Endogenous altruism, redistribution, and long term care*

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Abstract

This paper studies public provision of long term care insurance in a world in which family assistance is (i) uncertain and (ii) endogenous depending on the time parents spend raising their children. Public benefits will be paid in case of disability but cannot be combined with self-insurance or family aid. The benefits are provided equally to all recipients and financed by a proportional payroll tax. The paper shows that tax distortions imply full insurance is undesirable. It characterizes the optimal tax and identifies the elements that determine its size. Of crucial importance are the extent of under-insurance, the effect of the tax on the probability of altruism, the distortionary effect of the tax, and, with wage heterogeneity, the covariance between the social marginal utility of lifetime income and (i) earnings (positive effect) and (ii) the probability of altruism default (negative effect).
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1 Introduction

With life expectancy increasing in most countries, more and more people live longer and enter a lifespan where dependency is no longer an infrequent occurrence. The 80-plus population, people most at risk of suffering severe dependency and requiring long-term care (LTC), is increasing faster than any other age group. The provision of LTC thus represents a major challenge for most societies today and for decades to come. In these societies, most of the older people in need of LTC services continue to rely on informal home care or services provided by unpaid caregivers (usually nonprofessional family members, friends or other volunteers). Estimates both for the US and Europe are in the 80 to 90 percent range. However, changing social patterns including smaller families, mobility of the children and increased female labor force participation, are undermining the family's role in LTC provision and contributing to an ever increasing demand for formal care, whether public or private.[1]

It is important to point out that LTC is not the same things as health care. LTC is the provision of assistance and services to people who, because of disabling illnesses or conditions, have limited ability to perform basic daily activities such as bathing and preparing meals. The import of this is that traditional LTC consists of mainly custodial and non-skilled care. However, to address the often multiple chronic conditions associated with older populations it increasingly involves also some level of medical care that requires the expertise of skilled practitioners. Nevertheless long-term care can be provided at home, in the community, in assisted living facilities or in nursing homes.

The increasing need for LTC services combined with the decreasing availability of the family, the traditional provider of LTC, poses a huge economic challenge to our societies. It inflicts a pressing demand on the other two LTC providers—the market and the state—to offer either a substitute or a complement to what the family has thus far been providing by way of long term care.

[1] For a survey on LTC, see Cremer *et al.* (2012a) and Grabowski *et al.* (2012).

In this paper we study the role of social insurance and its appropriate design in a setting wherein family solidarity is unreliable. In other words, children may provide care to their dependant parents but this is not for certain. There are multiple reasons for this. They are demographic (childless families), societal (declining family norms), economic (increasing labor participation of women who in the past used to help their dependent parents). Whatever the reason, the possibility of solidarity default requires people to take appropriate steps such as purchasing private insurance, self-insuring and relying on public insurance or assistance schemes.

The uncertain nature of family aid is not just relevant within the context of LTC. It also has interesting implications for other public policy questions. These include the design of retirement policy (Chabbakatri *et al.*, 1993), and their political support (Leroux and Pestieau, 2011). Now this literature has shown that uncertain altruism increases retirement savings and enhances the support for pension schemes. But in industrialized countries, family solidarity is not the main mechanism for retirement support any more. On the other hand, family solidarity remains the main channel for the provision of LTC. A thorough study of the implications of uncertainty in this context is therefore of crucial importance.

To study the role and design of public LTC provision in a framework where family aid is uncertain, we consider a single generation of "parents" over their (two-period) life cycle. In the first period, they work, consume, and save for their retirement. In the second period, they are retired and *may* become dependent. The probability of this dependency is exogenously given. If a parent becomes dependent, a second source of uncertainly kicks in. Parents may not necessarily receive aid from their children. Moreover, this source of uncertainty is not exogenously given. Parents can influence it through investment of their time instilling family values in their children and in this way fostering family solidarity.[2]

[2] See on this Kotlikoff and Spivak (1981) or Cox and Stark (2005).

In studying the properties of a public LTC scheme, we focus on one generation's lifecycle. Members of this generation live two periods: a period of work and a period of retirement which can also be a period of dependency. They are also the ones who bear the cost of financing the LTC program which they do during their period of activity. No tax is imposed on their children to pay for their parents' LTC provision. In this way, we circumvent the issues associated with inter-generational wealth transfers. The role of children in our model is limited to their decision with regard to providing assistance to their parents. As a consequence, the welfare of the grown-up children does not figure in government's objective function; social welfare accounts only for the expected lifecycle utility of parents. Formally, we treat the children as if they live abroad and may or may not send remittances to their disabled parents.

Throughout the paper, we shall rule out private insurance markets. The main reason for this is that private insurance markets are not viable in this setting. Specifically, what is needed here is insurance against dependency *and* the failure of altruism. Now while private markets can typically provide insurance against dependency, this is not possible against altruism default (and to our knowledge no such coverage is ever provided). The problem is, of course, that such a scheme raises enormous moral hazard problems. The government, on the contrary, will have the possibility of indirectly covering against altruism default by awarding benefits that are mutually exclusive with family assistance. More explicitly, all dependent parents are entitled to a "limited" care facility if they want one. They also have the option of not asking for one using instead their own resources, and those of their children, to purchase whatever home care services they need (without any help from the government).

It is important to point out that it is the opting out feature of public provision that gives it an edge over private insurance markets. Public provision schemes that allow "topping out" do not enjoy this edge as they will be provided to all dependent parents and not just those who do not receive aid from their children. In a companion paper, Cremer et al. (2012) compare the two schemes. However, that paper treats the probability of altruism to be exogenous and allows for no heterogeneity amongst parents. Given these assumptions, the opting out scheme can, under some conditions, achieve an efficient outcome with full insurance. This is not possible when the default of altruism is endogenous. Parents can increase the likelihood of getting help from their children by spending more time with them. This creates a tradeoff for the parents between spending more time with their children versus more time working. The former increases the probability of having assistance in case of dependency, and the latter generates more labor income. We show that this trade-off will have a negative impact on the optimal level of insurance. Whereas our opting out scheme calls for full insurance when the probability of altruism is fixed, it will call for less than full insurance with an endogenous probability of altruism.

By introducing income heterogeneity amongst agents, we extend Cremer et al. (2012) in yet another direction. This leads to additional influences on the optimal tax rate. With identical individuals, the tax reflects three consideration. One is the extent of under-insurance which tends to increase the tax. The second is the effect of the tax on the probability of altruism which is also positive. The third is the distortionary effect of the tax; this tends to lower the tax. Heterogeneity brings about two additional consideration. Both are related to the redistributive objective. While the first is similar to a traditional linear income tax effect (because a uniform benefit is financed via a proportional income tax) and tends to increase the tax, the second term has a negative effect on the tax and is specific to the structure of the underlying LTC provision scheme. The reason is that the higher productivity parents benefit more from the public assistance program making the system regressive. This may appear surprising at first but it arises because these individuals face a higher probability of being abandoned by their children; the opportunity cost of their time being higher, they invest less time in their children.

2 The model with identical individuals

We concentrate on a single generation of parents and consider a two period setting. In period 1 the young working parent and a child coexist, but the parent makes all the decisions. In period 2, the parent is old and retired while the child is now a working adult who makes his decisions. Parents face two types of uncertainty. First, in old age they may be either "dependent" or "autonomous". The probability of dependency is exogenous and denoted by π. Second, the parents are uncertain about the perspective of obtaining help from their children in case they will be dependent in the second period. The probability that a child helps his dependent parent, p, is endogenous. It varies positively with the time parents spend raising their children, T. Consequently, with a probability of $1 - p(T)$, children "abandon" their parents in the sense that they do not provide them with any family LTC.

Parents' consumption when young and old is denoted by c and d respectively. Parents allocate their $\overline{T}$ units of time between caring for their children, T, and market labor, $\overline{T} - T$. Both usages of time bear no disutility. Preferences are quasilinear, with a constant marginal utility of c.

Denote the LTC services parents consume by x. These take either of two forms. One is home or nursing care provided, or financed, by children or the dependents' own resources. The other is the government provision of a limited facility. Government service cannot be topped up so that the two types of service are mutually exclusive. Government's provision, z, is financed by a proportional tax on the parent's wage w, at rate τ. The savings, s, of the recipients' of the government service are also taxed away.

Altruistic children give their parents a level of aid a. Consequently we have $x = s + a$ for parents who receive aid from their children and $x = z$ for the beneficiaries of government assistance. The level of aid is chosen by the children who, in the process, also choose between their own aid versus government assistance.

The parent's expected utility is given by,

$$\begin{aligned} EU &= w(1-\tau)\left(\overline{T}-T\right)-s+(1-\pi)U(s)+ \\ &\quad \pi\left[p(T)H(s+a)+(1-p(T))H(z)\right]. \end{aligned} \tag{1}$$

This specification already integrates the life cycle budget constraint and assumes that the interest rate is zero. Children also have quasilinear preferences represented by

$$u = y_c - a + \beta H(x), \tag{2}$$

where y_c represents their exogenous income while β is their degree of altruism. When $\beta = 0$ the child is not altruistic.

The timing of the game is as follows. In stage 1, the government sets its policy instruments z and τ. In stage 2, parents choose T and s. Finally, in stage 3, children choose the level of assistance a. At each stage there is full commitment. We solve this game by backward induction starting with the last stage when the grown-up children decide on the extent of their help to their parents, if any.

2.1 Stage 3: The child's choice

Define

$$a^* = \arg\max_a = y_c - a + \beta H(s+a),$$

that is the level of aid that an altruistic child provides if the parent does not consume government provided LTC service z. Defining

$$m(\beta) \equiv \left(H'\right)^{-1}\left(\frac{1}{\beta}\right), \tag{3}$$

we have[3]

$$a^* = m(\beta) - s. \tag{4}$$

[3] If s is "too large", $a^* = 0$; this is uninteresting and we will rule it out.

Altruistic children choose a^* if and only if it gives them more utility than the option of no assistance in which case the parent consumes z. Consequently, they compare

$$u = y_c - a^* + \beta H\left(s + a^*\right), \tag{5}$$

with

$$u = y_c + \beta H\left(z\right). \tag{6}$$

The child is indifferent between these alternatives if

$$\begin{aligned} H\left(z\right) &= H\left(s + a^*\right) - \frac{a^*}{\beta} \\ &= H\left(m\left(\beta\right)\right) - \frac{m\left(\beta\right) - s^*}{\beta}. \end{aligned} \tag{7}$$

Let $\widehat{z}$ denote the solution to equation (7). The children's choice then simply depends on the comparison of z and $\widehat{z}$. If $z \leq \widehat{z}$ altruistic children prefer to provide their own aid a^*, rather then entrust their parents to public assistance. On the other hand when $z > \widehat{z}$, altruistic children provide no aid. For all practical purposes this yields the same outcome as the one achieved under failure of altruism. In the remainder of the paper we concentrate on the equilibrium under which altruistic children provide aid a^*. Recall that this occurs with probability $p(T)$.

Using (5)–(6) it follows that

$$H\left(s + a^*\right) - H\left(z\right) > a^*/\beta > 0,$$

so that $s + a^* > z$. In other words, parents of altruistic children consume a higher level of LTC services then the parents who are abandoned by their offspring. Interestingly, this implies that parents who receive aid from their children will never ask the government for help. Put differently, the program is self-targeted.

2.2 Stage 2: The parent's choice

The expected utility of the parent in period one is equal to

$$EU = w(1-\tau)\left(\overline{T}-T\right) - s + (1-\pi)U(s) + \pi\left[p(T)H(m(\beta)) + (1-p(T))H(z)\right]. \tag{8}$$

The parent chooses s and T in order to maximize his expected utility. It follows from the first-order conditions of this problem, and assuming an interior solution for s^* and T^* that,[4]

$$(1-\pi)U'(s^*) = 1, \tag{9}$$

$$p'(T^*) = \frac{w(1-\tau)}{\pi\left[H(m(\beta)) - H(z)\right]}. \tag{10}$$

It follows from (9) that the parent's saving is fixed and does not vary with the tax τ and government provision z. On the other hand, we have from (10) that the time spent with children varies with τ and z: $T^* = T^*(\tau, z)$. To determine the direction of these variations, differentiate (10) partially with respect to τ and z. This yields

$$\frac{\partial T^*}{\partial \tau} = \frac{-w}{\pi\left[H(m(\beta)) - H(z)\right]p''(T^*)} > 0, \tag{11}$$

$$\frac{\partial T^*}{\partial z} = \frac{w(1-\tau)H'(z)}{\pi\left[H(m(\beta)) - H(z)\right]^2 p''(T^*)} < 0. \tag{12}$$

Intuitively, an increase in τ lowers the opportunity cost of spending time with children. This substitution effect has no countervailing income effect given our quasilinear specification so that T^* increases. On the other hand, an increase in z lowers the value of spending time with children and one reduces T^*.

[4] Observe that an interior solution for s is ensured as long as marginal utility of consumption is high at "low" consumption levels. However, to have an interior solution for T, it must be the case that $\partial(EU/\partial T)|_{T=0} > 0$. That is,

$$H(m(\beta)) - H(z) > \frac{w(1-\tau)}{\pi p'(0)}.$$

Otherwise, $T^* = 0$.

2.3 Stage 1: Optimal policy

The government's budget constraint is now written as

$$\tau w \left(\overline{T} - T^*\right) = \pi \left[1 - p\left(T^*\right)\right] \left(z - s^*\right), \tag{13}$$

where $T^* = T^*(\tau, z)$. The government chooses z to maximize

$$\begin{aligned} \pounds \quad &\equiv \quad w(1-\tau)\left(\overline{T} - T^*\right) - s^* + (1-\pi) U(s^*) + \\ &\quad\quad \pi \left[p(T^*) H(m(\beta)) + (1 - p(T^*)) H(z)\right], \end{aligned} \tag{14}$$

while allowing τ to adjust to variation in z according to equation (13). Differentiating $\pounds$ with respect to z, using the envelope theorem, yields

$$\begin{aligned} \frac{d\pounds}{dz} &= \left.\frac{\partial \pounds}{\partial z}\right|_{T^*} + \left.\frac{\partial \pounds}{\partial \tau}\right|_{T^*} \frac{d\tau}{dz} \\ &= \pi\left[1 - p(T^*)\right] H'(z) - w\left(\overline{T} - T^*\right)\frac{d\tau}{dz} \\ &= \pi\left[1 - p(T^*)\right]\left[H'(z) - \frac{z - s^*}{\tau}\frac{d\tau}{dz}\right]. \end{aligned} \tag{15}$$

Totally differentiating (13) with respect to z and deriving an expression for $d\tau/dz$, we show in the Appendix that

$$\frac{d\pounds}{dz} = \pi\left[1 - p(T^*)\right] \frac{-\left[1 - \frac{\pi p'(T^*)(z - s^*) - \tau w}{\pi[1 - p(T^*)]}\frac{\partial T^*}{\partial z}\right] + \left[1 + \frac{\pi p'(T^*)(z - s^*) - \tau w}{w\left(\overline{T} - T^*\right)}\frac{\partial T^*}{\partial \tau}\right] H'(z)}{1 + \frac{\pi p'(T^*)(z - s^*) - \tau w}{w\left(\overline{T} - T^*\right)}\frac{\partial T^*}{\partial \tau}}, \tag{16}$$

The optimal value of z is found by setting the expression for $d\pounds/dz$ in (16) equal to zero. We have

$$1 - \frac{\pi p'(T^*)(z - s^*) - \tau w}{\pi\left[1 - p(T^*)\right]}\frac{\partial T^*}{\partial z} = \left[1 + \frac{\pi p'(T^*)(z - s^*) - \tau w}{w\left(\overline{T} - T^*\right)}\frac{\partial T^*}{\partial \tau}\right] H'(z). \tag{17}$$

Equation (17) tells us that at the optimum value of z the marginal costs and benefits of a budget-balance increase in τ are equal. The left-hand side shows the net cost of raising one dollar through an increase in τ. This is less than one because of the endogeneity of T^*. The reason is that the concomitant increase in z as τ increases, lowers T^* resulting in a further increase in the government's tax revenue, thus reducing the net cost of raising a dollar from one. The right-hand side shows the net benefit of spending that dollar on z. Endogeneity of T^* also implies net benefits that falls short of $H'(z)$. With endogeneity, an increase in τ increases T^* so that the government raises less than a dollar to finance z.

Observe that in this interpretation, we have implicitly assumed that

$$\pi p'(T^*)(z - s^*) - \tau w < 0. \tag{18}$$

To see the reason for this, consider the effect of increasing τ while keeping z constant on the government's "budget surplus" $\tau w \left(\overline{T} - T^*\right) - \pi \left[1 - p(T^*)\right](z - s^*)$. This is equal to

$$w \left(\overline{T} - T^*\right) + \left[\pi p'(T^*)(z - s^*) - \tau w\right] \frac{\partial T^*}{\partial \tau}.$$

The component of this expression that arises due to endogeneity of T^* consists of two parts. One is a negative effect on the tax base as T^* increases; this is equal to $-\tau w \left(\partial T^* / \partial \tau\right)$. The other is a positive effect. As T^* increases, $p(T^*)$ increases and the government needs to support less people; this is given by $\pi p'(T^*)(z - s^*)\left(\partial T^* / \partial \tau\right)$. Inequality (18) states that the first effect dominates the second.

Finally, the endogeneity of T implies less than full insurance under the opting out scheme when we had full insurance with exogenous T. To see this, rearrange the optimality condition (17) and rewrite it as

$$H'(z) - 1 = -\left[\pi p'(T^*)(z - s^*) - \tau w\right] \left[\frac{1}{\pi \left[1 - p(T^*)\right]} \frac{\partial T^*}{\partial z} + \frac{H'(z)}{w \left(\overline{T} - T^*\right)} \frac{\partial T^*}{\partial \tau}\right]. \tag{19}$$

By substituting the expressions for $\partial T^* / \partial \tau$ and $\partial T^* / \partial z$, from (11) and (12), in (19) followed by a bit of algebraic manipulations, we prove in the Appendix that

$$H'(z) - 1 = -\frac{\left[\pi p'(T^*)(z - s^*) - \tau w\right]^2 H'(z)}{\left(\overline{T} - T^*\right) \tau \pi \left[H(m(\beta)) - H(z)\right]^2 p''(T^*)} \frac{(1 - \tau)}{\pi p'(T^*)} > 0. \tag{20}$$

Observe also that from (16),

$$\left.\frac{d\pounds}{dz}\right|_{z=s^*} = \pi\left[1-p\left(T^*\right)\right]\left[H'\left(s^*\right)-1\right] > 0.$$

Consequently, $H'(s^*) > 1$ ensures $g = z - s^* > 0$ and $\tau > 0$; we assume this is the case.

2.4 Stage 1: Optimal policy revisited

For the sake of interpretation, we present an alternative way of designing the optimal LTC policy. We express the government's problem by the Lagrangian

$$\begin{aligned}\pounds \equiv\ & w\left(1-\tau\right)\left(\bar{T}-T^*\right) - s^* + \left(1-\pi\right)U\left(s^*\right) + \\ & \pi\left[p\left(T^*\right)H\left(m\left(\beta\right)\right) + \left(1-p\left(T^*\right)\right)H\left(z\right)\right] + \\ & \mu\left[\tau w(\bar{T}-T^*) - \pi\left(1-p\left(T^*\right)\right)\left(z-s^*\right)\right].\end{aligned} \tag{21}$$

The first-order conditions of the government's problem with respect to τ and z, using the envelope theorem, are

$$\begin{aligned}\frac{\partial\pounds}{\partial\tau} &= w\left(\mu-1\right)\left(\bar{T}-T^*\right) - \mu\left[\tau w - \pi\left(z-s^*\right)p'\left(T^*\right)\right]\frac{\partial T^*}{\partial\tau} = 0, \\ \frac{\partial\pounds}{\partial z} &= \pi\left(1-p\left(T^*\right)\right)\left[H'\left(z\right)-\mu\right] - \mu\left[\tau w - \pi\left(z-s^*\right)p'\left(T^*\right)\right]\frac{\partial T^*}{\partial z} = 0.\end{aligned}$$

Substituting for $\pi(z-s^*)$ from the government's budget constraint (13) in the above first-order conditions, one can rewrite them as

$$\frac{\partial T^*}{\partial\tau} = \frac{-\left(\mu-1\right)\left(\bar{T}-T^*\right)}{\mu\tau\left[\frac{L^*}{1-p(T^*)}p'\left(T^*\right)-1\right]}, \tag{22}$$

$$\frac{\partial T^*}{\partial z} = \frac{-\pi\left(1-p\left(T^*\right)\right)\left[H'\left(z\right)-\mu\right]}{\mu\tau w\left[\frac{L^*}{1-p(T^*)}p'\left(T^*\right)-1\right]}. \tag{23}$$

Let T^c denote the compensated education time in an exercise in which when τ changes, z is adjusted to hold the parent's expected utility constant. We have

$$\frac{\partial T^c}{\partial\tau} = \frac{\partial T^*}{\partial\tau} + \frac{\partial T^*}{\partial z}\left.\frac{dz}{d\tau}\right|_{EU}.$$

Observe that $\partial T^c/\partial\tau$ is not an "ordinary" compensated derivative. This is because the compensation is a achieved through a variation in z, rather than in income as is the case for the standard Hicksian demand. Consequently, we cannot use duality theory to prove $\partial T^c/\partial\tau < 0$. To obtain this property additional restriction are needed, as shown in Appendix C.

Differentiating equation (21) with respect to τ and letting z adjust to keep EU constant results in[5]

$$\frac{dz}{d\tau}|_{EU} = \frac{w(\bar{T}-T^*)}{\pi(1-p\,(T^*))H'(z)}. \tag{24}$$

Substituting in the expression for $\partial T^c/\partial\tau$ leads to

$$\frac{\partial T^c}{\partial\tau} = \frac{\partial T^*}{\partial\tau} + \frac{w(\bar{T}-T^*)}{\pi(1-p\,(T^*))H'(z)}\frac{\partial T^*}{\partial z}. \tag{25}$$

Finally, substituting the expressions for $\partial T^*/\partial\tau$ and $\partial T^*/\partial z$ from (22)–(23) and rearranging the terms, we get

$$\tau = \frac{\left[1-\frac{1}{H'(z)}\right](\bar{T}-T^*)}{\left[1-\frac{\bar{T}-T^*}{1-p(T^*)}p'\,(T^*)\right]\frac{\partial T^c}{\partial\tau}}. \tag{26}$$

Even though our model concerns homogeneous individuals, this characterization is similar to that in the optimal linear income tax problem à la Sheshinski (1972). As in that problem, the denominator reflects efficiency considerations. The numerator reflects

[5] We have

$$\frac{d\pounds}{d\tau} = \frac{\partial\pounds}{\partial\tau} + \frac{\partial\pounds}{\partial z}\frac{dz}{d\tau}|_{EU} = 0.$$

Consequently,

$$\begin{aligned}\frac{dz}{d\tau}|_{EU} &= -\frac{\frac{\partial\pounds}{\partial\tau}}{\frac{\partial\pounds}{\partial z}} = -\frac{\frac{\partial\pounds}{\partial\tau}|_{s^*,T^*} + \frac{\partial\pounds}{\partial s^*}\frac{\partial s^*}{\partial\tau} + \frac{\partial\pounds}{\partial T^*}\frac{\partial T^*}{\partial\tau}}{\frac{\partial\pounds}{\partial z}|_{s^*,T^*} + \frac{\partial\pounds}{\partial s^*}\frac{\partial s^*}{\partial z} + \frac{\partial\pounds}{\partial T^*}\frac{\partial T^*}{\partial z}} \\ &= -\frac{-w(\bar{T}-T^*)}{\pi\,(1-p\,(T^*))\,H'\,(z)}.\end{aligned}$$

insurance considerations. In noting this analogy, observe also that[6]

$$1-\frac{(\bar{T}-T^{*})}{1-p\left(T^{*}\right)}p'\left(T^{*}\right)>0,$$

and recall that (19) implies $H'(z)>1$. A positive solution for τ (and thus for z) requires $\partial T^{c}/\partial\tau>0$.

We will see in the next section that this characterization is preserved when we introduce heterogeneous agents but equity terms also enter the picture.

Proposition 1 summarize the results of this section.

Proposition 1 *Assume there is no private insurance, government's assistance cannot be topped up, and the probability of a child helping his parents in case of dependency depends positively on the time the parents spend with that child when growing up.*

(i) Optimal private savings s^{*} and time spent with children T^{*} are characterized by equations (9)–(10). While $s^{*}$is fixed and depends only on the exogenously given probability of dependency, π, T^{*} varies positively with the tax rate τ and negatively with government provision z.

(ii) Assuming $H'\left(s^{*}\right)>1$, private savings are insufficient to satisfy long term care needs of the parents. The government should tax the labor income of the parents and use its proceeds to provide long term care for those who are not helped by their children.

(iii) The optimal value of z is characterized by the optimality condition (17).

(iv) There should be less than full insurance so that at the optimum,

$$H'\left(z\right)>1.$$

[6] We have from the government's budget constraint (13) that

$$\pi\left(z-s^{*}\right)=\frac{\tau w(\bar{T}-T^{*})}{1-p\left(T^{*}\right)}.$$

Substituting in inequality (18),

$$p'\left(T^{*}\right)\frac{\tau w(\bar{T}-T^{*})}{1-p\left(T^{*}\right)}-\tau w<0.$$

Simplifying leads to the indicated inequality.

(v) Equation (26) characterizes the optimal τ.

3 Heterogeneous individuals

We now turn to the case where individuals have different wages or productivities. Individuals of type i have a wage of w_i and represent a proportion n_i of the total population (the size of which is normalized to one). The game played within each family is exactly the same as in the previous section. Consequently, T_i^* and s_i^* are determined as described in Subsection 2.1 and 2.2 and all the properties an expression derived there continue to apply (except, of course that an index i has to be added where appropriate).

Three remarks are in order. First, quasi-linearity assumptions imply that, whenever children help their parents, $x_i = s_i^* + a_i = m(\beta)$. Additionally, s_i^* is determined through equation (9) independently of income so that $s_i^* = s^*$. Second, we assume a flat rate benefit z that is independent of the level of contributions $\tau w_i(\bar{T} - T_i^*)$ and only awarded to the parents who do not benefit from family solidarity. Consequently, we continue to have $x_i = z$ if LTC insurance is provided by the government (and $x_i = s^* + a = m(\beta)$ if it is provided by the altruistic children). In either case, LTC consumption remains the same across all dependent parents with different income levels. What varies amongst them is the time they spend with their children and thus their labor supply as well as their present day consumption levels. Third, to introduce a concern for redistribution, we consider a social welfare function that is a concave transformation of the parents' quasilinear utilities. That is, we will work with $V(EU_i)$, as opposed to EU_i, where EU_i denotes the (expected) lifecycle utility of a parent with productivity w_i and $V(\cdot)$ is a concave transformation. Observe also that EU_i is given by (8) as in the homogeneous individuals case, but indexed i for wage w_i.

Next, before setting up the government's optimization problem, to avoid cluttered notation, we introduce some additional notation and expressions that will simplify the presentation of subsequent expressions. We use H' for $H'(z)$, y_i for $w_i\left(\bar{T} - T_i^*\right)$ and p_i

for $p(T_i^*)$. Similar to the homogeneous individuals case, let T_i^c denote the compensated leisure when τ changes and z is adjusted to hold the parent i's expected utility constant. We can then write

$$\frac{\partial T_i^c}{\partial \tau} = \frac{\partial T_i^*}{\partial \tau} + \frac{\partial T_i^*}{\partial z} \left.\frac{dz}{d\tau}\right|_{EU_i}.$$

Introduce

$$C_i \equiv \left.\frac{dz}{d\tau}\right|_{EU_i} = \frac{y_i}{\pi(1-p_i)H'}.$$

to measure the amount of z one needs to give to parent i as a result of an increase in τ to keep his lifetime utility EU_i constant.[7] We can then rewrite the previous expression as

$$\frac{\partial T_i^c}{\partial \tau} = \frac{\partial T_i^*}{\partial \tau} + C_i \frac{\partial T_i^*}{\partial z}. \tag{27}$$

Next introduce

$$\breve{C} \equiv \frac{\sum n_i y_i}{\pi H' \sum n_i (1-p_i)}.$$

This is the sum of the numerator of C_i divided by the sum of the denominator ($\breve{C}$ is not equal to $\sum n_i C_i$ of course). Adding $(\breve{C} - C_i)\partial T_i^*/\partial \tau$ to both sides of (27)

$$\begin{aligned} \frac{\partial T_i^c}{\partial \tau} + \frac{\partial T_i^*}{\partial z}(\breve{C} - C_i) &= \frac{\partial T_i^*}{\partial \tau} + \breve{C}\frac{\partial T_i^*}{\partial z} \\ &\equiv \frac{\partial \breve{T}_i}{\partial \tau} \end{aligned} \tag{28}$$

Observe that $\partial \breve{T}_i/\partial\tau$ is yet another compensated derivative derived from $\partial T_i^c/\partial\tau$ by adding to it one more compensation term equal to the difference between $\breve{C}\,(\partial T_i^*/\partial z)$ and $C_i\,(\partial T_i^*/\partial z)$. We are now ready to set up the government's optimization problem.

The Lagrangian expression of the government is given by

$$\begin{aligned} \mathcal{L}_2 \;=\; & \sum n_i \{V[w_i(1-\tau)(\bar{T} - T_i^*) - s^* + (1-\pi)u(s^*) \\ & + \pi p(T_i^*)H(m(\beta)) + \pi(1-p(T_i^*))H(z)] + \mu(\tau w_i(\bar{T} - T_i^*) - \pi(1-p(T_i^*))(z - s^*)\}. \end{aligned} \tag{29}$$

[7] This term is the same as the expression (24) in the homogeneous individuals case, only indexed by parent i.

Differentiating (29) while making use of the envelope theorem yields the following first-order conditions

$$\frac{\partial \mathcal{L}_2}{\partial \tau} = \sum n_i \left\{ \left[\mu - V'(EU_i)\right] y_i - \mu \left[\tau w_i - \pi p_i'(z - s^*)\right] \frac{\partial T_i^*}{\partial \tau} \right\} = 0, \tag{30}$$

$$\frac{\partial \mathcal{L}_2}{\partial z} = \sum n_i \left\{ \left[V'(EU_i)H' - \mu\right] \pi(1 - p_i) - \mu \left[\tau w_i - \pi p_i'(z - s^*)\right] \frac{\partial T_i^*}{\partial z} \right\} = 0. \tag{31}$$

Appendix D shows that first-order conditions (30)–(31) can be combined as follows

$$\begin{aligned} \frac{\partial \mathcal{L}_2}{\partial \tau} + \frac{\partial \mathcal{L}_2}{\partial z}\breve{C} = &- E\left(V'(EU)\right)\left[y - (1-p)\frac{E\left(y\right)}{E\left(1-p\right)}\right] + \mu\left(\frac{H'-1}{H'}\right)E\left(y\right) \\ &- \mu\tau E\left(w\frac{\partial \breve{T}}{\partial \tau}\right) + \mu\pi(z - s^*)E\left(p'\frac{\partial \breve{T}}{\partial \tau}\right) = 0, \end{aligned} \tag{32}$$

where the operator E is used for the "expected value" (frequency-weighted summation over i). Introducing the cov$(\cdot)$ notation, familiar from the optimal linear income tax literature, we can rewrite expression (32) as

$$\begin{aligned} &- \mathrm{cov}(V', y) + \frac{E\left(y\right)}{E\left(1-p\right)}\mathrm{cov}(V', (1-p)) \\ &+ \mu\left(\frac{H'-1}{H'}\right)E\left(y\right) - \mu\tau E\left(w\frac{\partial \breve{T}}{\partial \tau}\right) + \mu\pi(z - s^*)E\left(p'\frac{\partial \breve{T}}{\partial \tau}\right) = 0, \end{aligned}$$

or

$$\tau = \frac{- \mathrm{cov}(V', y) + \frac{E(y)}{E(1-p)}\mathrm{cov}(V', (1-p)) + \mu\left(\frac{H'-1}{H'}\right)E\left(y\right) + \mu\pi(z - s^*)E\left(p'\frac{\partial \breve{T}}{\partial \tau}\right)}{\mu E\left(w\frac{\partial \breve{T}}{\partial \tau}\right)}. \tag{33}$$

In interpreting this formula, one caveat is in order. While expression (33) is valid regardless of the sign of the compensated derivative $\partial\breve{T}/\partial\tau$, its interpretation (and more fundamentally the existence of an interior solution) do depend on the sign of $\partial\breve{T}/\partial\tau$. In the linear optimal income tax problem, the sign of $\partial T^c/\partial\tau$ is unambiguously positive. This appears to be the "reasonable" case here as well; however, one cannot rule out the theoretical possibility of a negative sign for it; see Subsection 2.4 and Appendix C).

Consequently, one cannot be certain about the sign of $\partial \breve{T}_i/\partial\tau$ either. Nevertheless we shall concentrate on the case where $\partial \breve{T}_i/\partial\tau > 0$ which is the "reasonable" case, consistent with the traditional deadweight loss terms in optimal tax expressions. However, one has to keep in mind that this is an assumption and not a result. The relevance of our results have to be qualified accordingly.

First observe that when individuals are identical the covariance terms in (33) vanish while from the definitions of C_i and $\breve{C}$, $C_i = \breve{C}$ so that from (27) $\partial \breve{T}_i/\partial\tau = \partial T^c/\partial\tau$. Consequently, (33) simplifies to

$$\begin{aligned} \tau &= \frac{\left(\frac{H'-1}{H'}\right) y + \pi(z-s^*)p'\frac{\partial T^c}{\partial\tau}}{w\frac{\partial T^c}{\partial\tau}} \\ &= \frac{\left(\frac{H'-1}{H'}\right) y}{w\frac{\partial T^c}{\partial\tau}} + \frac{\pi(z-s^*)p'}{w}. \end{aligned} \tag{34}$$

One can easily show that (34) is equivalent to (26), the expression obtained in the previous section for the homogeneous individuals case.[8] We start by interpreting the terms that appear in (26), or equivalently (34), and are present even when there is no heterogeneity among parents. Then proceed with the interpretation of the terms that appear because of heterogeneity.

There are three terms. First is the insurance gap $(H'-1) > 0$; the larger this gap the higher is the tax. Second is the positive effect of the tax on the probability of solidarity, p'; this makes the public provision of LTC less costly and leads to a higher tax. Third is the distortionary effect of the tax on leisure, $\partial T^c/\partial\tau$, lowering tax revenues; this calls for a lower tax rate.[9]

[8] In (34) substitute $w\left(\overline{T}-T^*\right)$ for y, $\tau w\left(\overline{T}-T^*\right)/\left[1-p\left(T^*\right)\right]$ for $\pi\left(z-s^*\right)$, from the government's budget constraint (13), and simplify. This yields

$$\tau = \frac{\left(\frac{H'-1}{H'}\right)\left(\overline{T}-T^*\right)}{\frac{\partial T^c}{\partial\tau}} + \frac{\tau\left(\overline{T}-T^*\right)p'}{1-p\left(T^*\right)}.$$

Collecting the terms involving τ, and "solving" for τ, leads to (26).

[9] Recall that we assume $\partial T^c/\partial\tau > 0$.

Next turning to the general formula (33), there are two additional covariance terms pertaining to the redistributive role of the public scheme. First, there is the covariance between the social utility of income and earnings: $\mathrm{cov}(V', y) < 0$; this is the classical term of the linear income tax model. Given that it appears with a negative sign in the formula, it has a positive effect on the level of the tax. This is not surprising in that a uniform benefit is financed via a proportional income tax (similar to the demogrant of the linear income tax). Second, there is the covariance between the social utility of income and the probability of altruism default. One would expect that the more productive parents are more likely to be abandoned by their children. This follows from the fact that they face a higher opportunity cost in spending time with their children. Consequently, one would expect $\mathrm{cov}(V', (1-p)) < 0$ so that this term tends to decrease the level of τ. Put differently, to the extent that it is the high-income dependents, rather than low-income dependents, who demand the public benefit redistributive concerns call for a smaller tax.

The main results of this section are summarized in the following proposition.

Proposition 2 *Consider a redistributive version of the LTC public scheme of Proposition 1 consisting of a uniform benefit and a payroll tax. The public benefit is granted to dependent elderly who do not benefit from family help. Individuals differ in earnings and the tax has a positive effect on the time devoted to children. We show that the optimal payroll tax depends on five terms (and their sign is given under the assumption that time spent with children increases with the tax rate): (i) the extent of under-insurance (positive), (ii) the effect of the tax on the probability of altruism (positive), (iii) the distortionary effect of the tax (negative), (iv) the covariance between the social marginal utility of lifetime income and earnings (positive effect), and (v) the covariance between the social marginal utility of lifetime income and the probability of altruism default (negative).*

4 Conclusion

This paper has studied public provision of long term care insurance in a world in which family assistance is (i) uncertain and (ii) endogenous depending on the time parents spend raising their children. The increasing unreliability of families as the primary source of LTC provision is due to many factors. Some are exogenous but others are not and can be influenced by how parents raise their children. Inculcating a sense of family solidarity through spending time with them is a case in point. The paper has been mainly concerned with the implications of this factor. Another focus of the paper, hitherto unexplored, has been the implications of income heterogeneity amongst families for the design of a public insurance scheme.

We have adopted a rather simple framework with a linear payroll tax that finances a LTC flat rate benefit. This benefit while accessible to all dependent parents is self targeted such that only the disabled parents who are deprived from receiving aid from their children opt for it. Initially, to focus solely on the endogeneity issue, the paper has considered a setup with parents of identical productivities. Parents devoting time educating their children and shaping their values can mitigate the altruism default. This education time is diverted from market labor and therefore implies a tax distortion that makes provision of full insurance LTC undesirable. The optimal tax formula in this case is a trade-off between three consideration: under-insurance which tends to increase the tax, the effect of the tax on the probability of altruism which also increases the tax and the distortionary effect which tends to lower the tax.

Introducing earnings heterogeneity amongst parents, the paper has drawn attention to two other factors related to the government's redistributive objective. One is the presence of the covariance between the social utility of income and earnings with a negative sign; an effect present in the traditional linear income tax model calling for a higher tax rate. The other is the presence of the covariance between the social utility of income and the probability of altruism defaults appearing with a positive sign. It has a dampening effect on the size of the optimal tax. This consideration is new and arises from the interaction between endogeneity of the default of altruism and heterogeneity. The point is that more productive parents have a higher opportunity cost of time and tend to invest less time in their children. As a result, they face a higher probability of being abandoned by their children. They will thus benefit more from the public assistance program making the system regressive. Given this inherent regressivity, redistributive concerns call for a smaller tax.

Among themes for future research, one could think of enriching the tax instruments allowing for nonlinear income taxes. Another is to consider circumstances where individuals differ not only in earnings but also in the tightness of family solidarity and in the probability of dependency. In such a case, the correlation between those two individual characteristics would be crucial.

Appendix

A Derivation of (16)

First, totally differentiate (13) with respect to z to get an expression for $d\tau/dz$. We have,

$$\frac{d\tau}{dz} w\left(\overline{T} - T^*\right) - \tau w \frac{dT^*}{dz} = -\pi p'\left(T^*\right) \frac{dT^*}{dz}\left(z - s^*\right) + \pi\left[1 - p\left(T^*\right)\right], \tag{A1}$$

where

$$\frac{dT^*}{dz} \equiv \frac{\partial T^*}{\partial z} + \frac{\partial T^*}{\partial \tau}\frac{d\tau}{dz}. \tag{A2}$$

Substituting from (A2) into (A1) and solving for $d\tau/dz$ yields

$$\frac{d\tau}{dz} = \frac{\pi\left[1 - p\left(T^*\right)\right] - \left[\pi p'\left(T^*\right)\left(z - s^*\right) - \tau w\right]\left(\partial T^*/\partial z\right)}{w\left(\overline{T} - T^*\right) + \left[\pi p'\left(T^*\right)\left(z - s^*\right) - \tau w\right]\left(\partial T^*/\partial \tau\right)}. \tag{A3}$$

Next, substitute for $d\tau/dz$ from (A3) in (15) and simplify, using the government's budget constraint (13), to get

$$\begin{aligned} \frac{d\pounds}{dz} &= \pi\left[1 - p\left(T^*\right)\right]\left[\frac{\tau w\left(\overline{T} - T^*\right) H'(z) - \pi\left[1 - p\left(T^*\right)\right]\left(z - s^*\right) + \Gamma}{\tau w\left(\overline{T} - T^*\right) + \tau\left[\pi p'\left(T^*\right)\left(z - s^*\right) - \tau w\right]\left(\partial T^*/\partial \tau\right)}\right] \\ &= \pi\left[1 - p\left(T^*\right)\right]\left[\frac{H'(z) - 1 + \frac{\Gamma}{\tau w\left(\overline{T} - T^*\right)}}{1 + \frac{\pi p'\left(T^*\right)\left(z - s^*\right) - \tau w}{w\left(\overline{T} - T^*\right)}\frac{\partial T^*}{\partial \tau}}\right], \end{aligned} \tag{A4}$$

where

$$\Gamma \equiv \left[\pi p'\left(T^*\right)\left(z - s^*\right) - \tau w\right]\left[\tau H'(z)\frac{\partial T^*}{\partial \tau} + \left(z - s^*\right)\frac{\partial T^*}{\partial z}\right].$$

Substituting the value of Γ in (A4), using the government's budget constraint (13), after simplifications one arrives at (16).

B Derivation of (20)

Substitute the expressions for $\partial T^*/\partial \tau$ and $\partial T^*/\partial z$, from (11) and (12) in the last bracketed expression on the right-hand side of (19), while also substituting for $\pi\left[1 - p\left(T^*\right)\right]$

from the government's budget constraint (13), and for $\tau\left[H\left(m\left(\beta\right)\right)-H\left(z\right)\right]$ from equation (10). We have

$$\begin{aligned}
&\frac{1}{\pi\left[1-p\left(T^{*}\right)\right]}\frac{\partial T^{*}}{\partial z}+\frac{H'\left(z\right)}{w\left(\overline{T}-T^{*}\right)}\frac{\partial T^{*}}{\partial \tau}= \\
&\frac{1}{w\left(\overline{T}-T^{*}\right)}\left[\frac{\left(z-s^{*}\right)}{\tau}\frac{\partial T^{*}}{\partial z}+H'\left(z\right)\frac{\partial T^{*}}{\partial \tau}\right]= \\
&\frac{1}{\left(\overline{T}-T^{*}\right)}\frac{H'\left(z\right)}{\tau\pi\left[H\left(m\left(\beta\right)\right)-H\left(z\right)\right]^{2}p''\left(T^{*}\right)}\left\{\left(z-s^{*}\right)\left(1-\tau\right)-\tau\left[H\left(m\left(\beta\right)\right)-H\left(z\right)\right]\right\}= \\
&\frac{1}{\left(\overline{T}-T^{*}\right)}\frac{H'\left(z\right)}{\tau\pi\left[H\left(m\left(\beta\right)\right)-H\left(z\right)\right]^{2}p''\left(T^{*}\right)}\frac{\left(1-\tau\right)}{\pi p'\left(T^{*}\right)}\left[\pi p'\left(T^{*}\right)\left(z-s^{*}\right)-\tau w\right].
\end{aligned}$$

Substituting this expression in (19) yields (20).

C The sign of $\partial L^{c}/\partial\tau$

Substituting for $\partial L^{*}/\partial\tau$ and $\partial L^{*}/\partial z$ from (11)–(12) in (25),

$$\begin{aligned}
\frac{\partial L^{c}}{\partial\tau} &= \frac{w}{\pi\left[H\left(m\left(\beta\right)\right)-H\left(z\right)\right]p''\left(T^{*}\right)}-\frac{wL^{*}}{\pi(1-p\left(T^{*}\right))H'(z)}\frac{w\left(1-\tau\right)H'\left(z\right)}{\pi\left[H\left(m\left(\beta\right)\right)-H\left(z\right)\right]^{2}p''\left(T^{*}\right)} \\
&= \frac{w}{\pi\left[H\left(m\left(\beta\right)\right)-H\left(z\right)\right]p''\left(T^{*}\right)}\left\{1-\frac{wL^{*}}{\pi(1-p\left(T^{*}\right))H'(z)}\frac{\left(1-\tau\right)H'\left(z\right)}{\left[H\left(m\left(\beta\right)\right)-H\left(z\right)\right]}\right\}
\end{aligned}$$

To obtain $\partial T^{c}/\partial\tau<0$ we must have $\partial L^{c}/\partial\tau<0$, which in turn requires

$$1>\frac{wL^{*}}{\pi(1-p\left(T^{*}\right))}\frac{\left(1-\tau\right)}{\left[H\left(m\left(\beta\right)\right)-H\left(z\right)\right]},$$

or

$$1-\tau<\frac{\pi(1-p\left(T^{*}\right))\left[H\left(m\left(\beta\right)\right)-H\left(z\right)\right]}{wL^{*}}.$$

We also have

$$\tau wL^{*}=\pi\left[1-p\left(T^{*}\right)\right]\left(z-s^{*}\right)$$

$\Rightarrow$

$$1-\tau<\frac{\tau}{z-s^{*}}\left[H\left(m\left(\beta\right)\right)-H\left(z\right)\right]$$

$\Rightarrow$

$$\frac{1-\tau}{\tau} < \frac{H\left(m\left(\beta\right)\right) - H\left(z\right)}{z - s^{*}}$$

$\Rightarrow$

$$\frac{1}{\tau} < \frac{z - s^{*} + H\left(m\left(\beta\right)\right) - H\left(z\right)}{z - s^{*}}$$

$\Rightarrow$

$$\tau > \frac{z - s^{*}}{z - s^{*} + H\left(m\left(\beta\right)\right) - H\left(z\right)},$$

which is not necessarily satisfied.

D Derivation of expression (32)

Using equations (30)–(31), one obtains

$$\begin{aligned}
\frac{\partial \mathcal{L}_2}{\partial \tau} + \frac{\partial \mathcal{L}_2}{\partial z}\breve{C} \quad = \quad & \sum_i n_i \left\{ \mu \left[y_i - \frac{(1-p_i)\sum_i n_i y_i}{H' \sum_i n_i (1-p_i)} \right] \right. \\
& \left. - V'(EU_i) \left[y_i - \frac{(1-p_i)\sum_i n_i y_i}{\sum_i n_i (1-p_i)} \right] \right\} \\
& - \mu \sum_i n_i \left\{ \left[\tau w_i - \pi p_i (z - s^{*}) \right] \frac{\partial T_i}{\partial \tau} \right\}.
\end{aligned}$$

Using the operator E, this yields (32).

References

[1] Chakrabarti, S., W. Lord & P. Rangazas (1993). Uncertain altruism and investment in children, The American Economic Review 83(4), 994-1002.

[2] Cox, D. & O. Stark, (2005). On the demand for grandchildren: Tied transfers and the. demonstration effect. Journal of Public Economics, 89: 1665-1697

[3] Cremer, H., P. Pestieau, & G. Ponthière, (2012a) The economics of Long-Term-Care: A survey , forthcoming, Nordic Economic Policy Review.

[4] Cremer, H., F. Gahvari & P. Pestieau, (2012). Uncertain altruism and the provision of long term care, unpublished.

[5] Grabowski, D.C. E.C. Norton, & C.H. Van Houtven. 2012. Informal care. Chapter 30, pp. 318-328. In The Elgar Companion to Health Economics, Second Edition, A.M. Jones, ed. Northampton, MA: Edward Elgar.

[6] Kotlikoff, L. & A. Spivak, (1981) The family as an incomplete annuities market, Journal of Political Economy, 89,

[7] Leroux, M-L. & P. Pestieau,(2011) Social security and family support, CORE DP 2011/45.