(p, y^l + \pi^h p\overline{Q}) \gtreqqless \overline{Q}$ according to $\overline{Q} \lesseqqgtr Q^*$. The result on $t(N_{BC})$ then follows immediately from (6).

Proof of Proposition 1: Consider the Besley-and-Coate-equivalent conditional cash transfer policy for Q^{max} by setting $N = N_{BC}^{max} = \pi^h pQ^{max}$. The poor will be (weakly) better-off under this policy as compared to the original Besley and Coate solution. To see this, recall from Lemma 2 that resulting $\overline{q}(N_{BC}^{max}) = q(p, y^l + N_{BC}^{max})$ will be $\lesseqqgtr$ than Q^{max} according to $Q^{max} \gtreqqless Q^*$. If $Q^{max} > Q^*$, the *lower* $\overline{q}(N_{BC}^{max})$ offered (as compared to $\overline{Q} = Q^{max}$) makes the poor better off; if $Q^{max} < Q^*$, the *higher* $\overline{q}(N_{BC}^{max})$ offered makes the poor better off; if $Q^{max} = Q^*$, there will be no change in $\overline{q}(N_{BC}^{max})$ and the poor remain just as well-off (as compared to their position under Besley and Coate).

Next, one must check if the proposed policy change is feasible; that is, if it satisfies the incentive compatibility constraint (8) for the given values of $N = N_{BC}^{max}, t(N_{BC}^{max})$ and $T(N_{BC}^{max})$. To examine this, recall that Q^{max} satisfies (2), the incentive compatibility constraint for the rich under Besley and Coate, as an equality so that we have

$$u\left(y^h - T_{BC}^{max}, Q^{max}\right) = v\left(p, y^h - T_{BC}^{max}\right), \tag{A3}$$

where $T_{BC}^{max} = \pi^l pQ^{max}$. Now, with $T(N_{BC})$ under conditional cash transfers taking the same value as T_{BC}, the value of the right-hand side of (8) will be equal to that of the right-hand side of (A3). Turning to the left-hand side of (8), it will be less than the left-hand side of (A3) when $\overline{q}(N_{BC}^{max}) = q(p, y^l + N_{BC}^{max}) < Q^{max}$ (depicted in Figure 1) and more than it when $\overline{q}(N_{BC}^{max}) = q(p, y^l + N_{BC}^{max}) > Q^{max}$ (depicted in Figure 2). Consequently, the policy is feasible in the former case but not in the latter case.

Finally, observe that when $\overline{q}(N_{BC}^{max}) = q(p, y^l + N_{BC}^{max}) < Q^{max}$ and (8) is satisfied as a strict inequality, one can increase N over N_{BC}^{max} and make the poor even more well off. This proves that when $Q^{max} > Q^*$, one can always make the poor better off with a conditional cash transfer policy as compared to giving them Q^{max} under the Besley and Coate scheme. Secondly, when $Q^{max} < Q^*$so that $\overline{q}(N_{BC}^{max}) = q(p, y^l + N_{BC}^{max}) > Q^{max}$, the maximum N that one can achieve under the stipulated conditional cash transfer policy, N_{FB}^{max}, must be lower than N_{BC}^{max} in order to be incentive compatible. Now, transferring N_{FB}^{max} through conditional cash transfers instead of N_{BC}^{max} under Besley and Coate, with $N_{FB}^{max} < N_{BC}^{max}$, has two opposite implications. On the one hand, by transferring less resources to the poor, it would make them worse-off. On the other hand, by allowing the poor to purchase their most-preferred quality level from the market, it would make them better-off. The net effect is ambiguous and can go either way. An example at the end of the Appendix establishes this.

A2 Many income types

Assume there are H groups of peoples with incomes $y^1 < y^2 < \ldots < y^m < y^{m+1} < \ldots < y^H$, with the first m groups being designated as "poor" and the second $H - m$ groups as rich. By "designated" we mean the number of groups of people the government wishes to redistribute to. Let π^l denote the proportion of the l-type poor ($l = 1, 2, \ldots, m$), and π^h the proportion of the h-type rich ($h = m + 1, m + 2, \ldots, H$), in the population so that $\sum_{l=1}^{m} \pi^l + \sum_{h=m+1}^{H} \pi^h = 1$.

Providing one variety of q: Assume that only one variety of the indivisible good, coupled with one value of t, is offered to everyone. To characterize the utility possibility frontier, one determines $\overline{q}, t$ and T in order to maximize

$$\sum_{l=1}^{m} \gamma^l u(y^l - T + t, \overline{q}) + \sum_{h=m+1}^{H} \gamma^h v(p, y^h - T),$$

where γ^j's are positive constants such that $\sum_{j=1}^{H} \gamma^j = 1$, subject to the government's budget constraint

$$T - (p\overline{q} + t) \sum_{l=1}^{m} \pi^l \geq 0, \tag{A4}$$

and the "appropriate" incentive compatibility constraints for the m poor-type and the $H - m$ rich-type groups as discussed below.

Despite the existence of many groups of poor and rich people, our earlier assumption on the normality of q implies that we need only to consider two incentive compatibility constraints: that of the most wealthy poor and the one for the least wealthy rich. The following lemma establishes this point.

Lemma A1 *The single-crossing property. If the incentive compatibility constraint $v(p, y - T) \geq u(y - T + t, \overline{q})$ is binding for an individual with income y, it must be slack for all individuals with incomes greater than y: $v(p, z - T) > u(z - T + t, \overline{q})$, when $z > y$. Similarly, if $u(y-T+t, \overline{q}) \geq v(p, y-T)$ is binding for y, then $u(z-T+t, \overline{q}) > v(p, z-T)$, for $z < y$.*

Proof. Consider the downward incentive compatibility constraint (10),

$$\Delta \equiv v(p, y - T) - u(y - T + t, \overline{q}) \geq 0.$$

Substitute for t from the government's budget constraint (A4) into the above expression to arrive at

$$\Delta = v(p, y - T) - u\left(y + \frac{\sum \pi^h}{\sum \pi^l} T - p\overline{q}, \overline{q}\right). \tag{A5}$$

Partially differentiate (A5) with respect to y. We have,

$$\frac{\partial \Delta}{\partial y} = v_y(p, y - T) - u_c(y + T \sum \pi^h / \sum \pi^l - p\overline{q}, \overline{q}). \tag{A6}$$

Evaluate (A6) at the value for y that makes (10) binding so that $(c(p, y-T), q(p, y-T))$ and $(y+T\sum \pi^h / \sum \pi^l - p\overline{q}, \overline{q})$ are on the same indifference curve. Observe that $\overline{q}$ is less than efficient for the individual with income y so that $q(p, y-T) > \overline{q}$. To determine the sign of $v_y - u_c$ along an indifference curve as q increases, differentiate $u_c(c, q)$ with respect to q. We have,

$$\frac{\partial u_c}{\partial q} = u_{cc}\frac{\partial c}{\partial q} + u_{cq} = \frac{u_{cq}u_c - u_{cc}u_q}{u_c} > 0,$$

where the sign follows from normality of q. Consequently, $\partial \Delta / \partial y > 0$ which proves the first part of the Lemma. A similar argument establishes the second part and completes the proof. ∎

Armed with this lemma, the incentive compatibility constraints in this case are,

$$\begin{aligned} v(p, y^{m+1} - T) &\geq u(y^{m+1} - T + t, \overline{q}), && \text{(A7)} \\ u(y^m - T + t, \overline{q}) &\geq v(p, y^m - T). && \text{(A8)} \end{aligned}$$

The Lagrangian expression for this problem can then be written as

$$\begin{aligned} \mathcal{L} &= \sum_{l=1}^{m} \gamma^l u(y^l - T + t, \overline{q}) + \sum_{h=m+1}^{H} \gamma^h v(p, y^h - T) + \mu \left[T - (p\overline{q} + t) \sum_{l=1}^{m} \pi^l \right] \\ &+ \lambda^{m+1} \left[v(p, y^{m+1} - T) - u(y^{m+1} - T + t, \overline{q}) \right] + \lambda^m \left[u(y^m - T + t, \overline{q}) - v(p, y^m - T) \right], \end{aligned}$$

with the first-order conditions,

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial t} &= \sum_l \gamma^l u_c^l - \mu \sum_l \pi^l - \lambda^{m+1} u_c^{m+1} + \lambda^m u_c^m = 0, && \text{(A9)} \\ \frac{\partial \mathcal{L}}{\partial \overline{q}} &= \sum_l \gamma^l u_{\overline{q}}^l - p\mu \sum_l \pi^l - \lambda^{m+1} u_{\overline{q}}^{m+1} + \lambda^m u_{\overline{q}}^m = 0, && \text{(A10)} \\ \frac{\partial \mathcal{L}}{\partial T} &= -\sum_l \gamma^l u_c^l - \sum_h \gamma^h v_y^h + \mu - \lambda^{m+1}(v_y^{m+1} - u_c^{m+1}) - \lambda^m (u_c^m - v_y^m) = 0. && \text{(A11)} \end{aligned}$$

Now, given that $u_{\overline{q}}^l / u_c^l$ increases with y^l (because c^l increases and $\overline{q}$ remains same), one cannot have $u_{\overline{q}}^l / u_c^l = p$ for all values of l (if $m > 1$). Consequently, one cannot have first-best redistribution in this case. Moreover, it follows from equations (A9)–(A10) that

$$\frac{\sum_l \gamma^l u^l_{\overline{q}} + \lambda^m u^m_{\overline{q}}}{\sum_l \gamma^l u^l_c + \lambda^m u^m_c} = \frac{\mu p \sum_l \pi^l + \lambda^{m+1} u^{m+1}_{\overline{q}}}{\mu \sum_l \pi^l + \lambda^{m+1} u^{m+1}_c}. \tag{A12}$$

This equation tells us that $u^l_{\overline{q}}/u^l_c = p$, if there is one group of poor people (so that $l = 1 = m$) and $\lambda^{m+1} = \lambda^2 = 0$.

Providing many varieties of q: The above discussion alerts us to the possibility of first-best redistribution if one offers as many different bundles of quality and cash as there are poor groups of individuals. Let the government offer q at differentiated quality levels. Specifically, let $\overline{q}_l$ and t_l denote the in-kind and conditional cash transfers to the l-type poor. On the basis of Lemma A1,[18] one can now limit the number of incentive compatibility constraints that has to be taken into account. It will be sufficient to ensure that an individual with income level y^{k+1} does not participate in a cash-cum-in-kind-transfer scheme characterized by $(\overline{q}_k, t_k)$ but that a person with income y^k does (for all $k = 1, 2, \ldots, H-1$.) If these conditions are satisfied, no individuals with incomes greater than y^{k+1} would participate in the $(\overline{q}_k, t_k)$ program. And if the person with y^k chooses $(\overline{q}_k, t_k)$, he will not choose the bundle that is meant for individuals with higher income levels.

To characterize the utility possibility frontier then, one has to determine $\overline{q}_l, t_l$, for $l = 1, 2, \ldots, m$, and T in order to maximize

$$\sum_{l=1}^{m} \gamma^l u(y^l - T + t_l, \overline{q}_l) + \sum_{h=m+1}^{H} \gamma^h v(p, y^h - T),$$

subject to the government's budget constraint,

$$T - \sum_{l=1}^{m} \pi^l (p\overline{q}_l + t_l) \geq 0,$$

[18] This lemma was proved for one variety of q. But a similar result holds with many varieties of q as well.

and the incentive compatibility constraints,

$$
\begin{aligned}
u(y^l - T + t_l, \overline{q}_l) &\geq u(y^l - T + t_{l-1}, \overline{q}_{l-1}), \qquad l = 2, 3, \ldots, m, \\
v(p, y^{m+1} - T) &\geq u(y^{m+1} - T + t_m, \overline{q}_m), \\
u(y^l - T + t_l, \overline{q}_l) &\geq u(y^l - T + t_{l+1}, \overline{q}_{l+1}), \qquad l = 2, 3, \ldots, m, \\
u(y^m - T + t_m, \overline{q}_m) &\geq v(p, y^m - T).
\end{aligned}
$$

Let λ_l denote the downward incentive compatibility constraint for an individual with income l choosing the bundle $(\overline{q}_l, t_l)$ over the bundle $(\overline{q}_{l-1}, t_{l-1})$ for $l = 2, 3, \ldots, m$, with λ^{m+1} corresponding to not participating in the program (and thus enjoying a utility level of $v(p, y^{m+1} - T)$ over the choice of $(\overline{q}_m, t_m)$. Similarly, let δ_l $(l = 1, 2, \ldots, m-1)$ denote the upward incentive compatibility constraint for an individual with income l choosing the bundle $(\overline{q}_l, t_l)$ over the bundle $(\overline{q}_{l+1}, t_{l+1})$, with δ^m corresponding to the choice of $(\overline{q}_m, t_m)$ over not participating (and thus enjoying a utility level of $v(p, y^m - T)$. Denote the utility of a person with income y^k who chooses the bundle $(\overline{q}_j, t_j)$ by $u^{k,j}$.

The Lagrangian expression for this problem is

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{l=1}^{m} \gamma^l u(y^l - T + t_l, \overline{q}_l) + \sum_{h=m+1}^{H} \gamma^h v(p, y^h - T) + \mu \left[T - \sum_{l=1}^{m} \pi^l (p\overline{q}_l + t_l) \right] \\
+ \; & \sum_{l=2}^{m} \lambda^l \left[u(y^l - T + t_l, \overline{q}_l) - u(y^l - T + t_{l-1}, \overline{q}_{l-1}) \right] + \lambda^{m+1} \left[v(p, y^{m+1} - T) \right. \\
- \; & \left. u(y^{m+1} - T + t_m, \overline{q}_m) \right] + \sum_{l=1}^{m-1} \delta^l \left[u(y^l - T + t_l, \overline{q}_l) - u(y^l - T + t_{l+1}, \overline{q}_{l+1}) \right] \\
+ \; & \delta^m \left[u(y^m - T + t_m, \overline{q}_m) - v(p, y^m - T) \right].
\end{aligned}
$$

Rearranging the terms, one can rewrite the Lagrangian expression as

$$\begin{aligned}
\mathcal{L} &= (\gamma^1+\delta^1)u(y^1-T+t_1,\overline{q}_1) - \lambda^2 u(y^2-T+t_1,\overline{q}_1) + \mu T - \mu\pi^1(p\overline{q}_1+t_1) \\
&+ \lambda^{m+1} v(p, y^{m+1}-T) - \delta^m v(p,y^m-T) + \sum_{h=m+1}^{H} \gamma^h v(p,y^h-T) \\
&+ \sum_{l=2}^{m} \Big[(\gamma^l+\lambda^l+\delta^l) u(y^l-T+t_l,\overline{q}_l) - \lambda^{l+1} u(y^{l+1}-T+t_l,\overline{q}_l) \\
&- \delta^{l-1} u(y^{l-1}-T+t_l,\overline{q}_l) - \mu\pi^l(p\overline{q}_l+t_l) \,\Big].
\end{aligned}$$

The first-order conditions are, for all $l = 2, 3, \ldots, m$,

$$\frac{\partial \mathcal{L}}{\partial t_1} = (\gamma^1+\delta^1)u_c^1 - \lambda^2 u_c^{2,1} - \mu\pi^1 = 0, \tag{A13}$$

$$\frac{\partial \mathcal{L}}{\partial t_l} = (\gamma^l+\lambda^l+\delta^l)u_c^l - \lambda^{l+1} u_c^{l+1,l} - \delta^{l-1} u_c^{l-1,l} - \mu\pi^l = 0, \tag{A14}$$

$$\frac{\partial \mathcal{L}}{\partial \overline{q}_1} = (\gamma^1+\delta^1)u_{\overline{q}}^1 - \lambda^2 u_{\overline{q}}^{2,1} - \mu p\pi^1 = 0, \tag{A15}$$

$$\frac{\partial \mathcal{L}}{\partial \overline{q}_l} = (\gamma^l+\lambda^l+\delta^l)u_{\overline{q}}^l - \lambda^{l+1} u_{\overline{q}}^{l+1,l} - \delta^{l-1} u_{\overline{q}}^{l-1,l} - \mu p\pi^l = 0, \tag{A16}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial T} &= -(\gamma^1+\delta^1)u_c^1 + \lambda^2 u_c^{2,1} + \mu - \lambda^{m+1} v_y^{m+1} + \delta^m v_y^m - \sum_h \gamma^h v_y^h \\
&- \sum_{l=2}^{m} \Big[(\gamma^l+\lambda^l+\delta^l)u_c^l - \lambda^{l+1} u_c^{l+1,l} - \delta^{l-1} u_c^{l-1,l} \Big] = 0.
\end{aligned} \tag{A17}$$

Dividing (A15) by (A13) and (A16) by (A14) yield, for all $l = 2, 3, \ldots, m$,

$$\begin{aligned}
\frac{u_{\overline{q}}^1}{u_c^1} &= \frac{p\mu\pi^1 + \lambda^2 u_{\overline{q}}^{2,1}}{\mu\pi^1 + \lambda^2 u_c^{2,1}} = p + \frac{\lambda^2 u_c^{2,1}(u_{\overline{q}}^{2,1}/u_c^{2,1} - p)}{\mu\pi^1 + \lambda^2 u_c^{2,1}}, \\
\frac{u_{\overline{q}}^l}{u_c^l} &= \frac{p\mu\pi^l + \lambda^{l+1} u_{\overline{q}}^{l+1,l} + \delta^{l-1} u_{\overline{q}}^{l-1,l}}{\mu\pi^l + \lambda^{l+1} u_c^{l+1,l} + \delta^{l-1} u_c^{l-1,l}} \\
&= p + \frac{\lambda^{l+1} u_c^{l+1,l}(u_{\overline{q}}^{l+1,l}/u_c^{l+1,l} - p) + \delta^{l-1} u_c^{l-1,l}(u_{\overline{q}}^{l-1,l}/u_c^{l-1,l} - p)}{\mu\pi^l + \lambda^{l+1} u_c^{l+1,l} + \delta^{l-1} u_c^{l-1,l}}.
\end{aligned}$$

If none of the incentive compatibility constraints are binding, $\lambda^{l+1} = \delta^l = 0$, for all $l = 1, 2, \ldots, m$, and we have a first-best solution.[19] In the second-best solution, one cannot

[19] A first-best solution can be constructed directly in an analogous manner to the two-income type

a priori determine the direction of distortion in the consumption of the indivisible good by the various groups of poor people. It will all depend on which incentive compatibility constraints are binding and which are not.

There arises an additional complication here if one wishes also to redistribute between the various rich groups. This seem a natural concern when there are "rich" people of various degrees. Of course, one way of doing this is to designate only one group of individuals as rich, with the aim of redistributing from this group to all other groups. This would require the government to offer $H-1$ different $\overline{q}_l, t_l$ bundles. Leaving this case aside, the possibility of first-best redistribution between the rich groups hinges crucially on the type of information available to the government. If quality levels are publicly observable, one can impose a nonlinear tax conditioned on the purchase of q^h ($h = m+1, m+2, \ldots, H$). However, if this were the case, one could use the same scheme to effect first-best redistribution between the rich and poor as well. Other tax schemes, like linear commodity taxation, which does not rest on public observability of

---

case. Let N_{max} denote the value of $N > 0$ that satisfies the downward incentive compatibility constraint of an individual with income y^{m+1},

$$v\left(p, y^{m+1} - \frac{\sum \pi^l}{\sum \pi^h} N\right) \geq u\left(y^{m+1} + N - pq(p, y^m + N), q(p, y^m + N)\right),$$

as an equality. Then set, for any $0 < N < N_{max}$, and all $l = 1, 2, \ldots, m$,

$$\overline{q}_l = q(p, y^l + N), \qquad t_l = \frac{N}{\sum \pi^h} - p\overline{q}_l, \qquad T = \frac{\sum \pi^l}{\sum \pi^h} N.$$

With all poor groups receiving the same net transfer N, every group will be happiest with his own bundle of $(t_l, \overline{q}_l)$ with $\overline{q}_l$ being the bundle l would buy for himself if he were to receive N in cash. The bundles also satisfy the government's budget constraint, and we have a first-best allocation.

Observe also that other first-best allocations will be made feasible by allowing N to vary for different people. Redefine N_{max} accordingly, and let $0 < N^m < N^{m-1} < \ldots < N^1 < N_{max}$ denote the net transfers to poor groups with incomes $y^m > y^{m-1} > \ldots > y^1$. One needs to set, for $l = 1, 2, \ldots, m$,

$$\overline{q}_l = q(p, y^l + N^l), \qquad t_l = N^l + \frac{\sum \pi^l N^l}{\sum \pi^h} - p\overline{q}_l, \qquad T = \frac{\sum \pi^l N^l}{\sum \pi^h},$$

such that

$$u(y^l - T + t_l, \overline{q}_l) \geq u(y^l - T + t_{l-1}, \overline{q}_{l-1}).$$

Observe also that, given the normality of q, if l does not choose the $(\overline{q}_{l-1}, t_{l-1})$ bundle, he will not choose the bundles for groups $l-2, l-1, \ldots, 1$ either.

consumption levels, coupled with identical lump-sum rebates, can achieve some degree of redistribution between the rich groups but they will be second best.

Proposition A1 summarizes our results in this case.

Proposition A1 *Assume there are many groups of poor and many groups of rich people. Then:*

(i) If only one variant of the indivisible good is provided publicly, the cash-cum-in-kind-transfer scheme is second-best (although it will dominate the Besley and Coate scheme).

(ii) If the indivisible good is provided publicly in as many variants as the designated number of poor groups, with each variant being combined with a different level of conditional cash transfers, the cash-cum-in-kind-transfer scheme consists of first- and second-best solutions.

A3 An Example

Assume there are equal numbers of rich and poor persons ($\pi^h = \pi^l = .5$) who have identical Cobb-Douglas preferences given by

$$u = (cq)^{0.25}.$$

Further, set $p = 1$. This utility function yields the demand functions $c = q = 0.5y$, for any net income level y.

1. *Besley and Coate solution:* It is simple to show that,

$$Q^{min} = \frac{2y^l}{9};\; Q^{max} = \frac{2y^h}{9};\; \widetilde{Q} = \left(1 - \sqrt{0.5}\right) y^l;\; Q^* = \frac{2y^l}{3};\; \widehat{Q} = y^l,$$

with

$$u^l(\overline{Q}) = \left[(y^l - 0.5\overline{Q})\overline{Q}\right]^{0.25}. \tag{A18}$$

Observe that $Q^{max} < \widetilde{Q}$ if $y^h < 4.5\left(1 - \sqrt{0.5}\right) y^l$, and $Q^{max} > \widehat{Q}$ if $y^h > 4.5y^l$. We also have $y^h \gtreqless 3y^l \Rightarrow Q^{max} \gtreqless Q^*$.

2. *First-best conditional cash transfers:* We now have,

$$\overline{q}(N) = q(p, y^l{+}N) = 0.5(y^l{+}N); \;\; t(N) = 0.5(3N{-}y^l); \;\; T(N) = N; \;\; N_{FB}^{max} = \frac{(y^h - y^l)^2}{4y^h},$$

with

$$u^l(N) = \Big[0.5(y^l + N)\Big]^{0.5}. \tag{A19}$$

Comparing the maximum attainable utility levels for the poor under the first-best conditional cash transfers (i.e. when $N = N_{FB}^{max} = (y^h - y^l)^2/4y^h$) and under the Besley and Coate scheme (i.e. when $\overline{Q} = Q^{max} = 2y^h/9$), we have

$$u^l(N_{FB}^{max}) - u^l(Q^{max}) = \left[\frac{y^l}{2} + \frac{(y^h - y^l)^2}{8y^h}\right]^{0.5} - \left[\left(y^l - \frac{y^h}{9}\right)\frac{2y^h}{9}\right]^{0.25}.$$

One can easily establish that

$$\begin{array}{lrcl}
\text{Case (i):} & y^h > 3y^l & \Rightarrow & u^l(N_{FB}^{max}) > u^l(Q^{max}), \\
\text{Case (ii):} & y^h = 3y^l & \Rightarrow & u^l(N_{FB}^{max}) = u^l(Q^{max}), \\
\text{Case (iii):} & 1.42002y^l < y^h < 3y^l & \Rightarrow & u^l(N_{FB}^{max}) < u^l(Q^{max}), \\
\text{Case (iv):} & y^l < y^h < 1.42002y^l & \Rightarrow & u^l(N_{FB}^{max}) > u^l(Q^{max}).
\end{array}$$

The first inequality shows that, as demonstrated formally in the text, whenever $Q^{max}(= 2y^h/9) > Q^*(= 2y^l/3)$, one attains a higher maximum utility for the poor under cash transfers. Case (ii) shows that the two solutions are identical when $Q^{max} = Q^*$. The last two inequalities indicate that when $Q^{max} < Q^*$, either policy may result in the maximum attainable utility for the poor.

3. *Second-best conditional cash transfers:* Denote the solutions under the second-best conditional cash transfers by SB. Assume that $y^l = 1$, and generate an example of the above four cases by setting values for y^h that are greater than 3, equal to 3, between 1.42 and 3, and between 1 and 1.42. The *maximum* attainable utility levels for the poor, under Besley and Coate scheme, first-best conditional cash transfers and second-best conditional cash transfers, are then calculated as:

- Case (i), $y^h = 5$:

$$u^l(Q^{max}) = 0.838389, \quad u^l(N_{FB}^{max}) = 0.948683, \quad (u^l)_{SB}^{max} = 1.00576.$$

- Case (ii), $y^h = 3$:

$$u^l(Q^{max}) = 0.816497, \quad u^l(N_{FB}^{max}) = 0.816497, \quad (u^l)_{SB}^{max} = 0.850719.$$

- Case (iii), $y^h = 2$:

$$u^l(Q^{max}) = 0.766776, \quad u^l(N_{FB}^{max}) = 0.750, \quad (u^l)_{SB}^{max} = 0.768101.$$

- Case (iv), $y^h = 1.4$:

$$u^l(Q^{max}) = 0.715932, \quad u^l(N_{FB}^{max}) = 0.717137, \quad (u^l)_{SB}^{max} = 0.723182.$$

It is also interesting to note that when $y^h = 2$ (i.e. in case (iii)), if one sets $\gamma^l = 0.811446$, then $u^l_{SB} = 0.766776$ (as opposed to $u^l_{SB} = 0.768101$ which is attained when $\gamma^l = 1$). This coincides with the Besley and Coate solution where $u^l(Q^{max})$ attains its *maximal* value also at 0.766776.