# Income tax reform in France: a case study* 

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#### Abstract

This paper calibrates the graduated income tax system currently in place in France while assuming that the number of earning-ability types in the economy are four. It also computes the optimal linear and nonlinear income tax schedules for this economy. Its main finding is that while an optimal linear income tax is (in most scenarios) welfare superior to the current tax system, the welfare gain may be small. On the other hand, an optimal general income tax leads to substantial welfare gains over the present system.


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[^0]
## 1 Introduction

There are at least three different conceptual issues in the tax reform debate. One is using income or consumption as the tax base. The second is the various tax codes, deductions, exemptions, shelters and the like that often result in differential tax treatment of different sources of income or consumption. The third is the number of the tax rates faced by individuals. These issues are often mixed by the advocates of a flat tax system who generally want to replace income with consumption as the tax base, broaden the tax base by simplifying the existing tax codes, and have a single rate. There is no reason for this. A better understanding of the merits and problems of any tax reform proposal requires one to study these questions separately.

This paper is concerned only with the question of a single rate for the French economy. This is not to suggest that the other two issues are not important. In fact, we believe they are more important. The question of the number of tax rates is much simpler to address. It is often argued that a flat tax is "simpler". There may be some truth in this statement if by simpler one means that it makes it simpler for the taxpayers to always know what marginal tax rates they face. Although, it should not be too difficult for taxpayers to find out their marginal tax rates even if the tax rates vary with income. Moreover, leaving this aside, we do not think that in the age of computers, implementing a tax system with many rates is any more difficult than implementing a system with differential rates. Consequently, in trying to answer this particular question, one important factor is whether one can achieve the same amount of redistribution with a flat tax as one is currently doing without increasing the distortionary effects of the tax system. Put differently, how much do we lose in terms of efficiency if we want to achieve the same redistributive objectives by opting for a single tax rate?

To be sure, one can devise different flat tax systems generating the same tax revenue but each with a different degree of redistributive power. The degree of progressivity depends on the size of the tax rate and the size of the exemptions. To make it easiest for the flat tax to dominate the current tax system, we assume that the flat tax is chosen optimally. Even so, we are able to show that while the flat tax is generally
welfare superior to the current tax system in France, the welfare gain appears to be rather small. The exceptions arises when the inequality aversion index is extremely high. With this combination of parameters, the optimal tax rate is high so that a linear income tax is very progressive and brings about a significant welfare improvement. By way of comparison, we then ask how much improvement one can make over the current system if one adopts an optimal general income tax. The answer is quite a bit.

Using French data on households' income, employment and consumption, the paper models an economy consisting of four groups of individuals who differ in earning abilities. The four groups are identified as "managerial staff", "intermediate-salaried employees", "white-collar workers" and "blue-collar workers". The data enable us to determine the groups's earning abilities, their labor supply and their net-of-tax wages. The data also enable us to calibrate a CES utility function for the households. Optimal tax policies are derived by maximizing an iso-elastic social welfare function.

## 2 The model

The economy consists of four groups of individuals who differ in earning abilities. Normalize the population size to one and denote the fraction of people of type $j$ in total population by $\pi^{j}$. Each person is endowed with one unit of time. He has preferences over labor supply, $L$, and one composite consumption good, $x$. The consumption good is produced by a linear technology subject to constant returns to scale in a competitive environment. The producer price of consumer good is normalized at one. All individual types have CES preferences in goods and labor supply with an identical elasticity of substitution between leisure and consumption goods, $\rho$. Let $w$ denote wage and $I=w L$ income. The preferences for a person of type $j$ can then be represented by

$$
\begin{equation*}
\mho^{j}=\mathbf{U}\left(x, \frac{I}{w^{j}}\right)=\left[a x^{\frac{\rho-1}{\rho}}+(1-a)\left(1-\frac{I}{w^{j}}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \quad j=1,2,3,4, \tag{1}
\end{equation*}
$$

where $a$ is a constant.
Each consumer chooses his consumption level by maximizing (1) subject to his budget constraint. The budget constraint will be nonlinear when the income tax schedule is nonlinear. However, for the purpose of uniformity in exposition, we always characterize
the consumer's choice, even when he faces a nonlinear budget constraint, as the solution to an optimization problem in which he faces a possibly type specific linearized budget constraint. To do this, introduce a "virtual income", $G^{j}$, into each type's budget constraint. Denote the $j$-type's marginal income tax rate by $t^{j}$ and let $w_{n}^{j}=w^{j}\left(1-t^{j}\right)$. We can then write $j$ 's budget constraint as

$$
\begin{equation*}
p x^{j}=G^{j}+M^{j}+w_{n}^{j}\left(\frac{I^{j}}{w^{j}}\right), \tag{2}
\end{equation*}
$$

where $p$ is the consumer price of $x, G^{j}$ is the income adjustment term (virtual income) needed for linearizing the budget constraint (or the lump sum rebate if the tax function is linear), and $M^{j}$ is the individual's exogenous income. Note also that $I^{j}=w^{j} L^{j}$ so that $w_{n}^{j}\left(I^{j} / w^{j}\right)=w_{n}^{j} L^{j}$.

In calibrating the parameters of the utility function (1), we use the procedure described in Cremer et al. (2003). The data come from the "Institut National de la Statistique et des Etudes Economiques" (INSEE). ${ }^{1}$ The data are collected through different surveys taken on representative samples of households. Consumption data are from "Enquête Budget des Familles". ${ }^{2}$ This survey uses a sample of 10,240 households who are grouped into eight categories. Out of the eight categories, only four report any wage incomes. They are classified as: "managerial staff", "intermediate-salaried employees", "white-collar workers" and "blue-collar workers". These categories constitute the four types of individuals in our model. The data covers 117 consumption goods which we aggregate into a single consumption good $(x)$. Data on wages are from the "Enquête emploi", ${ }^{3}$ this survey uses tax returns of a large sample of households. The data are available for the four categories of households we consider. The most recent year for which all the data we need are available is 2006 . Table 1 provides a summary of the data. We consider two alternative values for $\rho, 0.6649$ and 1.3889. ${ }^{4}$ The first value of $\rho$ is our estimate from Cremer et al. (2003); the second is the highest value of $\rho$ found in the literature. ${ }^{5}$ Based on these values of $\rho$, we compute $a$ using the 2006 INSEE data.

[^1]Table 1. Data Summary: 2006
(monetary figures in euro)

|  | $(1)$ |  | $(3)$ | $(4)$ |
| :--- | :---: | :---: | :---: | :---: |
|  | Managerial Staff | Intermediary Level | White Collars | Blue Collars |
| $\pi^{j}$ | $(\%)$ | 16.89 | 25.84 | 32.16 |
| $p x$ | 44424.0 | 33671.3 | 26021.4 | 25.12 |
| $L$ | 0.526250 | 0.463750 | 0.416250 | 26002.0 |
| $w$ | 73079.3 | 44800.1 | 30355.8 | 0.456250 |
| $t$ | $(\%)$ | 14.00 | 5.50 | 0.00 |
| $w_{n}$ | 62848.2 | 42336.1 | 30355.8 | 30837.6 |
| $M$ | 7724.9 | 13072.6 | 13385.8 | 0.00 |
| $G$ | 3625.2 | 965.4 | 0.0 | 11937.6 |

The result is $a=0.5200$ when $\rho=0.6649$ and $a=0.6264$ when $\rho=1.3889$.

## 3 The French benchmark tax system

The French benchmark tax system is a simplified version of the French economy. It differs from the "real" French tax system in two key assumptions. The first is that the population is comprised of only four types of households; the second is that all households work. Specifically we solve the model of Section 2 using the calibrated parameters values there and the actual observed values for the tax rates in France. However, rather than using the calculated values of the exogenous incomes, we set all $M^{j}$ 's equal to zero. We do this to make the comparison with the optimal tax structure, which assumes no exogenous income, meaningful. ${ }^{6}$ The solution is presented in Table 2.

The values reported in Table 2 differ from the actual observed values given in Table 1 in two important respects. First, labor supply for all types are somewhat higher in the benchmark system than their actual observed values. These differences reflect the labor supplies of people we have ignored by assuming that the economy consists of only four types on individuals. The second difference appears in consumption levels. The assumption of no capital income results in expenditure levels that are lower than the observed ones. The variable $A T$ in Table 2 refers to the average income taxes. Observe

[^2]Table 2. The current system: 2006
(monetary figures in euro)

|  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) |
|  | Managerial Staff | Intermediary Level | White Collars | Blue Collars |
| $\pi^{j} \quad(\%)$ | 16.89 | 25.84 | 32.16 | 25.12 |
| $p x$ | 36696.8 | 25310.6 | 18556.9 | 18812.7 |
| $L$ | 0.526214 | 0.575047 | 0.611313 | 0.610058 |
| $w$ | 73079.3 | 44800.1 | 30355.8 | 30837.6 |
| $t$ (\%) | 14.00 | 5.50 | 0.00 | 0.00 |
| $w_{n}$ | 62848.2 | 42336.1 | 30355.8 | 30837.6 |
| $G$ | 3625.2 | 965.4 | 0.0 | 0.0 |
| AT (\%) | 4.6 | 1.8 | 0.0 | 0.0 |

Monetary figures in euro
that the benchmark system is progressive with the average tax payments increasing from $0 \%$ for Type 4 to $4.6 \%$ for Type 1 .

Finally, in comparing the current tax system with optimal linear and nonlinear tax systems, we shall assume that the government's external revenue requirement (expenditures on non-transfer payments) is equal to $20 \%$ of France's GDP (at the first-best allocations). This corresponds to approximately the actual value of these expenditures in France.

## 4 Welfare

The derivations of optimal flat and nonlinear income tax systems are based on the assumption that the society's welfare is measured by an iso-elastic social welfare function of the form

$$
\begin{align*}
W & =\frac{1}{1-\eta} \sum_{j=1}^{4} \pi^{j}\left(\mho^{j}\right)^{1-\eta}, \quad \eta \neq 1 \quad \text { and } \quad 0 \leq \eta<\infty  \tag{3}\\
& =\sum_{j=1}^{4} \pi^{j} \ln \mho^{j}, \quad \eta=1
\end{align*}
$$

where $\eta$ is the "inequality aversion index". The value of $\eta$ dictates the desired degree of redistribution in the economy: The higher is $\eta$ the more the society cares about equality. ${ }^{7}$

[^3]In choosing a value for $\eta$ for our optimal tax calculations, we have been guided by the observed degree of redistribution in the existing French tax system. Using Bourguignon and Spadaro (2000), Cremer et al. (2003) show that the French tax system is rationalized by a value of $\eta$ between 0.1 and 1.9.

To make welfare comparisons across types, we report the equivalent variation, $E V$, of a policy change from the "benchmark allocation" $b$ to a tax alternative. Thus, for each type $j=1,2,3,4$, we calculate an $E V^{j}$ from the following relationship

$$
\mathbf{v}\left(w_{n, b}^{j}, G_{b}^{j}+E V_{i}^{j}\right)=\mathbf{v}\left(w_{n, i}^{j}, a_{i}^{j}\right),
$$

where subscript $b$ denotes the benchmark, subscript $i$ refers to one of the two tax options of optimal flat tax or optimal nonlinear income tax, and $a_{i}^{j}$ stands for the lump-sum income of individual $j$ under tax system $i$. Observe that $a_{i}^{j}$ is the "virtual income" under a general income tax.

Finally, to make welfare comparisons for the society, we use the concept of the "social equivalent variation", $E V^{s}$. We define this analogously to $E V$ from the following relationship,

$$
\frac{1}{1-\eta} \sum_{j=1}^{4} \pi^{j} \mathbf{v}\left(w_{n, b}^{j}, G_{b}^{j}+E V^{s}\right)^{1-\eta}=\frac{1}{1-\eta} \sum_{j=1}^{4} \pi^{j} \mathbf{v}\left(w_{n, i}^{j}, a_{i}^{j}\right)^{1-\eta} .
$$

## 5 The optimal linear income tax

The flat tax alternative considered is the linear income tax system with a lump-sum element. This is a more powerful system than a flat tax with an exemption level. Denote the tax rate by $t$ and the lump-sum element by $a$. Normalize the consumer price of $x$ to one. ${ }^{8}$ An individual of type $j$ would then face a budget constraint $x=w^{j}(1-t) L+a=$ $w_{n}^{j} L+a$. Maximizing (1) with respect to $x$ and $L$ subject to this budget constraint yields

$$
\begin{equation*}
x^{j}=\mathbf{x}\left(w_{n}^{j}, a\right) ; \quad L^{j}=\mathbf{L}\left(w_{n}^{j}, a\right) . \tag{4}
\end{equation*}
$$

Using (4), one derives the $j$-type's indirect utility function: $\mathbf{v}\left(w_{n}^{j}, a\right)$.

[^4]Table 3
The optimal linear income tax $(\rho=0.665)$

|  |  | $\eta=0.035$ | $\eta=0.1$ | $\eta=0.5$ | $\eta=1.0$ | $\eta=5.0$ | $\eta=10.0$ | $\eta=30$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $t(\%)$ | 13.17 | 13.96 | 18.16 | 22.20 | 36.58 | 41.93 | 44.93 |  |
| $G$ |  | 2657.8 | 2834.0 | 3751.0 | 4604.1 | 7359.2 | 8249.3 | 8710.3 |
| $A T^{1}(\%)$ | 6.3 | 6.7 | 8.4 | 10.2 | 16.8 | 19.5 | 21.1 |  |
| $A T^{2}$ | $(\%)$ | 2.6 | 2.7 | 3.1 | 3.4 | 4.8 | 5.4 | 5.7 |
| $A T^{3}$ | $(\%)$ | -1.8 | -2.1 | -3.5 | -4.9 | -10.8 | -13.4 | -14.9 |
| $A T^{4}$ | $(\%)$ | -1.6 | -1.8 | -3.2 | -4.5 | -10.0 | -12.4 | -13.9 |
| $E V^{1}$ | -645.4 | -776.7 | -1493.6 | -2219.7 | -5153.3 | -6421.1 | -7185.6 |  |
| $E V^{2}$ | -269.0 | -293.2 | -433.3 | -588.7 | -1354.3 | -1751.8 | -2010.4 |  |
| $E V^{3}$ | 272.7 | 310.8 | 505.6 | 679.6 | 1149.3 | 1240.1 | 1263.5 |  |
| $E V^{4}$ | 237.9 | 273.9 | 457.2 | 619.8 | 1047.1 | 1121.2 | 1135.0 |  |
| $E V^{s}$ | 8.3 | 12.2 | 45.9 | 104.6 | 626.1 | 965.5 | 1204.1 |  |

Monetary figures in euro

The government's problem is to choose $t$ and $a$ in order to maximize

$$
\begin{equation*}
\frac{1}{1-\eta} \sum_{j=1}^{4} \pi^{j} \mathbf{v}\left(w_{n}^{j}, a\right)^{1-\eta} \tag{5}
\end{equation*}
$$

subject to its revenue constraint

$$
\begin{equation*}
\sum_{j=1}^{4} \pi^{j}\left[t w^{j} L^{j}-a\right] \geq \bar{R} \tag{6}
\end{equation*}
$$

where $\bar{R}$ is the government's external revenue requirement.
Two parameters determine the specifics of the optimal tax solution. One is the elasticity of substitution between consumption and leisure, $\rho$. The higher is this parameter, the more distortionary is a given tax rate. Thus as $\rho$ increases, the optimal tax rate decreases. As stated earlier, our calculations are based on two extreme values of $\rho$ found in the literature. ${ }^{9}$ The other parameter is $\eta$, the inequality aversion index. The higher is $\eta$, the more redistributive is the tax system leading to a higher tax rate. Table 3 reports the optimal income tax rate, $t$, and the corresponding lump sum element of the income tax schedule for $\rho=0.665$ at different values of $\eta$. As expected, $t$ increase with $\eta$ varying from $13.17 \%$ for $\eta=0.035$ to $51.7 \%$ for $\eta=30$.

[^5]The average tax payments show the extent of the progressivity of the optimal flat tax. When the inequality aversion index is lowest, the flat tax whose average tax rates vary from $-1.8 \%$ to only $6.3 \%$ is slightly more progressive than the current tax system with rates from $0 \%$ to $4.6 \%$. Given this higher level of tax progressivity, it is not surprising that this tax system is more beneficial to the poor as compared to the rich. The equivalent variation values show how the rich and the poor fare as compared to the present system. Observe also that $E V^{s}$ is positive which means that the society as a whole is better off.

As $\eta$ increases, the gain to the low-ability types and the loss to the high-ability types increase. At $\eta=1.0$, the flat tax has average tax rates of $-4.5 \%,-4.9 \%, 3.4 \%$, and $10.2 \% ; E V^{s}=104.6$ and the society as a whole is better off. Observe that, except for very large values of $\eta$, welfare improvement remains modest (although gains and losses for different income groups are significant).

Similarly, Table 4 reports the optimal income tax rate and the corresponding lump sum element of the income tax schedule for $\rho=1.39$. This time around, $t$ varies between $8.82 \%$ and $33.93 \%$ as compared to $13.17 \%$ to $44.93 \%$ range we reported in Table 3. Indeed, every single tax rate associate with a particular value of $\eta$ is now smaller than previously. The reason is the much higher value of $\rho$, the elasticity of substitution between leisure and consumption goods ( 1.39 versus $\rho=0.665$ ). A higher value of $\rho$ implies that a particular tax rate will be more distortionary. Consequently, the optimal tax rate at every value of $\eta$ is now smaller than previously. As far as welfare comparisons are concerned, the general pattern appears to be the same as previously. That is, the rich (Types 1 and 2) lose while the poor (Types 3 and 4) gain from a switch to the optimal linear income tax. The only exception is that Type 1 gains at the two low values of $\eta$ ). Again, as previously, the losses of the rich and the gains of the poor increase with $\eta$ which is not surprising. At the social level, $E V^{s}$ is negative initially but turns positive, with the gains becoming larger, as $\eta$ increases.

Table 4
The optimal linear income tax $(\rho=1.389)$

|  | $\eta=0.035$ | $\eta=0.1$ | $\eta=0.5$ | $\eta=1.0$ | $\eta=5.0$ | $\eta=10.0$ | $\eta=30$ |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $t(\%)$ | 8.82 | 9.43 | 12.67 | 15.80 | 26.96 | 31.06 | 33.93 |  |
| $G$ | 1525.6 | 1661.6 | 2361.5 | 3002.2 | 4971.1 | 5557.3 | 5917.0 |  |
| $A T^{1}(\%)$ | 5.5 | 5.8 | 7.4 | 9.1 | 15.2 | 17.7 | 19.4 |  |
| $A T^{2}$ | $(\%)$ | 2.9 | 3.0 | 3.3 | 3.6 | 5.0 | 5.6 | 6.0 |
| $A T^{3}$ | $(\%)$ | -0.7 | -1.0 | -2.6 | -4.2 | -11.3 | -14.5 | -17.0 |
| $A T^{4}$ | $(\%)$ | -0.5 | -0.8 | -2.3 | -3.8 | -10.4 | -13.3 | -15.7 |
| $E V^{1}$ | 243.3 | 101.9 | -664.3 | -1431.6 | -4425.2 | -5648.8 | -6549.0 |  |
| $E V^{2}$ | -305.0 | -326.9 | -455.4 | -601.0 | -1329.0 | -1697.2 | -1993.0 |  |
| $E V^{3}$ | 63.1 | 102.7 | 302.4 | 477.7 | 927.8 | 1007.6 | 1029.4 |  |
| $E V^{4}$ | 34.6 | 72.1 | 261.0 | 425.7 | 836.5 | 901.3 | 912.3 |  |
| $E V^{s}$ | -9.7 | -9.5 | 4.6 | 43.2 | 475.0 | 766.2 | 974.8 |  |

Monetary figures in euro

## 6 The optimal general income tax

We now turn to the case of a general income tax $T(I)$, which imposes no a priori restriction on the functional form of the tax function. To determine the optimal tax function we proceed in the usual way by determining first the optimal incentive compatible allocation; see e.g., Stiglitz (1987). From this solution we can then deduce the properties of the implementing income tax function and specifically the marginal and average tax rates faced by the different types of individuals. ${ }^{10}$

With this in mind, we derive $x^{j}$ and $I^{j}$ as the solution to the following problem for the government. Maximize

$$
\begin{equation*}
\frac{1}{1-\eta} \sum_{j=1}^{4} \pi^{j} \mathbf{U}\left(x^{j}, \frac{I^{j}}{w^{j}}\right)^{1-\eta}, \tag{7}
\end{equation*}
$$

with respect to $x^{j}$ and $I^{j}$, subject to the resource constraint

$$
\begin{equation*}
\sum_{j=1}^{4} \pi^{j}\left(I^{j}-x^{j}\right) \geq \bar{R}, \tag{8}
\end{equation*}
$$

[^6]and the incentive compatibility constraints, for $j \neq k ; j, k=1,2,3,4$,
\[

$$
\begin{equation*}
\mathbf{U}\left(x^{j}, \frac{I^{j}}{w^{j}}\right) \geq \mathbf{U}\left(x^{k}, \frac{I^{k}}{w^{j}}\right) . \tag{9}
\end{equation*}
$$

\]

Having determined the optimal allocations $\left(x^{j}, I^{j}\right)$, we then determine $t^{j}$, the $j$-type's marginal income tax rate required to implement these allocations. Moreover, if implementation is to be carried out through a menu of linear income tax schedules (possibly truncated), we calculate the required lump-sum tax to be levied on the $j$-type, $a^{j}$, from equation (2). ${ }^{11}$

Table 5 reports the marginal income tax rates for $\rho=0.665$ at different values of $\eta$. Not surprisingly, the optimal marginal income tax rate is always zero for the highest ability type. This particular result must be treated with great care. Had we been able to identify a small bracket of top-skills people, the marginal income tax rate would have been zero only for that small bracket and not the very sizable group we have. Consequently, in practice, so many people at the top should not face a zero marginal income tax rate. More interestingly, leaving the top segment aside, the rest of the optimal marginal income tax schedule is U-shaped; that is, as income increases, it first declines and then increases. This pattern is robust to variations in $\eta$ and remains unchanged for all values of $\eta$ that we report. It is consistent with the finding of Diamond (1998) who showed the theoretical possibility of having a U-shaped marginal tax rate if preferences are quasi-linear.

The average tax rates indicate how powerful the general income tax is in achieving progressivity. Even when $\eta$ takes its lowest value, average tax rates stand at $-14.87 \%$,$14.87 \%, 3.7 \%$, and $22.09 \%$. Contrast this with the current average tax rates of $0 \%, 0 \%$, $1.8 \%$, and $4.6 \%$. Note also the comparison with average tax rates of $-1.6 \%,-1.8 \%, 2.6 \%$, and $6.3 \%$ under the optimal flat tax system with the same values of $\rho$ and $\eta$. In $E V$ terms, and as compared to the current system, the two poor groups (Types 3 and 4) gain 2,528 and 2,549 euro each, while the two rich groups lose 556 and 7,286 euro each. The gain to the society amounts to 468 euro.

[^7]Table 5
The optimal general income tax ( $\rho=0.665$ )

|  |  | $\eta=0.035$ | $\eta=0.1$ | $\eta=0.5$ | $\eta=1.0$ | $\eta=5.0$ | $\eta=10.0$ | $\eta=30$ |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $t^{1}(\%)$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  |
| $t^{2}$ | $(\%)$ | 14.81 | 15.34 | 18.30 | 21.38 | 34.07 | 39.51 | 42.87 |
| $t^{3}$ | $(\%)$ | 6.35 | 6.72 | 8.79 | 11.08 | 22.59 | 29.50 | 37.66 |
| $t^{4}(\%)$ | 10.59 | 10.92 | 12.85 | 14.98 | 25.74 | 32.24 | 39.93 |  |
| $A T^{1}(\%)$ | 22.09 | 22.19 | 22.78 | 23.42 | 26.47 | 28.13 | 29.69 |  |
| $A T^{2}$ | $(\%)$ | 3.73 | 3.73 | 3.68 | 3.63 | 3.33 | 3.38 | 4.31 |
| $A T^{3}$ | $(\%)$ | -14.87 | -14.97 | -15.55 | -16.19 | -19.63 | -22.04 | -25.58 |
| $A T^{4}$ | $(\%)$ | -14.87 | -14.97 | -15.55 | -16.19 | -19.63 | -22.04 | -25.58 |
| $E V^{1}$ | -7286.10 | -7333.26 | -7602.75 | -7897.19 | -9331.17 | -10131.92 | -10892.95 |  |
| $E V^{2}$ | -556.43 | -560.26 | -585.91 | -621.24 | -909.18 | -1170.80 | -1566.52 |  |
| $E V^{3}$ | 2549.91 | 2561.91 | 2628.21 | 2696.20 | 2958.60 | 3053.86 | 3108.22 |  |
| $E V^{4}$ | 2528.00 | 2539.09 | 2600.11 | 2662.20 | 2893.93 | 2969.84 | 3000.31 |  |
| $E V^{s}$ | 468.23 | 496.08 | 662.35 | 857.25 | 1939.39 | 2543.18 | 3023.79 |  |

Monetary figures in euro

To put these numbers into perspective, recall that under a flat tax system with the same values of $\rho$ and $\eta$, the two poor groups gain 238 and 273 euro each while the two rich groups lose 269 and 645 euro each, resulting in a gain of 8 euro to the society. While the pattern of gain and losses are similar it remains that the differences are quite spectacular. Put differently, while the flat tax appears to be a step in the right direction, it is a "small" step compared to the welfare improvement that can be achieved through a general income tax.

Table 5 also shows that average tax rate is negative for the two poor groups and positive for the two rich groups at all values of $\eta$. Thus the two poor groups always receive an income subsidy from the government. The equivalent variation values indicate the extent to which, in comparison to the present system, the two poor groups become better off and the two rich groups worse off. The society is also always better off. Moreover, as $\eta$ increases, the tax system becomes more progressive. This shows itself in the fact that average tax rates and equivalent variation values increase in absolute value for all types as $\eta$ increases.

The marked difference between redistributive and welfare properties of non-linear
and linear income tax schedules contradicts the earlier results of Slemrod et al. (1994) and Saez (2001, 2002). They find that an optimal linear income tax captures most of the redistributive potential of the optimal general income tax. The reason for this difference is that these authors work with a continuum distribution of skills. We, on the other hand, derive a discrete distribution for skills based on actual French data. This difference has its most dramatic effect on the tax treatment of the richest people. Whereas the zero marginal income tax rate at the top is irrelevant for continuous distribution of skills, it applies in our setup with full force. Recall that we not only have a top-skills group but a sizable one at $16.89 \%$. Both of these approaches are extreme. Ideally, one wants to derive a discrete skills distribution which is based on actual data, but also allows for a lot more groups than four which we have. Unfortunately, the available data does not allow us to consider this more realistic setting. ${ }^{12}$

The huge welfare gains associated with the optimal non-linear tax system are due to the starkly different pattern of optimal marginal tax rates from those currently in place in France. In the current system, marginal tax rate is very low initially and then continuously increase. On the other hand, the optimal tax pattern requires the marginal income tax rates to decline in income initially before starting to increase. Thus the most empirically relevant policy conclusion here is a reorientation of the existing marginal income tax rates to the optimal U-shaped pattern that we have derived here. Table 6 presents the same picture as Table 5 for the higher value of $\rho=1.39 .{ }^{13}$ Here too the optimal marginal income tax rates is U-shaped for all values of $\eta$ except for the highest ability types who should face a zero marginal income tax rate. The two poor

[^8]Table 6
The optimal general income tax ( $\rho=1.389$ )

|  |  | $\eta=0.035$ | $\eta=0.1$ | $\eta=0.5$ | $\eta=1.0$ | $\eta=5.0$ | $\eta=10.0$ | $\eta=30$ |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $t^{1}(\%)$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  |
| $t^{2}$ | $(\%)$ | 10.32 | 10.78 | 13.36 | 16.02 | 26.90 | 31.44 | 34.06 |
| $t^{3}$ | $(\%)$ | 4.40 | 4.67 | 6.23 | 7.95 | 16.69 | 22.07 | 28.72 |
| $t^{4}(\%)$ | 6.99 | 7.25 | 8.74 | 10.38 | 18.74 | 23.90 | 30.32 |  |
| $A T^{1}(\%)$ | 19.90 | 20.01 | 20.64 | 21.33 | 24.54 | 26.22 | 27.73 |  |
| $A T^{2}$ | $(\%)$ | 3.17 | 3.16 | 3.10 | 3.01 | 2.50 | 2.48 | 3.73 |
| $A T^{3}$ | $(\%)$ | -15.12 | -15.26 | -16.07 | -16.97 | -21.93 | -25.61 | -31.55 |
| $A T^{4}$ | $(\%)$ | -15.12 | -15.26 | -16.07 | -16.97 | -21.93 | -25.61 | -31.55 |
| $E V^{1}$ | -6828.15 | -6886.02 | -7215.18 | -7572.11 | -9269.66 | -10178.74 | -11000.42 |  |
| $E V^{2}$ | -389.33 | -392.95 | -418.40 | -455.46 | -776.81 | -1074.46 | -1534.94 |  |
| $E V^{3}$ | 2342.51 | 2356.81 | 2435.17 | 2514.36 | 2808.73 | 2911.10 | 2970.36 |  |
| $E V^{4}$ | 2323.94 | 2337.37 | 2410.70 | 2484.30 | 2749.47 | 2833.21 | 2868.61 |  |
| $E V^{s}$ | 432.74 | 459.90 | 622.35 | 813.07 | 1863.03 | 2433.33 | 2887.21 |  |

Monetary figures in euro
groups have a negative, and the two rich groups a positive, average tax rate. The switch from the current system makes the two poor groups better off and the two rich groups worse off at all values of $\eta$. The society always becomes better off. Furthermore, the average tax rates and equivalent variation values increase in absolute value for all types as $\eta$ increases. The basic difference with the numbers in Table 5 is that the marginal income tax rate is now smaller for all types at every value of $\eta$ (except, of course, for the highest ability type). In consequence, at every value of $\eta$, the equivalent variation numbers are smaller in absolute value.

## 7 Conclusion

This paper has studied the properties of two alternative tax structures to the current graduated tax system in France. The comparisons are based on a calibrated and simplified model of the French economy containing four types of households with CES preferences. We have found that the switch to a flat tax leads at best to a small (and possibly negative) welfare improvement unless the inequality aversion index for the society is extremely high. On the other hand, moving to an optimal general income tax
improves welfare over the current system tremendously. To generate this, the marginal income tax rates in France that are currently set to increase with income should be replaced with one that is U-shaped.

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[^1]:    ${ }^{1}$ All data are publicly available at http://www.insee.fr/. A file with a list of the detailed links to the data used is available from the authors upon request.
    ${ }^{2}$ Survey of family budgets.
    ${ }^{3}$ Employment survey.
    ${ }^{4}$ This implies labor supply elasticities of 0.0546 for $\rho=0.6649$ and of 0.5611 for $\rho=1.3889$.
    ${ }^{5}$ The value of $\rho$ in the literature is in the range $[0.61,1.39]$.

[^2]:    ${ }^{6}$ If there is an exogenous income, the optimal tax scheme simply taxes it away.

[^3]:    ${ }^{7}$ As is well-known, $\eta=0$ implies a utilitarian social welfare function and $\eta \rightarrow \infty$ a Rawlsian.

[^4]:    ${ }^{8}$ With the demand functions for goods and supply of labor being homogeneous of degree zero in the prices and wage, there is an extra degree of freedom in setting one of the consumer prices (in addition to one of the producer prices).

[^5]:    ${ }^{9}$ The in-between values of $\rho$ yield the same general picture.

[^6]:    ${ }^{10}$ To be precise these are the tax rates associated with the income levels earned by the different types. The implementing tax function depends on income and not on the type per se. As usual in discrete type models, the solution provides a full characterization of the tax function only at some points (four in our case). For other income levels there are degrees of freedom in designing the implementing tax function (which is thus not uniquely defined).

[^7]:    ${ }^{11}$ This will be the value that $G^{j}+M^{j}$ has to take in order that equation (2) is satisfied at the given value of $t^{j}$.

[^8]:    ${ }^{12}$ As in virtually all optimal tax studies, the underlying model of the economy is a Walrasian general equilibrium model (with linear technologies). This means that (involuntary) unemployment is ignored. The presence of involuntary unemployment is likely to affect the optimal progressivity of the income tax but it is not clear a priori in which direction. Sørensen (1999) points out that according to modern labour market theories an increased tax progressivity is likely to reduce unemployment. However, it also tends to reduce work effort and labor productivity. Simulations presented by this author suggest that the optimal degree of progressivity of the income tax under unemployment could be quite large. Consequently, one would expect the superiority of the nonlinear tax to be reinforced if unemployment is accounted for. The validation of this conjecture is, however, a challenging task which we have to leave for future research.
    ${ }^{13}$ The referee pointed out that we have many reasons to believe that, the elasticity of substitution between leisure and consumption, $\rho$, is heterogenous across types, and that this may also reinforce the desirability of nonlinear taxation over a flat tax scheme. We thank the referee for this observation which adds further strength to our conclusion.

