Second-best Pigouvian taxation: a clarification

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June 2012
Revised September 2013

*I thank the Editor and two anonymous referees for helpful comments.

Abstract

This paper argues that the search for a "purely environmental" component of a tax on goods or factors of production that impact the environment—separate from its redistributive and distortive effects—is fraught with difficulties. The quest is often impossible because of the interconnectedness between labor supply, consumption decisions and the environmental quality. The paper differentiates between two conceptualization for "the Pigouvian tax" that have been employed in the literature and argues that each has tried to isolate the environmental component in its own way. One conceptualization, due to Cremer *et al.* (1998), does so by ruling out direct feedback from changes in environmental quality on the incentive effect of the tax. In the second conceptualization, due to Bovenberg and van der Ploeg's (1994), incentive effects are ruled out by making consumers' valuation of environmental quality independent of the labor supply. This is achieved by assuming separability between labor supply and other goods (including environmental equality). To convey its message, the paper studies the properties of optimal polluting and non-polluting non-labor input taxes in a Mirrleesian model with endogenously determined wages.

JEL classification: H21, Q50.

Keywords: Pigouvian taxes, second-best taxes, environmental quality.

1 Introduction

The Pigouvian prescription for correcting an externality is to levy a tax on it equal to its marginal social damage. This is a first-best remedy that should generally be modified in the second-best. Sandmo (1975) made this point in a pioneering work nearly four decades ago and initiated a vast literature on second-best emission taxes.[1] The main vexing question is whether one should tax emissions at an amount equal to, greater than, or smaller than the marginal social damage. This question is intricately related to the question of double dividend in environmental economics. Those who believe in a strong double dividend and think that environmental taxes reduce the existing tax distortions in the economy call for environmental taxes that exceed the marginal social damage of emissions. On the other hand, those who think that environmental taxes exacerbate the existing tax distortions argue for taxes that fall short of the marginal social damage of emissions. Either way, in recent years, the question has become particularly relevant for policy makers in the industrialized world where a consensus is emerging on the necessity of carbon taxes, or taxes on energy consumption, to fight global warming (aimed at curtailing CO_2 emissions, from fossil-fuel combustion, deemed as the most important of greenhouse gases).

One important point which has gone unnoticed in this literature is that in second-best environments, particularly those inhabited by heterogeneous agents, the concept of the marginal social damage of emissions, which the contributors to this literature have identified as the "Pigouvian tax," is not uniquely defined. One problem is how to translate the disutility of emission damage into dollars: to divide the disutility of emission damage by private marginal utility of income or by the shadow cost of public funds. This problem applies equally to economies with homogeneous and heterogeneous agents. A second problem concerns aggregation across heterogeneous agents. Specifically, there are at least two definitions of the Pigouvian tax in the literature and a tax which can be termed Pigouvian under one definition will not necessarily be Pigouvian under the

[1] See, among others, Bovenberg and van der Ploeg (1994), Bovenberg and de Mooij (1994, 1997), Fullerton (1997), Schöb (1997), Cremer, Gahvari and Ladoux (1998, 2001), Cremer and Gahvari (2001), Boadway and Tremblay (2008), Micheletto (2008), and Gahvari (2010, 2012). See also the survey by Bovenberg and Goulder (2002) and the references therein.

other definition. Naturally, this creates some confusion not only theoretically but also policy-wise when one compares and contrasts the optimal tax with the Pigouvian tax.[2] The purpose of this note is to clarify these definitions and draw a link between them and the different preference separability assumptions different authors have employed.[3]

To be as general as possible, I study this issue within the context of a Mirrleesian optimal tax problem with endogenous wages. I shall also confine my discussion to input taxes. However, the findings apply to models with Ramsey taxation as well as polluting good taxes. The general point that I will try to convey is this. The concept of the Pigouvian tax that originates with Bovenberg and van der Ploeg (1994), Bovenberg and de Mooij (1994), and Fullerton (1997) is intimately linked to preferences being weakly separable in labor supply and other goods (including emissions). On the other hand, the concept of the Pigouvian tax that originates with Cremer *et al.* (1998) is linked to the weak-separability of preferences between emissions and other goods (including labor supply).

2 The model

Consider an open economy which uses four factors of production to produce a composite consumption good, c. All markets are competitive. The factors of production consist of skilled labor, L^s, unskilled labor, L^u, and two non-labor inputs: one that is non-polluting, K, and the other which is polluting as in energy, E. The production technology, $O = \mathbf{O}(L^s, L^u, K, E)$, exhibits constant returns to scale with diminishing marginal product for all factors. Skilled and unskilled labor, who are *not* perfect substitutes, come from domestic sources with their wage rates, w^s and w^u, determined endogenously. The non-labor inputs K and E are imported from outside at the fixed world prices of r and p.

All types of workers have identical preferences defined over the composite consumption good, c, the worker's labor supply, h, and aggregate emissions. Emissions come

[2]Some tentative numerical estimates of, and comparisons between, the optimal emission tax versus the Pigouvian tax are presented in Cremer, Gahvari and Ladoux (2003, 2011).

[3]In particular, I do not make any claims to the originality of my second-best tax characterizations.

from the use of energy in production. The unit of measurement for emissions is chosen such that a unit of energy creates a unit of emissions. Thus E also represents aggregate emissions. Represent preferences by the utility function $U = \mathbf{U}(c, h, E)$ where U is increasing in c, decreasing in h and in E, and strictly quasi-concave.

Finally, denote the proportion of each worker type in total population by π^j, $j = s, u$. Normalizing the population size at one, the j-type's (aggregate) labor supply entering the production function, L^j, is related to a j-type worker's labor supply, h^j, according to $L^j = \pi^j h^j$.

2.1 The optimal tax problem

Assume, as is standard in the optimal income tax literature, that an individual's income, $I \equiv wh$, is publicly observable but that his type and labor supply are not. Given that types are unobservable, one cannot tax wage rates directly in production. Consequently, assuming competitive markets and profit maximization, factor payment by firms to each type of labor input must be equal to its marginal product. That is, denoting the partial derivative of $\mathbf{O}(L^s, L^u, K, E)$ with respect to L^j, $j = s, u$, by an L^j-subscript it must be the case that $w^j = \mathbf{O}_{L^j}(L^s, L^u, K, E)$ and w^j is determined endogenously within the model.

Rewrite preferences as a function of observables, c, I, and E: $U = \mathbf{U}(c, I/w, E)$. Let

$$\mathbf{U}^j(c, I, E) \equiv \mathbf{U}\left(c, \frac{I}{w^j}, E\right),$$

with the notation $u^j \equiv \mathbf{U}^j\left(c^j, I^j, E\right) = \mathbf{U}\left(c^j, I^j/w^j, E\right)$ and $u^{jk} \equiv \mathbf{U}^j\left(c^k, I^k, E\right) = \mathbf{U}\left(c^k, I^k/w^j, E\right)$. Thus, in the language of the optimal income tax theory, u^j is the utility of the j-type worker at the $\left(c^j, I^j\right)$ bundle the government wants him to have, and u^{jk} is the utility of the j-type worker at the $\left(c^k, I^k\right)$ bundle the government wants type $k \neq j$ to have. Let γ^j's denote positive constants with the normalization that $\gamma^s + \gamma^u = 1$. Constrained Pareto-efficient allocations are found by maximizing $\gamma^s u^s + \gamma^u u^u$ subject to the economy's resource constraint,[4]

$$\mathbf{O}\left(L^s, L^u, K, E\right) \geq \pi^s c^s + \pi^u c^u + rK + pE + \overline{R}, \tag{1}$$

where $\overline{R}$ is the government's external expenditures (non-transfers), the self-selection constraints,

$$u^s \quad \geq \quad u^{su}, \tag{2}$$

$$u^u \quad \geq \quad u^{us}, \tag{3}$$

and two other constraints that ensure the equality of the firms' factor payments to labor inputs with their marginal products:

$$w^s - \mathbf{O}_{L^s}\left(\cdot\right), \tag{4}$$

$$w^u - \mathbf{O}_{L^u}\left(\cdot\right). \tag{5}$$

The resource constraint ensures that the output of the economy, $\mathbf{O}\left(L^s, L^u, K, E\right)$, suffices to finance domestic consumption, $\pi^s c^s + \pi^u c^u$, imports, $rK + pE$, and government's consumption, $\overline{R}$. The self-selection, or incentive compatibility constraints, ensure that the desired allocations (constrained Pareto-efficient allocations) can be decentralized through appropriate tax/transfer policies (i.e. chosen voluntarily by utility-maximizing individuals facing their budget constraints). Imposing conditions (4)–(5), the equality of wage payments to marginal products, as constraints on the optimal tax problem distinguishes the method of the proof given in this paper from the traditional Stiglitz (1982) approach and makes the derivations much simpler.

[4] One can also interpret γ^s and γ^u as welfare weights associated with u^s and u^u in a utilitarian social welfare maximization problem. The set of (constrained) Pareto-efficient allocations correspond to the set of allocations defined by different values of γ^s and γ^u as the welfare weights change (γ^u/γ^s goes from zero to infinity). To see the equivalence, recall that the Pareto-efficient allocations are often find by maximizing the utility of one agent subject to keeping the utility of other agents fixed plus the economy's resource constraint (informational constraints are also included when describing constrained Pareto-efficient allocations). Thus, in the stated problem of this paper, one maximizes u^s subject to $u^u \geq \overline{u}$ plus constraints (1)–(5). This is summarized by the Lagrangian

$$\begin{aligned}\mathcal{L} =& u^s + \eta\left(u^u - \overline{u}\right) + \lambda\left(u^s - u^{su}\right) + \mu\left[\mathbf{O}\left(\cdot\right) - \pi^s c^s - \pi^u c^u - rK - pE - \overline{R}\right] \\ &+ \delta^s\left[w^s - \mathbf{O}_{L^s}\left(\cdot\right)\right] + \delta^u\left[w^u - \mathbf{O}_{L^u}\left(\cdot\right)\right].\end{aligned}$$

Comparing this expression with the Lagrangian expression (6) in the text reveals that the two are equivalent except for the normalization rule. Whereas in the problem leading to Lagrangian (6) $\gamma^s + \gamma^u$ is normalized to one; here one implicitly normalizes $\gamma^s = 1$ while denoting γ^u/γ^s by η.

Assume that in equilibrium $w^s \geq w^u$ and the government wishes to redistribute from skilled to unskilled workers.[5] One can then ignore the "upward" self-selection constraint corresponding to unskilled- "mimicking" skilled-workers, $u^u \geq u^{us}$, and summarize the optimal tax problem by the Lagrangian[6]

$$\begin{aligned}\mathcal{L} =& \gamma^s \mathbf{U}\left(c^s, \frac{I^s}{w^s}, E\right) + \gamma^u \mathbf{U}\left(c^u, \frac{I^u}{w^u}, E\right) + \\ & \lambda\left[\mathbf{U}\left(c^s, \frac{I^s}{w^s}, E\right) - \mathbf{U}\left(c^u, \frac{I^u}{w^s}, E\right)\right] + \\ & \mu\left[\mathbf{O}\left(L^s, L^u, K, E\right) - \pi^s c^s - \pi^u c^u - rK - pE - \overline{R}\right] + \\ & \delta^s\left[w^s - \mathbf{O}_{L^s}\left(L^s, L^u, K, E\right)\right] + \delta^u\left[w^u - \mathbf{O}_{L^u}\left(L^s, L^u, K, E\right)\right], \end{aligned} \tag{6}$$

where $\lambda, \mu, \delta^s, \delta^u$ are the Lagrangian multipliers associated with the downward self-selection constraint, the resource constraint, and the two labor demand constraints.

The following Proposition provides the characterization of the optimal input taxes.[7]

Proposition 1 *In the optimal tax problem of Section 2.1 input taxes on non-polluting capital K and polluting energy E are characterized by*

$$\mathbf{O}_K(\cdot) - r = \frac{\delta^s \mathbf{O}_{L^s K}(\cdot) + \delta^u \mathbf{O}_{L^u K}(\cdot)}{\mu}, \tag{7}$$

$$\mathbf{O}_E(\cdot) - p = \frac{\delta^s \mathbf{O}_{L^s E}(\cdot) + \delta^u \mathbf{O}_{L^u E}(\cdot)}{\mu} - \frac{\gamma^s u_E^s + \gamma^u u_E^u}{\mu} - \frac{\lambda\left(u_E^s - u_E^{su}\right)}{\mu}, \tag{8}$$

$$= \frac{\delta^s \mathbf{O}_{L^s E}(\cdot) + \delta^u \mathbf{O}_{L^u E}(\cdot)}{\mu} - \left(\pi^s \frac{u_E^s}{u_c^s} + \pi^u \frac{u_E^u}{u_c^u}\right) - \frac{\lambda}{\mu} u_c^{su}\left(\frac{u_E^u}{u_c^u} - \frac{u_E^{su}}{u_c^{su}}\right). \tag{9}$$

Proof. The optimal tax structure is found by differentiating the Lagrangian expression (6) with respect to $c^j, I^j, w^j, K,$ and E (for $j = s, u$). However, for our purposes, it suffices to derive the derivatives of the Lagrangian with respect to $c^j, K,$ and E only.

[5] Gaube (2005) shows that wage endogeneity may lead to second-best solutions other than the traditional "redistributive" and "regressive" cases. Given the note's goals, I nevertheless concentrate on the $w^s \geq w^u$ case. See Micheletto (2004) for the characterization of the optimal redistributive policy with endogenous wages.

[6] In the language of the optimal income tax problem, mimicking refers to the act of one agent choosing the bundle intended for another type, thus misrepresenting his true type in the tax design problem.

[7] One subscript on a variable denotes its partial derivative with respect to that argument, and two subscripts denote second partial derivatives with respect to those two arguments.

These are given by

$$\frac{\partial \mathcal{L}}{\partial c^s} = (\gamma^s + \lambda) u_c^s - \mu \pi^s = 0, \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial c^u} = \gamma^u u_c^u - \lambda u_c^{su} - \mu \pi^u = 0, \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial K} = \mu \left[\mathbf{O}_K(\cdot) - r \right] - \delta^s \mathbf{O}_{L^s K}(\cdot) - \delta^u \mathbf{O}_{L^u K}(\cdot) = 0, \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial E} = \gamma^s u_E^s + \gamma^u u_E^u + \lambda \left(u_E^s - u_E^{su} \right) + \mu \left[\mathbf{O}_E(\cdot) - p \right] - \delta^s \mathbf{O}_{L^s E}(\cdot) - \delta^u \mathbf{O}_{L^u E}(\cdot) = 0. \tag{13}$$

To prove (7)–(8), divide the first-order conditions (12)–(13) by μ and rearrange the terms.

To prove (9), write the second and third expressions on its right-hand side of (8) as (ignoring their signs),

$$\frac{\gamma^s u_E^s + \gamma^u u_E^u}{\mu} + \frac{\lambda \left(u_E^s - u_E^{su} \right)}{\mu} = (\gamma^s + \lambda) \frac{u_c^s}{\mu} \frac{u_E^s}{u_c^s} + \gamma^u \frac{u_c^u}{\mu} \frac{u_E^u}{u_c^u} - \lambda \frac{u_E^{su}}{\mu}.$$

Next, from (10) $(\gamma^s + \lambda) u_c^s = \mu \pi^s$ and from (11) $\gamma^u u_c^u = \lambda u_c^{su} + \mu \pi^u$. Substitute into the above expression to rewrite it as

$$\begin{aligned} \frac{\gamma^s u_E^s + \gamma^u u_E^u}{\mu} + \frac{\lambda \left(u_E^s - u_E^{su} \right)}{\mu} &= \pi^s \frac{u_E^s}{u_c^s} + \left(\frac{\mu \pi^u + \lambda u_c^{su}}{\mu} \right) \frac{u_E^u}{u_c^u} - \lambda \frac{u_E^{su}}{\mu} \\ &= \left(\pi^s \frac{u_E^s}{u_c^s} + \pi^u \frac{u_E^u}{u_c^u} \right) + \frac{\lambda}{\mu} u_c^{su} \left(\frac{u_E^u}{u_c^u} - \frac{u_E^{su}}{u_c^{su}} \right) \end{aligned}$$

Substituting this expression into the right-hand side of (8) yields equation (9). ■

In comparing the tax formula for E with that for K, one notes that (i) they have one term that has the same format in both,[8] and (ii) the tax on E, the polluting input, includes two additional terms. This result mirrors Sandmo's (1975) "additive property" derived for taxation of *goods* when wages are constant. There is an "add-on" tax for externality correction given by the additional terms on the right-hand sides of (8)–(9). Observe also that with exogenously fixed wage rates, $\mathbf{O}_{L^s K}(\cdot) = \mathbf{O}_{L^u K}(\cdot) = \mathbf{O}_{L^s E}(\cdot) = \mathbf{O}_{L^u E} = 0$; but the additional terms on the right-hand sides of (8)–(9) do not change.[9]

[8] This term is represented by $\left[\delta^s \mathbf{O}_{L^s K}(\cdot) + \delta^u \mathbf{O}_{L^u K}(\cdot) \right] / \mu$ in (7), and $\left[\delta^s \mathbf{O}_{L^s E}(\cdot) + \delta^u \mathbf{O}_{L^u E}(\cdot) \right] / \mu$ in (8) and (9). Obviously, while $\mathbf{O}_{L^s K}(\cdot)$ has the same format as $\mathbf{O}_{L^s E}(\cdot)$, they are not identical. The same observation applies to $\mathbf{O}_{L^u K}(\cdot)$ and $\mathbf{O}_{L^u E}(\cdot)$.

[9] Indeed, with $\mathbf{O}_{L^s K}(\cdot) = \mathbf{O}_{L^u K}(\cdot) = \mathbf{O}_{L^s E}(\cdot) = \mathbf{O}_{L^u E} = 0$, equation (8) collapses to equation (17), given (16), in Cremer and Gahvari (2001).

Consequently, wage endogeneity affects only the tax component that is common between non-polluting and polluting inputs. It does not affect the structure of the two additional terms.[10]

3 Polluting factor taxes and the structure of preferences

I start by presenting the two prevailing definitions of the Pigouvian tax in the literature. Both definitions equate the Pigouvian tax with the marginal social damage of emissions. They differ in their conceptualization, or measurement, of the marginal social damage.

One definition, due to Cremer *et al.* (1998), discounts the marginal disutility of emissions by the shadow cost of public funds to the society, μ.[11] This conception of the *social* cost takes into account the fact that the society might assign different weights to the disutility of emissions experiences by different individual types (type j has the weight γ^j). It also takes into account the fact that, in the absence of lump-sum taxes, the cost of a dollar to the society, μ, is not the same as its private cost to individuals (u_c^j for individual j). Specifically,

Definition 1 *The Pigouvian tax, τ^{CG}, is the weighted average of the marginal environmental damage of emissions discounted by the shadow cost of public funds, μ. Formally,*

$$\tau^{CG} \equiv \sum_j \gamma^j \left(\frac{-u_E^j}{\mu} \right). \tag{14}$$

[10] Thus, with exogenous wages, $\mathbf{O}_K(\cdot) = r$ and there should be no taxes on non-polluting inputs. With endogenous wages, on the other hand, this is no longer the case because $\mathbf{O}_{L^sK}(\cdot) \neq 0$ and $\mathbf{O}_{L^uK}(\cdot) \neq 0$. In this note, I am interested only in the difference between non-polluting and polluting inputs. I will thus not discuss the properties of the common element between them which constitutes the tax on the non-polluting factor.

[11] The resource constraint (1)

$$\mathbf{O}(\cdot) \geq \pi^s c^s + \pi^u c^u + rK + pE + \overline{R},$$

can also be written as

$$[\mathbf{O}(\cdot) - (rK + pE)] - (\pi^s c^s + \pi^u c^u) \geq \overline{R}.$$

Observe that $\mathbf{O}(\cdot) - (rK + pE)$ is aggregate domestic expenditures (incomes) and $\pi^s c^s + \pi^u c^u$ is the aggregate domestic *private* sector consumption. Put differently, the left-hand side of the latter inequality shows tax revenues so that this constraint can be considered as the government's budget constraint. This is why one can refer to μ as both the shadow price for the resource constraint as well as the shadow cost of public funds.

The second definition, due to Bovenberg and van der Ploeg (1994), is motivated by Samuelson's condition for optimal provision of public goods. The idea here is to sum the private costs of the emission damage over all individuals. Thus marginal disutility of emissions for each individual, $-u_E^j$, is discounted by his own marginal utility of income, or marginal utility of the numeraire consumption good, u_c^j, and then summed over all individuals. According to this conceptualization,

Definition 2 *The Pigouvian tax, τ^{BV}, is the sum of all individuals' marginal rate of substitution between a reduction in environmental damage and the numeraire consumption good. Formally,*

$$\tau^{BV} \equiv \sum_j \pi^j \left(\frac{-u_E^j}{u_c^j} \right). \tag{15}$$

3.1 First-best

Observe first that the two definitions are the same in the first-best. To show this, set $\lambda = 0$ in equations (10) and (11). It then follows that in the first-best, one has

$$\mu = \frac{\gamma^j u_c^j}{\pi^j}.$$

Substituting for μ from this relationship into the definition of τ^{CG} in equation (14) yields equation (15) which is the definition of τ^{BV}.

3.2 Weak-separability in emissions

Assume preferences are weakly separable in emissions and other goods (including labor supply). Under this hypothesis, the utility function is of the form $U = \mathbf{U}(\mathbf{F}(c, I/w), E)$, with the notation $u^j = \mathbf{U}\left(\mathbf{F}\left(c^j, I^j/w^j\right), E\right)$ and $u^{jk} = \mathbf{U}\left(\mathbf{F}\left(c^k, I^k/w^j\right), E\right)$. The following lemma proves that under this assumption the last expression on the right-hand side of (8) vanishes.

Lemma 1 *In the optimal tax problem of Proposition 1, assume preferences are weakly separable in emissions and other goods (including labor supply). Then*

$$u_E^s - u_E^{su} = 0.$$

Proof. With $\lambda > 0$, the "downward" incentive-compatibility constraint is binding so that $u^s = u^{su}$, or

$$\mathbf{U}\left(\mathbf{F}\left(c^s, \frac{I^s}{w^s}\right), E\right) = \mathbf{U}\left(\mathbf{F}\left(c^u, \frac{I^u}{w^s}\right), E\right).$$

It follows from this expression that

$$\mathbf{F}\left(c^s, \frac{I^s}{w^s}\right) = \mathbf{F}\left(c^u, \frac{I^u}{w^s}\right).$$

In turn, this equality implies that

$$u_E^s = \mathbf{U}_E\left(\mathbf{F}\left(c^s, \frac{I^s}{w^s}\right), E\right) = \mathbf{U}_E\left(\mathbf{F}\left(c^u, \frac{I^u}{w^s}\right), E\right) = u_E^{su}.$$

■

Armed with Lemma 1, one can simplify the optimal tax on the polluting input (8) to

$$\mathbf{O}_E(\cdot) - p = \frac{\delta^s \mathbf{O}_{L^s E}(\cdot) + \delta^u \mathbf{O}_{L^u E}(\cdot)}{\mu} - \frac{\gamma^s u_E^s + \gamma^u u_E^u}{\mu}. \tag{16}$$

Comparing this relationship with the optimal tax on non-polluting input K in equation (7) reveals that the only difference is the inclusion of the last expression on the right-hand side of (16) which is not present in (7). This additional expression is equal to τ^{CG} and we have,

Corollary 1 *In the optimal tax problem of Proposition 1, assume preferences are weakly separable in emissions and other goods (including labor supply). Then the "tax difference" between polluting input and non-polluting input is equal to τ^{CG}, the Pigouvian tax à la Cremer et al. (1998).*[12]

3.3 Weak-separability in labor supply

Assume now that preferences are weakly separable in labor supply and other goods (including emissions). Under this hypothesis, $U = \mathbf{U}(\mathbf{G}(c, E), I/w)$ with the notation $u^j = \mathbf{U}\left(\mathbf{G}\left(c^j, E\right), I^j/w^j\right)$ and $u^{jk} = \mathbf{U}\left(\mathbf{G}\left(c^k, E\right), I^k/w^j\right)$. Under this type of

[12] By referring to the last expression on the right-hand side of (16) as the "tax difference," I do not mean to imply that the first expressions on the right-hand sides of (7) and (16) take the same values; only that they have identical formulas.

separability, the marginal rate of substitution between E and c is given by

$$\frac{u_E}{u_c} = \frac{\mathbf{G}_E\left(c,E\right)}{\mathbf{G}_c\left(c,E\right)},$$

which is independent of the labor supply. Thus skilled and unskilled workers have the same marginal rate of substitution between c and E if they choose the same value for c. Consequently, the last expression on the right-hand side of (9) vanishes. This simplifies the expression for the optimal tax on polluting input (9) to

$$\mathbf{O}_E\left(\cdot\right) - p = \frac{\delta^s \mathbf{O}_{L^s E}\left(\cdot\right) + \delta^u \mathbf{O}_{L^u E}\left(\cdot\right)}{\mu} - \left(\pi^s \frac{u_E^s}{u_c^s} + \pi^u \frac{u_E^u}{u_c^u}\right). \tag{17}$$

Comparing (17) with (7) indicates that the two differ only in the inclusion of the last expression on the right-hand side of (17) which is not present in (7). This additional expression is equal to τ^{BV} and we have,

Corollary 2 *In the optimal tax problem of Proposition 1, assume preferences are weakly separable in labor supply and other goods (including emissions). Then the "tax difference" between polluting input and non-polluting input is equal to* τ^{BV}*, the Pigouvian tax à la Bovenberg and van der Ploeg (1994).*

3.4 Strong separability

If both separability conditions hold, both characterizations (16) and (17) remain valid so that $\tau^{CG} = \tau^{BV}$ and the two definitions of the Pigouvian tax coincide. Under this circumstance, preferences will be strongly separable in goods, labor supply, and emissions: $U = \mathbf{\Phi}\left(c\right) - \mathbf{\Psi}\left(h\right) - \boldsymbol{\phi}\left(E\right)$.

3.5 Non-separability

If neither of the separability assumptions hold the optimal emission tax is not levied purely for externality correction. The add on tax has a redistributive component measured by the last term on the right-hand side of (8) or (9). This component imposes a distortion on the use of polluting input in order to affect the level of emissions. The reason for it is that as long as this distortion hurts the skilled labor more than the unskilled labor, it can slacken the otherwise binding incentive compatibility constraint of skilled towards unskilled workers and effect more redistribution towards the unskilled.

4 Concluding remarks

This note has clarified the two definitions of the Pigouvian tax that exist in the literature. It has done so by deriving the properties of optimal non-labor input taxes, one polluting and the other non-polluting, in a Mirrleesian optimal tax model with endogenously determined wages. The results, however, apply to models with Ramsey taxes and the "tax difference" between polluting and non-polluting goods. Specifically, it has shown that the optimal "add-on" tax for externality correction when preferences are separable in emissions and other goods (including labor supply) is what Cremer *et al.* (1998) have dubbed Pigouvian. On the other hand, the optimal add-on tax when preferences are separable in labor supply and other goods (including emissions) is what Bovenberg and van der Ploeg (1994) have termed Pigouvian.

The lesson that emerges is that naming a particular component of a tax that impacts the environment "Pigouvian" is somewhat arbitrary. Leaving the question of the environment aside, whether or not one wants to supplement a general income tax with an additional tax depends on the trade-off between its cost and benefit. The cost is that of introducing a new distortion. The benefit is a possible lowering of the distortion in the labor market (as some of the required tax revenue will now be raised from an additional source), while achieving a more desired redistribution than otherwise would not be attainable (through impacting the incentive compatibility constraints). When taxing goods or factors of production that impact the environment, the same considerations play a role on top of the environmental consequences of the tax. However, these components are not quite separable or additive; they do interact with one another. The Pigouvian tax construct purports to leave out the interactions and isolate the environmental effects.

With the interconnectedness between consumption, labor supply decisions and the environment, however, one cannot generally achieve this separation in toto. There will be effects emanating directly from marginal changes in environmental quality itself as well as from the differences in the valuation of environmental quality that are brought about by changes in the labor supply. These impact incentive compatibility constraints and thus redistribution.[13] In one definition, Cremer *et al.*'s (1998), the direct feedback from changes in environmental quality is taken out from the equation by ruling out any complementarity and/or substitutability relationships between environmental quality and goods/factors of production. In the second definition, Bovenberg and van der Ploeg's (1994), it is the complementarity and/or substitutability relationship between labor supply and goods/factors of production that is ruled out. As a result, in setting the tax rates, one need not take into account the distributional effects originating from the labor market.[14]

[13] If there are other taxes in the system, these changes also affect the extent of the welfare cost associated with those taxes.

[14] Based on this latter type of separability, Kaplow (1996) argues that environmental taxes should be Pigouvian even in a tax reform exercise wherein the tax rates are suboptimal. His definition of Pigouvian is the same as Bovenberg and van der Ploeg's (1994). The argument assumes that the government is able to adjust its income policy whenever it introduces or changes an environmental tax. This raises the following question: If the government is indeed able to adjust its income policy at will, why not set it optimally so that the question becomes one of tax design as in Bovenberg and van der Ploeg (1994) rather than one of tax reform (which is by definition constrained)?

Kaplow's (1996) finding has also been interpreted to mean that the "marginal cost of public funds" is equal to one. This interpretation is unwarranted; see Gahvari (2006). More recently Jacobs (2010) also argues that the marginal cost of public funds is always equal to one. However, Jacobs faults the literature for using a "wrong" concept of the marginal cost of public funds and advocates using a different definition. In his case, the marginal cost of public funds is indeed always equal to one but definitionally. It also requires that the government tax tools include an *optimized* uniform cash rebate (or lump-sum tax).

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