Pensions and fertility: in search of a link

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Abstract

In overlapping generations models with endogenous fertility wherein the retired partake of consumption but do not contribute to production (through their labor), fertility has a positive and a negative externality. These can be internalized through a child allowance (or tax) or a linkage between pension benefits and the number of children. The prescription rest crucially on the assumption that no parents are better than others in raising their children and that fertility can be perfectly controlled. When either of these two assumptions are violated, the case for such policy recommendations are greatly weakened.

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1 Introduction

The relationship between fertility and pensions is a complex one. It is often argued that in the presence of a pay-as-you-go (PAYGO) social security system, there is a positive externality associated with having children which, if not corrected, implies that the number of children in a decentralized economy would be suboptimal. At the same time, it is also argued that a declining fertility will shrink the tax base and undermine the financial solvency of the PAYGO pension systems. To counter this, many economists have recently advocated a policy of linking pension benefits, and/or contributions, to the number of children.[1] I will reexamine both of these issues, the purported link and the policy recommendation to overcome the externality problem, in this paper.

The externality is generally stated in terms of the rate of return to PAYGO pension plans. This rate of return, it is argued, depends on the average number of children. The higher the number of children, the higher will be the available tax revenues—levied on the children when they grow up—to finance the pensions of the retired population. Hence when a particular parent has more children, he bestows benefits on all parents. On the other hand, the cost of rearing children is borne entirely privately.

This argument is somewhat misleading in that it connotes that the externality exists because of the PAYGO institution. Yet the same externality is at work in a fully funded system (as long as the economy's resource constraint is satisfied on a per period basis). The crucial point is that the retired partake of consumption but do not contribute to production (through their labor, that is). This implies that, given the same number of young people and the same amount of capital, a smaller population of the old people allows the per capita consumption of the old to increase without any effect on the per capita consumption of the young. Observe also that a smaller population size of older to younger people means a higher fertility rate. There is, however, a downside to this as well. A smaller population of older people results in a smaller level of aggregate

[1] See, among others, Van Groezen *et al.* (2000, 2003), Bental (1989), Kolmar (1997), Abio *et al.* (2002), Fenge and Meier (2003).

capital and aggregate output (assuming a constant savings rate). These two offsetting effects have been referred to in the literature as "intergenerational transfer" and "capital dilution" effects; see Michel and Pestieau (1993) and Cigno (1993).

A second and related issue that I shall discuss concerns the "quality" of children and their human capital accumulation through educational decisions of their parents. The point is that the number of workers is not the only thing that matters for the viability of the pension system. Availability of resources to fund pensions depends not just on the number of the workers, but also on their productivity. The more productive the children, the higher will be their ability to produce and to pay taxes. This reinforces the public good nature of a family's child-rearing activities.[2]

I will not distinguish between quality and quantity decisions in this paper. The key distinguishing element between the two is one of timing. The number of children born is known early; the quality of children, i.e. their future earning capacity, is determined much later. To account for both features, one needs a model with at least three periods of decision making. This makes the presentation of the issues more complicated than necessary.[3] I thus do not specifically distinguish between fertility and educational investment decisions. Instead, I lump the investments in quantity and quality together as if one decision determines both. This simplifies the modeling substantially by allowing me to concentrate on a setting with two periods of decision making. In so doing, I use the concept of number of children in efficiency units that is widely used in growth theory.

Two issues with far reaching implications arise in assessing the relationship between the quantity/quality of children and pensions, as well as in formulating policy recommendations. These are heterogeneity of parents in child-rearing, or in tastes for children, and the uncertainty associated with the number or quality of children they will end up with. Heterogeneity arises because child rearing, like any other activity, requires spe-

[2] On this, see Cigno *et al.* (2003).

[3] Cremer *et al.* (2008b) have recently studied this problem. Cigno and Luporini (2003) too have such a model; however they do not optimize over tax instruments.

cific skills not shared equally by everyone. Uncertainty arises because one can not fully control the future quality of children through education. Infertility, premature death, misplanning, and multiple births imply that one can not fully control quantity either.

Heterogeneity matters because it introduces an element of redistribution into the picture. In consequence, one may want to redistribute towards parents who find themselves to be poor because child rearing is too costly for them. Under this circumstance, there may exist a tradeoff between the two objectives of having the "right" quality/quantity of children and redistribution. The question of the tradeoff is all the more pertinent when the parents' child-rearing ability is not publicly observable and we have an adverse selection problem.

Uncertainty matters because it introduces an element of insurance into the picture. In consequence, one may want to insure parents against the risk of becoming poor because of ending up with too many children.[4] Providing benefits independently of the number of children can be viewed as a mechanism to insure parents against becoming destitute. This raises the question of balancing the benefits of providing insurance to the population against the costs of their having a suboptimal number of children (because of the externality problem). The question of the tradeoff is all the more pertinent when the parents' effort (investment level) is not publicly observable and we have a moral hazard problem.

Ideally, one wants to have one single model that can address all these issues together. It is easier, however, to study them one by one and introduce the needed features as the discussion proceeds. This is the path that I will pursue. I start with a short background. I then present a basic overlapping generations model with endogenous fertility that underlies most of my discussion. Initially, I assume that fertility is deterministic. This highlights the essence of the externality argument and the policy recommendation for subsidizing children. Next, I introduce heterogeneity into the model and, with it, the

[4] I am implicitly assuming here that children are not expected to support their parent in retirement (as would be the case with a PAYGO pension plan is place). Otherwise the risk one faces in becoming poor will be associated with having too few or less able children; see Sinn (2004).

associated adverse selection problem. I follow this by introducing uncertainty and the associated moral hazard problem into the basic model. In each case, I explain what the additional complications and insights are, what they imply about a link between pensions and fertility, and how they might affect the simple policy recommendation of child subsidies.

2 Background

The starting place for the literature on the externality associated with fertility decisions and its subsidization is Samuelson (1975). Although the literature on optimal population size is a much older one, Samuelson (1975) was the first paper that studied the problem within the context of an overlapping generations model—the model that underlies much of the recent literature on this issue.[5] To be sure, Samuelson did not model fertility endogenously. Instead, he was interested in the welfare ranking of the steady states that emerge under different fertility rates (which he considered to be exogenous). He did this on the basis of three different models each yielding a different answer.

The first model Samuelson considered was the neoclassical Solow-Swan growth model. Let y, k, c denote the steady state values of output, capital, and consumption, all in per capita terms. Denote a representative parent's number of children by n so that the population growth rate is $n-1$. Aggregate production function for the economy is represented by a smooth, twice-differentiable, and strictly concave function $y = f(k)$. Given any n, per capita consumption is then equal to $c = f(k) - (n-1)\,k$ where $(n-1)\,k$ is the net addition to capital per period. The value of k that maximizes c is

$$c(n) \equiv \max_{k} \;\; \{c = f(k) - (n-1)\,k\}\,. \tag{1}$$

The optimization over k yields the "golden rule" condition: $f'(k) = (n-1)$. The optimal value of n is then found from the maximization of $c(n)$. It follows from the

[5] Dasgupta (1969) is an interesting paper set within the context of the neoclassical growth models. See Meade (1955) for an analysis of the earlier static question of the "optimum size of population."

envelope theorem that

$$\frac{dc(n)}{dn} = \frac{\partial c}{\partial n} = -k^* < 0. \tag{2}$$

That is, given any value for n, one can always find a smaller value for it that increases per capita consumption. Thus, in this model, as population declines per capita consumption increases as people live off "narrowing" of capital.

Next, Samuelson studied the problem using his own version of an overlapping generations model first presented in Samuelson (1958). In this model, one transfers resources across time through a storage technology. Let r denote the rate of return on storage, c denote present consumption, and d future consumption. Each generation is born with a certain level of exogenous income, y, in its first period of life (and receives nothing further in the second period). There are two steady-state solutions in this model. In one, people use the storage technology to transfer resources to the second period. Their optimization problem is one of choosing c and d to maximize $u(c,d)$ subject to $c + d/(1+r) = y$. This can be written as

$$u(r) = \max_{d} \left\{ u(c,d) = u\left(y - \frac{d}{1+r}, d \right) \right\}. \tag{3}$$

The second steady-state solution is the one in which the young finance the consumption of the old. This is summarized by

$$u(n) = \max_{d} \left\{ u(c,d) = u\left(y - \frac{d}{n}, d \right) \right\}. \tag{4}$$

Observe that the first-steady state solution is independent of n, while the second is increasing in n. Specifically,

$$\frac{du(n)}{dn} = \frac{\partial u}{\partial n} = \frac{d}{n^2} u_c\left(c,d\right) > 0. \tag{5}$$

Samuelson (1958) called $n-1$ the "biological rate of interest." One wants this rate, which is an alternative to the rate of return on storage, to be as high as possible.

The third model Samuelson studied was Diamond's (1965) version of the overlapping generations model. Here, as with the standard neoclassical growth model, production uses labor and capital. People transfer resources across time through purchases of capital goods which earns them a rate of return that, unlike the Samuelson's model, is not fixed and depends on the capital labor ratio in production. The economy's resource constraint in per capita terms is now given by

$$f(k) \geqq c + \frac{d}{n} + (n-1)\,k. \tag{6}$$

The maximal steady state, for a given n, is

$$u(n) \equiv \max_{k,d} \left\{ u(c,d) = u\left(f(k) - (n-1)\,k - \frac{d}{n}, d \right) \right\}. \tag{7}$$

The optimal value of n is that which maximizes $u(n)$. *If* the solution to this problem is interior, it yields the goldenest of the golden rules and what Samuelson (1975) described as the "Serendipity Theorem."

To understand the characteristics of the optimum value of n, differentiate $u(n)$ with respect to n. The envelope theorem implies that

$$\frac{du(n)}{dn} = \frac{\partial u}{\partial n} = \left(-k + \frac{d}{n^2} \right) u_c\,(c,d)\,. \tag{8}$$

This equation tells that there are two effects on the steady-state utility associated with increasing n. One effect, d/n^2, is positive implying that increasing n enhances the steady state utility: A higher value for n means there will be a higher number of future working people who can support a retired person. This is called the "intergenerational transfer" effect. The second effect, $-k$, is negative implying that increasing n lowers the steady state utility: A higher value for n also means a higher number of future working people thus requiring a higher amount of capital to maintain the capital labor ratio. This called "capital dilution" effect.

Now, from Euler's theorem, $f(k) = w + rk$. Substituting for $f(k)$ in equation (6) for the economy's resource constraint and rearranging terms, one gets

$$\frac{d}{n} = w + rk - c - (n-1)\,k.$$

This, in turn, implies that

$$-k + \frac{d}{n^2} = \frac{w - c - nk + [r - (n-1)]\,k}{n}. \tag{9}$$

Substituting from equation (9) into (8), while noting that at the golden rule $r = (n-1)$, results in

$$\frac{du(n)}{dn} = \left(\frac{w - c - nk}{n}\right) u_c\,(c, d)\,. \tag{10}$$

The first-order condition for the optimum of n thus requires $w - c - nk = 0$. Observe also that at the laissez faire solution, $w - c - nk = 0$. This tells us that optimal n, if it is imposed, will be decentralized without any lump-sum taxes and transfers on either the young or the old.

Finally, of course, one has to make sure that the second-order condition is also satisfied when $du(n)/dn = 0$. Deardorff (1976) gave an example with Cobb-Douglas preferences wherein the first-order condition yielded a minimum value for the steady-state utility. Michel and Pestieau (1993) provided sufficient conditions for an interior n to yield a maximum. These are complementarity between capital and labor in production and complementarity between first and second period consumption in preferences.

3 Endogenous and deterministic fertility

The models with endogenous fertility are essentially an outgrowth of the third model that Samuelson (1975) considered, namely the Diamond's (1965) two-period overlapping generations model. In these models, unlike Diamond (1965) and Samuelson (1975), individuals decide how many children they will have, if any, upon considering the cost and the benefits of having children. Much of my discussion in this paper is based on a model of this type also used by van Groezen *et al.* (2003). I will add other features to it as I proceed and when the question to be studied warrants it.

Assume that a parent's utility depends on the number of his children. For simplicity, also assume that preferences are strongly separable in consumption when young, c, consumption when old, d, and the number of children, n. They are represented by the utility function

$$u(c) + v(d) + \varphi(n), \tag{11}$$

where $u(\cdot), v(\cdot)$, and $\varphi(\cdot)$ are strictly concave and twice differentiable. Inclusion of the number of children in a parent's utility function reflects a special kind of altruism on the part of parents towards their children.[6] It is an example of models with "one-sided" altruism where children do not care about their parents; Abel (1988). If children exhibit altruism towards their parents, then parents may want to have children not just because of their love for children, but also for the possibility that the children would take care of the parents when they retire. Put differently, parents may want to have children as a source of insurance for old age.[7]

The welfare criterion I use, following Samuelson (1975), is steady-state utility maximization. This is equivalent to using a social welfare function defined over (undiscounted) average utilities of all future generations. The criterion poses some interesting philosophical questions even if fertility were given exogenously. Inclusion of the utility of unborn children, using average versus sum of utilities, and the extent to which the utilities of future generations should be discounted, are three such questions. The conceptual issues become formidable when one deal with endogenous fertility. If parents care about the utility of their children, then the utility of the children is already taken care of through the parents' utility. Should we include them again? And what does one mean by Pareto efficiency in this situation? Dasgupta (1995) argues that we do not even have the right vocabulary to address these issues. Some recent papers have tried to tackle this problem and to come up with acceptable definitions of Pareto efficiency; see e.g. Golosov *et al.* (2007) and Michel and Wigniolle (2007).

[6] Becker and Barro (1988) consider a richer setup where a parent's utility depends not only on the number of children but also on the children's utility or consumption.

[7] I also ignore the use of one's family as a mechanism for intergenerational transfers as in Cigno (1993). This and the other issues mentioned are interesting but go beyond the points of this paper.

3.1 First-best steady state

Let a denote the cost of raising a child. To account for this, modify the previously defined steady-state resource constraint (6) to

$$f(k) \geqq c + an + \frac{d}{n} + (n-1)\,k. \tag{12}$$

The optimization problem facing us is now choosing c, d, n, and k to maximize the utility function (11) subject to the resource constraint (12). Denote the Lagrangian expression associated with this problem by $\mathcal{L}$ and the Lagrangian multiplier associated with the resource constraint by μ. The first-order conditions to this problem are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial k} &= \mu\left[f'(k) - n + 1\right] = 0, \\
\frac{\partial \mathcal{L}}{\partial c} &= u'(c) - \mu = 0, \\
\frac{\partial \mathcal{L}}{\partial d} &= v'(d) - \frac{\mu}{n} = 0, \\
\frac{\partial \mathcal{L}}{\partial n} &= \varphi'(n) + \mu\left(a - \frac{d}{n^2} + k\right) = 0.
\end{aligned}$$

Eliminating μ between these equations yields:

$$f'(k) = n - 1, \tag{13}$$

$$\frac{v'(d)}{u'(c)} = \frac{1}{n}, \tag{14}$$

$$\frac{\varphi'(n)}{u'(c)} = a + \left(k - \frac{d}{n^2}\right). \tag{15}$$

These derivations follow those of van Groezen *et al.* (2003). Equation (13) is the golden rule condition, equation (14) equates the marginal rate of substitution between future and present consumption to its societal cost, and equation (15) equates the marginal rate of substitution between the number of children and current consumption to its social cost. Observe that the last equation includes not only a, the cost of raising a child, but also $k - d/n^2$. Recall that k is the capital dilution effect and $-d/n^2$ is the intergenerational transfer effect that also existed in Samuelson (1975).

3.2 Decentralization

Assume the government uses a lump-sum tax on the young, T, pension benefits paid to the old, P, and a per-child tax, t (a per-child allowance when $t < 0$), to decentralize the first-best allocation characterized above. Given these fiscal instruments, a young parent maximizes utility, as given by equation (11), subject to his intertemporal budget constraint

$$w - T \geqq c + \frac{d - P}{1 + r} + (a + t)n. \tag{16}$$

Manipulating the first-order conditions to the parent's optimization problem yields

$$\begin{aligned} \frac{v'(d)}{u'(c)} &= \frac{1}{1+r}, \\ \frac{\varphi'(n)}{u'(c)} &= a + t. \end{aligned}$$

It follows from these equations that decentralization requires the government to set T and P such that the golden rule condition is satisfied: $1 + r = n$. Additionally, t must be set to satisfy

$$t = k - \frac{d}{n^2}, \tag{17}$$

where k, d, and n are set at their first best values. Observe that the value of t is equal to the combined effect of the capital dilution effect k, a negative externality requiring a Pigouvian tax, and the intergenerational transfer effect $-d/n^2$, a positive externality requiring a Pigouvian subsidy.

3.3 The sign of t

In Samuelson's (1958) overlapping generations model, there is no capital accumulation and no capital dilution effect. This means that $t = -d/n^2$ is always negative and attaining the first best requires a subsidy (a child allowance). In models with capital accumulation à la Diamond (1965), t can take positive as well as negative values. To see this, consider the budget constraint of the old given by

$$d - P = (1 + r)\, kn.$$

Substituting for k from this equation into equation (17) yields

$$t = \frac{[n - (1+r)]\, d - nP}{n^2\,(1+r)}.$$

At the golden rule, $n = 1 + r$ so that

$$t = \frac{-P}{n^2}$$

It then follows from the government's budget constraint $n\,(T + tn) = P$ that $T = 2P/n$.

Observe that it is possible for P to be positive as well as negative. As with the exogenous fertility case, if at the laissez faire $1 + r < n$, then one wants to set P to be positive. This is when laissez faire entails "too much" capital. A positive P would then transfer resources to the old, lowers capital intensity, and increases r. Interestingly, though, one would also set t to be negative in this case which tends to encourage n (contrary to what one may expect). The reverse happens if laissez faire entails "too little" capital so that $1 + r > n$. In this case, one sets P to be negative and transfers resources to the young. This enhances capital intensity thus reducing r. With $p < 0$, t is set at a positive value which tends to discourage n. Finally, if laissez faire coincides with the golden rule so that $1 + r = n$, then $P = T = t = 0$.

3.4 Relating pensions to fertility

The decentralization procedure I discussed above is based on three instruments: a child tax (or subsidy), an old-age pension, and a lump-sum tax on the young. Interestingly, the decentralization can also be achieved using only an old-age pension scheme coupled with a lump-sum tax on the young. To see this, recall that the representative individual's intertemporal budget constraint is given by

$$w - c - \frac{d - P}{1 + r} - (a + t)n - T = 0.$$

One can rewrite this as

$$w - c - \frac{d - [P - t\,(1+r)\,n]}{1 + r} - an - T = 0.$$

This immediately tells us that if t^*, P^*, T^* implements the first-best allocation, then $\widehat{t} = 0, \widehat{P} = P^* + \tau(1+r)n$; where $\tau = -t^* = d/n^2 - k$ with d, n, and k set at their first-best values, and $\widehat{T} = T^*$ will also implement the same allocation. Put differently, relating pensions to fertility is an *alternative* to a child subsidy scheme.

4 Parents' heterogeneity and redistribution

Thus far I have assumed that the economy is inhabited by identical individuals. While it is true that at any moment of time agents differ in age, everyone is nevertheless identical from a life-cycle perspective. In reality, however, parents differ in many respects. Most pertinent to our analysis, they are likely to differ in the ability to raise children (of a certain quality) and possibly in tastes for children too. Such differentials open up the possibility that the government may want to redistribute resources towards parents who face higher costs in child rearing (the low-ability parents). If this is the case, linking pension benefits positively to fertility will have adverse redistributive effects as it penalizes high-cost families. This then raises the question of a potential conflict between the government's redistributive aims and its efficiency enhancing goal (correcting the externality inherent in fertility decisions).

To address this question, consider again the two-period overlapping generations model in the steady state. However, as in Cremer *et al.* (2008a), assume now that parents have differential abilities in raising children and that this is the only source of heterogeneity among individuals. Specifically, assume that each generation consists of two types of individuals characterized by their cost of raising a child, a_j, $j = l, h$.[8] Let $0 < a_h < a_l$ so that the h-type is the more able parent. Denote the proportion of the h-type parents by π_h and the l-type parents by π_l where $\pi_h + \pi_l = 1$. There is no correlation between a parent's type and the type of his children. Thus while different types of parents may have different number of children, the proportion of h-types and

[8] As it is standard, I assume a world of single parents. One could also use as decision unit a couple acting cooperatively.

l-types in total population remains the same.

4.1 Utilitarian first best

Let n_j denote the number of children a j-type parent has. The population growth rate is

$$\overline{n} = \pi_l n_l + \pi_h n_h.$$

Introduce

$$\begin{aligned} \overline{c} &\equiv \pi_l c_l + \pi_h c_h, \\ \overline{d} &\equiv \pi_l d_l + \pi_h d_h, \\ \overline{an} &\equiv \pi_l \left(a_l n_l\right) + \pi_h \left(a_h n_h\right), \end{aligned}$$

so that $\overline{c}$ denotes average (over the two types) consumption of the young, $\overline{d}$ denotes average consumption of the old, and $\overline{an}$ denotes average expenditures on children,. The economy's resource constraint in per capita terms is then represented by

$$f(k) \geqq \overline{c} + \overline{an} + \frac{\overline{d}}{\overline{n}} + \left(\overline{n} - 1\right) k. \tag{18}$$

Consequently, first-best allocations of this model are characterized by the solution to the Lagrangian

$$\mathcal{L} = \sum_{j=l}^{h} \pi_j \left[u\left(c_j\right) + v\left(d_j\right) + \varphi(n_j)\right] + \mu \left[f(k) - \overline{c} - \overline{an} - \frac{\overline{d}}{\overline{n}} - \left(\overline{n} - 1\right) k\right].$$

Manipulating the first-order conditions of the above optimization problem, one gets

$$f'(k) = \overline{n} - 1 \tag{19}$$

$$c_h = c_l \equiv c, \tag{20}$$

$$d_h = d_l \equiv d, \tag{21}$$

$$\frac{v'(d)}{u'(c)} = \frac{1}{\overline{n}}, \tag{22}$$

$$\frac{\varphi'\left(n_j\right)}{u'(c)} = a_j + k - \frac{d}{\overline{n}^2}, \quad j = l, h. \tag{23}$$

Equation (19) is the familiar golden rule result where $\overline{n}$ replaces n. Equations (20)–(21) tell us that consumption levels are equalized across types. This is to be expected with a utilitarian social welfare function and identical tastes. Equation (22) equates an individual's marginal rate of substitution between future and current consumption to its cost to the society. Finally, equation (23) relates a j-type individual's marginal rate of substitution between having a child and current consumption to its social cost. Observe that not all the costs to the society are borne by the individual himself. The j-type parent pays only a_j for raising each child. There is thus an externality in choosing n_j equal to

$$k - \frac{d}{\overline{n}^2}.$$

This is the same term that showed up in the model with identical individuals, except that $\overline{n}$ has replaced n; see equation (15). As in that model, k reflects the capital dilution effect and $-d/\overline{n}^2$ the intergenerational transfer effect.

4.2 Decentralization

Assume the following fiscal instruments are available to the government: a type-specific child tax (or allowance), t_j, a type-specific old-age pension, P_j, and a type-specific lump-sum tax on the young, T_j. The optimization problem of the j-type parent, $j = l, h$, in the face of these instruments is summarized by the Lagrangian

$$\mathcal{L}_j = u\left(c_j\right) + v\left(d_j\right) + \varphi(n_j) + \mu_j \left[w - T_j - c_j - \frac{d_j - P_j}{1+r} - \left(a_j + t_j\right) n_j\right].$$

Simplifying the first-order conditions of this problem yields

$$\frac{v'\left(d_j\right)}{u'\left(c_j\right)} = \frac{1}{1+r},$$
$$\frac{\varphi'\left(n_j\right)}{u'\left(c_j\right)} = a_j + t_j.$$

Compare the last equation with (23) which gives the social cost of $\varphi'\left(n_j\right)/u'\left(c_j\right)$. This suggests that one should set the per-child tax equal to

$$t = k - \frac{d}{\overline{n}^2}.$$

Note that the tax is the same for all types because the externality imposed on others is symmetric in nature. The cost of having a child, which varies across parents, is paid by the parent himself. Observe also that, as previously, t may take positive as well as negative values. The previous discussion on the sign of t applies.

4.2.1 Taxes and pensions

The values of P_j and T_j are chosen to achieve the redistributive aims of the government and to ensure that the economy is on its golden rule path. Considering the redistributive aims, I have already shown that with a utilitarian social welfare function, redistribution requires equalization of consumption levels for the two individual types. Now to have both types choose the same c and d, they should have the same resources to spend on these two consumption goods. This is achieved by having (T_h, P_h) and (T_l, P_l) satisfy the following equation,

$$\left(T_h - \frac{P_h}{1+r}\right) - \left(T_l - \frac{P_l}{1+r}\right) = (a_h + t)\, n_h - (a_l + t)\, n_l, \tag{24}$$

so that any difference in the expenditures of the two types on children is paid for by the government.

Additionally, to satisfy the golden rule condition, intergenerational transfers must be arranged in a particular fashion. To discern the relationship between (T_h, T_l) and (P_h, P_l) recall that each period's capital stock, $\overline{n}\, k$, is related to the savings of the young in the previous period according to,

$$\overline{n}k = \sum_{j=l}^{h} \pi_j \left[w - T_j - c_j - (a_j + t_j)\, n_j\right]. \tag{25}$$

Second, recall also that the young's intertemporal budget constraint is given by,

$$\sum_{j=l}^{h} \pi_j \left[w - T_j - c_j - (a_j + t_j)\, n_j\right] = \sum_{j=l}^{h} \pi_j \frac{d - P_j}{1+r}. \tag{26}$$

Now substituting $k - d/\overline{n}^2$ for t_j in equation (25) rewrites it as

$$\sum_{j=l}^{h} \pi_j T_j = w + \frac{d}{\overline{n}} - c - \sum_{j=l}^{h} \pi_j a_j n_j - 2\overline{n}k. \tag{27}$$

Next combining equations (25) and (26) while setting $1 + r = \overline{n}$, as required by the golden rule condition, yields

$$\sum_{j=l}^{h} \pi_j P_j = d - \overline{n}^2 k. \tag{28}$$

One thus ends up with the three equations (24), (27), and (28) to determine the values of T_l, T_h, P_l and P_h.

The existence of an extra degree of freedom in setting the values of the four fiscal instruments T_l, T_h, P_l, P_h suggests that there are many different ways through which decentralization can take place. In particular, one can set $T_h = T_l$ and have P_j vary across types according to

$$P_h - P_l = -(1 + r)\left[(a_h n_h - a_l n_l) + (n_h - n_l)\left(k - \frac{d}{\overline{n}^2}\right)\right],$$

where the values of r, k, d, n_l, and n_h are all set at their first-best values. Alternatively, one can give identical pensions ($P_h = P_l$) to the retired and let T_j depend on one's type. That is,

$$T_h - T_l = (a_h n_h - a_l n_l) + (n_h - n_l)\left(k - \frac{d}{\overline{n}^2}\right).$$

There also exists a different mechanism for decentralization; one that embeds child taxes (or allowances) into pensions and links pensions directly to fertility. Observe that whether one uses the mechanism that sets $T_h = T_l$ or the alternative mechanism that sets $P_h = P_l$, one must accompany each with a child tax given by $t^* = k - d/n^2$. However, one can levy this t^* indirectly through the pension system and have no direct tax on children. To see how this works, rewrite the j-type individual's budget constraint as

$$w - T_j - c_j - \frac{d_j - [P_j - t_j n_j (1 + r)]}{1 + r} - a_j n_j = 0.$$

One can then immediately see that if t^*, P_j^*, T_j^* implements the first-best allocation, $\widehat{t} = 0, \widehat{P}_j = P_j^* + \tau(1+r)\, n_j$; where $\tau = -t^* = d/n^2 - k$ with d, n, and k set at their first-best values, and $\widehat{T}_j = T_j^*$ will also implement the same allocation.

The upshot of this section is that the core message and the policy recommendations of the endogenous fertility model with identical individuals remains intact as long as one is in a first-best environment. Heterogeneity of agents does not seem to change this message. However, with agents' heterogeneity, it is hard to imagine that the environment remains one of the first best. I discuss the implications of leaving this environment next.

5 Adverse selection

If parents' child-rearing abilities are publicly observable, as was the case in the previous section, there is no conflict between externality-correction and redistribution. However, such individual characteristics are seldom publicly observable. There exists asymmetric information between parents and policy makers regarding this characteristic. Under this circumstance, the government is not able to distinguish between those individuals who have a small number of children due to high costs and those with low costs who try to free ride on the system. This implies that the often recommended child allowance policy, or linking pension benefits positively to fertility, penalizes high-cost families who have few children. Such policies thus have adverse redistributive impacts that make them undesirable. Consequently, the externality-correcting property of relating pension benefits to the number of children has to be balanced against its adverse redistributive potential.

5.1 The second-best policy

In order to zero in on the implications of adverse selection, it is easiest to ignore the problem of capital accumulation and work with Samuelson's original version of the overlapping generations model. This is Cremer *et al.*'s (2008a) setup on which I draw heavily for the results of this subsection. In this alternative framework, every person starts life with the same exogenous income I. People finance their old-age consumption either through investment in a storage technology with a fixed rate of return r or through a PAYGO pension scheme with a biological rate of return $\overline{n}-1$. I assume that $\overline{n}-1>r$ so that PAYGO is the preferred option.

The agency problem that arises in this setup is because a_j, the j-type's cost of raising a child, is publicly unobservable. This means that any system of differential pensions and taxes that the government wants to impose must be incentive compatible. The unobservability of types requires $a_j n_j$ and c_j to also be unobservable. Otherwise, one could infer the value of a_j. The second-period consumption level d_j, however, can be observable—an assumption that I maintain throughout this section.

To write the second-best problem in terms of observable variables, one must replace c_j by T_j (the first-period tax levied on the j-type). With PAYGO being the preferred option for financing old-age consumption, one never wants to use the storage technology. The optimal policy sets each type's pension benefits exactly equal to that type's consumption when retired. That is, $P_j = d_j$. Given this, consider a direct revelation mechanism that offers individuals the choice between two allocations: (T_l, P_l, n_l) or (T_h, P_h, n_h). Let

$$c_{jk} = I - a_j n_k - T_k,$$
$$U_{jk} = u\left(c_{jk}\right) + v(P_k) + \varphi(n_k),$$

so that c_{jk} and U_{jk} ($j \neq k = l,h$) denote the consumption and the utility of a j-type who "mimics" a k-type. The government's budget constraint is given by

$$\sum_{j=l}^{h} \pi_j \left(T_j - \frac{P_j}{\overline{n}}\right) = 0.$$

One can then summarize the second-best problem through the Lagrangian

$$\mathcal{L} = \sum_j \pi_j \left[U_j + \mu\left(T_j - \frac{d_j}{\overline{n}}\right)\right] + \lambda_h(U_h - U_{hl}) + \lambda_l(U_l - U_{lh}).$$

The first-order conditions for this problem are

$$\frac{\partial \mathcal{L}}{\partial T_l} = -(\pi_l + \lambda_l)u'(c_l) + \pi_l \mu + \lambda_h u'(c_{hl}) = 0, \tag{29}$$

$$\frac{\partial \mathcal{L}}{\partial T_h} = -(\pi_h + \lambda_h)u'(c_h) + \pi_h \mu + \lambda_l u'(c_{lh}) = 0, \tag{30}$$

$$\frac{\partial \mathcal{L}}{\partial d_l} = (\pi_l + \lambda_l - \lambda_h)v'(d_l) - \frac{\pi_l \mu}{\overline{n}} = 0, \tag{31}$$

$$\frac{\partial \mathcal{L}}{\partial d_h} = (\pi_h + \lambda_h - \lambda_l)v'(d_h) - \frac{\pi_h \mu}{\overline{n}} = 0, \tag{32}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n_l} &= -(\pi_l + \lambda_l)\left[u'(c_l)a_l - h'(n_l)\right] + \pi_l \mu \frac{\pi_l d_l + \pi_h d_h}{\overline{n}^2} \\ &\quad + \lambda_h \left[u'(c_{hl})a_h - h'(n_l)\right] = 0,\end{aligned} \tag{33}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n_h} &= -(\pi_h + \lambda_h)\left[u'(c_h)a_h - h'(n_h)\right] + \pi_h \mu \frac{\pi_l d_l + \pi_h d_h}{\overline{n}^2} \\ &\quad + \lambda_l \left[u'(c_{lh})a_l - h'(n_h)\right] = 0.\end{aligned} \tag{34}$$

There are two possible distortionary regimes. In one regime $\lambda_h > 0$ and $\lambda_l = 0$ so that one redistributes from the low-cost (high ability) to high-cost (low ability) parents. In the other regime $\lambda_l > 0$, $\lambda_h = 0$ and the redistribution is from the high-cost to low-cost parents. Observe that either regime is likely in that one type is not inherently "richer" or "poorer" in the absence of government intervention. That depends on which type spends more on children (i.e. on the size of $a_h n_h$ relative to $a_l n_l$). This in turn depends on the elasticity of substitution between consumption and fertility.

5.1.1 Regime 1: $\lambda_h > 0$ and $\lambda_l = 0$

Manipulating the first-order conditions (29)–(32), one can show that

$$\frac{v'(d_h)}{v'(d_l)} = \frac{\pi_h \pi_l + \pi_h(\lambda_l - \lambda_h)}{\pi_h \pi_l + \pi_l(\lambda_h - \lambda_l)}, \tag{35}$$

$$v'(d_l) \geq \frac{1}{\overline{n}}u'(c_l), \tag{36}$$

$$v'(d_h) \leq \frac{1}{\overline{n}}u'(c_h). \tag{37}$$

With $\lambda_h > 0$ and $\lambda_l = 0$, it follows from (35) and the concavity of $v(\cdot)$ that $d_h > d_l$. However, equations (36) and (37) do not allow one to determine the relative size of c_h

and c_l.

Regarding n_j, one can show from manipulating the first-order conditions (30) and (34) that the marginal rate of substitution for the h-type (those one distributes away from) is equal to

$$\frac{h'(n_h)}{u'(c_h)} = a_h - \frac{\overline{d}}{\overline{n}^2},$$

where $\overline{d} \equiv \pi_l d_l + \pi_h d_h$. The first term in the right-hand side is the cost of raising a child which the h-type pays himself. The second term is the familiar intergenerational transfer term that requires fertility to be subsidized. Recall that, in Samuelson's overlapping generations model, there is no capital accumulation and thus no capital dilution effect. The intergenerational transfer effect is the only source of externality.

Similarly, from equations (29) and (33) one can show that the marginal rate of substitution for the l-type (those one distributes towards) is equal to

$$\frac{h'(n_l)}{u'(c_l)} = a_l - \frac{\pi_l \mu}{(\pi_l - \lambda_h)u'(c_l)} \frac{\overline{d}}{\overline{n}^2} - \frac{\lambda_h}{\pi_l - \lambda_h} \frac{u'(c_{hl})a_h - u'(c_l)a_l}{u'(c_l)}. \tag{38}$$

The first term in the right-hand side of (38) is the l-type's cost of child rearing. The second term is the required Pigouvian subsidy (adjusted by the fact that the "marginal cost of public fund" is no longer equal to one). The third term is the distortion aimed at relaxing the binding incentive constraint of h-type households (who are hurt by redistribution). Observe that $u'(c_{hl})a_h - u'(c_l)a_l < 0$ and the last term on the right-hand side of (38) is positive. Incentive considerations requires n_l to be distorted downward. Intuitively, one wants to prevent the h-type from mimicking the l-type in this case. Hence one wants to make the l-type's allocation less attractive to the h-type. Now with $a_h < a_l$, the h-type tends to prefer, relative to the l-type, a larger n. Hence one must distort n_l downward. There is a conflict between externality correction (requiring a subsidy to induce a higher value for n_l) and incentive (requiring a tax to induce a lower value for n_l) terms.

5.1.2 Regime 2: $\lambda_l > 0$ and $\lambda_h = 0$

With $\lambda_l > 0$ and $\lambda_h = 0$, it now follows from (35) and the concavity of $v(\cdot)$ that $d_h < d_l$. Moreover, making use of (36) and (37), one obtains that $c_h < c_l$.

Turning to n_j, manipulating the first-order conditions (29) and (33) shows that the marginal rate of substitution for the l-type (those one distributes away from) is equal to

$$\frac{h'(n_l)}{u'(c_l)} = a_l - \frac{\overline{d}}{\overline{n}^2}.$$

This tells us that, as with regime 1, one wants to subsidize the fertility choice of the type one distributes away from by the full extent of the positive externality it accords others.

As far as the h-types are concerned (those one distributes towards), using (30) and (34), their marginal rate of substitution between fertility and consumption is

$$\frac{h'(n_h)}{u'(c_h)} = a_h - \frac{\pi_h \mu}{(\pi_h - \lambda_l)u'(c_h)} \frac{\overline{d}}{\overline{n}^2} - \frac{\lambda_l}{\pi_h - \lambda_l} \frac{u'(c_{lh})a_l - u'(c_h)a_h}{u'(c_h)}. \quad (39)$$

In this regime, however, the incentive term $u'(c_{lh})a_l - u'(c_h)a_h > 0$ so that the last term on the right-hand side of (39) is negative. Incentive considerations now require n_h to be distorted upward. Intuitively, one wants to prevent the l-type from mimicking the h-type in this case. Hence one wants to make the h-type's allocation less attractive to the l-type. But, with $a_l > a_h$, the l-type tends to prefer, relative to the h-type, a smaller n. Hence one must distort n_h upward. This will reinforce the externality correction (requiring a Pigouvian marginal subsidy) and the net effect requires a marginal subsidy on n_h.

The upshot of the discussion in this section is that, in the second best, children are taxed (or subsidized) *non-linearly*. The marginal subsidy on one type—those one distributes away from—is equal to the positive externality it bestows on others. The marginal subsidy on the other type—those one distributes towards—is either less or more than the externality (because of incentive considerations). In the former case, the subsidy may even turn to a tax. Algebraically, denoting the tax schedule by $t(n)$, either $t'(n_h) = \overline{d}/\overline{n}^2 < 0$ and an ambiguous sign for $t'(n_l)$; or $t'(n_l) = \overline{d}/\overline{n}^2 < 0$ and $t'(n_h) < t'(n_l) < 0$.

6 Endogenous and partly stochastic fertility/quality

What truly determines fertility, and what accounts for the observed evolution in fertility behavior, are still open questions. What is clear though is that no one can fully control fertility. The actual number of children in a family does not necessarily coincide with the number the parents initially intended to have. Infertility, premature death, misplanning and multiple births are some of the reasons explaining this gap. Similarly, one cannot completely determine the future earning ability of his children simply by investing in their education and training. There exists a gap between intended and actual quantity, and quality, of children. I do not specifically distinguish between fertility and educational decisions. Instead, I lump the investments in quantity and quality together as if one decision determines the two. In doing so, I use the concept of number of children in "efficiency units" which is widely used in growth theory.

Consider again the two-period overlapping generations model with capital accumulation in the steady state. Depart from the traditional model by assuming that fertility is in part stochastic. To make matters simple, following Cremer *et al.* (2003, 2006), assume that a parent can have either n_l or n_h children, with $n_h > n_l$. The actual realization of n_j depends on an initial "investment in children," e, and on some random shock. Thus when a parent invests e, he will have n_h children with probability $\pi(e)$ where $0 \leqslant \pi(e) \leqslant 1$ and $\pi'(e) > 0$ $(\pi''(e) < 0$ and $\pi(0) > 0)$. Naturally, the probability of having n_l children is given by $1 - \pi(e)$. Whenever it makes the notation simple, I substitute $\pi_h(e)$ for $\pi(e)$ and $\pi_l(e)$ for $1 - \pi(e)$. The cost of having children is not limited to the initial investment e. As previously, there are other costs borne in the first period that vary proportionately to the actual number of children (at the rate of

$a \geq 0$).

Given the same utility function as before, the expected utility of the young—the future parents—is written as

$$U = \sum_{j=l}^{h} \pi_j (e) \left[u (c_j) + v (d_j) + \varphi(n_j) \right].$$

Define

$$\begin{aligned} \overline{n} (e) &= \pi_l (e) n_l + \pi_h (e) n_h, \\ \overline{c} (e) &= \pi_l (e) c_l + \pi_h (e) c_h, \\ \overline{d} (e) &= \pi_l (e) d_l + \pi_h (e) d_h, \end{aligned}$$

to denote the average number of children (over the two states of the world), average first-period consumption, and average second-period consumption. One can then write the economy's resource constraint at the steady-state, on a per capita basis, as

$$f(k) \geqq \overline{c} (e) + e + a\overline{n} (e) + \frac{\overline{d} (e)}{\overline{n} (e)} + \left[\overline{n} (e) - 1 \right] k.$$

6.1 The utilitarian first-best

Assume that the social planner has perfect information, particularly with respect to the individuals' investment levels in children e, and that he controls all the relevant variables in the economy: k, e, c_j, and d_j, $j = l, h$. He sets these variables to maximize the expected lifetime utility in the steady-state subject to the economy's resource constraint. This problem is summarized by the Lagrangian

$$\begin{aligned} \mathcal{L} &= \sum_j \pi_j (e) \left[u (c_j) + v (d_j) + \varphi(n_j) \right] + \\ & \quad \mu \left[f(k) - \overline{c} (e) - e - a\overline{n} (e) - \frac{\overline{d} (e)}{\overline{n} (e)} - \left(\overline{n} (e) - 1 \right) k \right]. \end{aligned}$$

The corresponding first order conditions can be rearranged as

$$f'(k) = \overline{n}(e) - 1, \tag{40}$$

$$c_h = c_l \equiv c, \tag{41}$$

$$d_h = d_l \equiv d, \tag{42}$$

$$\frac{v'(d)}{u'(c)} = \frac{1}{\overline{n}(e)}, \tag{43}$$

$$\frac{[\varphi(n_h) - \varphi(n_l)]\, \pi'(e)}{u'(c)} = 1 + (n_h - n_l)\left(a + k - \frac{\overline{d}}{\overline{n}^2}\right)\pi'(e). \tag{44}$$

Equation (40) characterizes the golden rule condition for this setup with $\overline{n}(e)$ replacing n. Equations (41)–(42) tell us that consumption levels are equalized across the two states. Equation (43) equates the marginal rate of substitution between future and current consumption to its cost to the society. Finally, equation (44) equates the marginal benefit of increasing investment in fertility to its social costs.

Observe that, from the perspective of an individual, $\overline{n}(e)$ is given. The individual's cost of increasing investment in fertility is its direct purchasing cost which is one; and this is the only cost that he pays. The additional terms

$$(n_h - n_l)\left(a + k - \frac{\overline{d}}{\overline{n}^2}\right)\pi'(e)$$

that appear on the right-hand side of (44) reflect externality costs. Observe that the term $(n_h - n_l)\,\pi'(e)\,a$ in above is the additional cost imposed on the society due to increasing the number of children from n_l to n_h as an individual increases his investment in fertility. This is a negative externality. The second and the third terms $(n_h - n_l)\,\pi'(e)\,k$ and $(n_h - n_l)\,\pi'(e)\,d/\overline{n}^2$ are the previously noted capital dilution and the intergenerational transfer effects associated with increasing investment in fertility.

6.2 Decentralization

Assume that the government has at its disposal state-contingent lump-sum taxes, T_j, pensions, P_j, $j = l, h$, and a per unit tax on investment in fertility, t. The optimization problem of the representative individual in this environment will then be summarized by the Lagrangian

$$
\begin{aligned}
\mathcal{L} &= \sum_{i=l}^{h} \pi_j(e) \Bigg\{ u(c_j) + v(d_j) + \varphi(n_j) + \\
&\quad \mu_j \left[w - T_j - c_j - \frac{d_j - P_j}{1+r} - a\; n_j - (1+t)\, e \right] \Bigg\},
\end{aligned}
$$

yielding the first-order conditions

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_l} &= \pi_l(e) \left[u'(c_l) - \mu_l \right] = 0, \\
\frac{\partial \mathcal{L}}{\partial d_l} &= \pi_l(e) \left[v'(d_l) - \frac{\mu_l}{1+r} \right] = 0, \\
\frac{\partial \mathcal{L}}{\partial c_h} &= \pi_h(e) \left[u'(c_h) - \mu_h \right] = 0,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial d_h} &= \pi_h(e) \left[v'(d_h) - \frac{\mu_h}{1+r} \right] = 0, \\
\frac{\partial \mathcal{L}}{\partial e} &= \Bigg\{ u(c_h) + v(d_h) + \varphi(n_h) + \mu_h \left[w - T_h - c_h - \frac{d_h - P_h}{1+r} - a\; n_h - (1+t)\, e \right] \\
&\quad - u(c_l) - v(d_l) - \varphi(n_l) - \mu_l \left[w - T_l - c_l - \frac{d_l - P_l}{1+r} - a\; n_l - (1+t)\, e \right] \Bigg\} \pi'(e) \\
&\quad - \pi_l(e)\, \mu_l\, (1+t) - \pi_h(e)\, \mu_h\, (1+t) = 0.
\end{aligned}
$$

To have the above equations yield the first-best solution, first one has to make sure that the individual purchases the same amount of c and the same amount of d in both states of the world. This is ensured if he has the same amount of resources to spend on present and future consumption in each state of the world. To satisfy this requirement, (T_l, P_l) and (T_h, P_h) must be related according to

$$
\left(T_h - \frac{P_h}{1+r} \right) - \left(T_l - \frac{P_l}{1+r} \right) = a\,(n_l - n_h)\,. \tag{45}
$$

In this case, the first-order condition $\partial \mathcal{L} / \partial e = 0$ is simplified to

$$
\left[\varphi(n_h) - \varphi(n_l) \right] \pi'(e) = (1+t)\, u'(c)\,.
$$

Comparison of this equation with the social optimum (44) shows that to attain the command optimum, one must set

$$t = (n_h - n_l)\left(a + k - \frac{\overline{d}}{\overline{n}^2}\right)\pi'(e)\,. \tag{46}$$

This makes perfect sense in light of the externality discussion above.

As in previous models, lump-sum taxes and pensions must also be set in such a way as to satisfy the golden rule condition. Two conditions are of particular interest. One relates each period's capital stock, $\overline{n}\;k$, to the savings of the young in the previous period,

$$\overline{n}(e)\,k = \sum_{j=l}^{h}\left[w - c - (1+t)\,e - T_j - an_j\right]\pi_j(e)\,.$$

The other is the young's intertemporal budget constraint,

$$\sum_{j=l}^{h}\pi_j(e)\left[w - c - (1+t)\,e - T_j - a\;n_j\right] = \sum_{j=l}^{h}\pi_j(e)\,\frac{d - P_j}{1+r}.$$

Substituting for t from (46) in the first equation and combining the first and the second equations while setting $1 + r = \overline{n}(e)$ yield the following two equations

$$\begin{aligned}\sum_{j=l}^{h}\pi_j(e)\,T_j &= w - c - e - (a+k)\,\overline{n}(e) - (n_h - n_l)\left(a + k - \frac{\overline{d}}{\overline{n}^2}\right)\pi'(e)\,e, && (47)\\ \sum_{j=l}^{h}\pi_j(e)\,P_j &= d - \overline{n}^2 k. && (48)\end{aligned}$$

With the three equations (45), (47), and (48) to determine the values of T_l, T_h, P_l and P_h, there exists one extra degree of freedom in setting these instruments. In particular, one can give retired people identical pensions ($P_h = P_l$) and have T_j depend on the number of children. This case corresponds to a policy of giving the young child supplements. Alternatively, one can impose a tax on the young regardless of how many children they have ($T_h = T_l$) and give the old pension benefits that depend on the number of children.

To sum, decentralization requires a Pigouvian tax on investment in fertility equal to (46). This is then coupled by giving every retired person the same pension benefits while taxing the young on the basis of the number of children they have. That is,

$$T_h - T_l = -a\,(n_h - n_l)\,.$$

Alternatively, one levies the same tax on the young regardless of the number of children they have, and give the retired pension benefits that vary with their number of children according to

$$P_h - P_l = a\overline{n}\,(n_h - n_l)\,.$$

It is important to emphasize two points here. It is true that the size of t varies with the difference between n_h and n_l. But t is not given per child; it is a tax on fertility investment. Secondly, it is also true that either the young receive a "child allowance" or the old get pensions that vary with the number of children they have. But neither is given to encourage fertility per se. They are levied only for their income effects to ensure equalization of consumption over the states of the world; namely for their insurance property.

7 Moral hazard

Stochastic fertility (or educational attainment) added a dimension of insurance to the problem. Yet, as long as investment in fertility is publicly observable, by appropriately taxing (or subsidizing) it, one attains both objectives of a "correct" level of fertility and full insurance. Without observability, a moral hazard problem arises in the choice of fertility investment. The government may then try to correct the problem by taxing (or subsidizing) one's number of children which is observable (as opposed to fertility investment which is not). This creates a tradeoff between achieving the correct level of investment, through a subsidy on the number of children, and achieving full insurance.

To address this question, I revert back to Samuelson's version of the overlapping generations model which allows close scrutiny with less distraction. Assume, as in

Cremer *et al.* (2006), that everyone starts life with the same exogenous income I. There is a storage technology with a fixed rate of return, r. Assume r is "small" enough to have a PAYGO pension system as the superior mechanism for financing consumption during retirement. Continue to assume that the number of children n_j $(j = l, h)$ is publicly observable.

Set up a two-stage optimal tax problem when the instruments do not include a tax on fertility investment e. The young's problem, in the second stage, when facing the policy instruments T_l, T_h, P_l and P_h, is to choose c_l, c_h, d_l, d_h and e to maximize

$$U = \big(1 - \pi(e)\big)[u(c_l) + v(d_l)] + \pi(e)[u(c_h) + v(d_h)].$$

The optimization problem is subject to, for $j = l, h$,

$$\begin{aligned} c_j &= I - e - T_j - a\, n_j, \\ d_j &= P_j, \end{aligned}$$

where the second constraint ensures that all old-age consumption is financed from pensions alone (which is the optimal policy given the assumption on r). The first-order condition with respect to e, assuming an interior solution, is[9]

$$\Big\{u(c_h) + v(d_h) - \big[u(c_l) + v(d_l)\big]\Big\}\pi'(e) - \big(1 - \pi(e)\big)u'(c_l) - \pi(e)u'(c_h) = 0. \qquad (49)$$

The first expression on the left-hand side of (49) measures the benefit (for the individual) of increasing e, while the second expression measures the cost. Not surprisingly, an interior solution requires marginal benefits to equal marginal costs. The solution to the individual's problem, denoted by $\widetilde{e}(T_l, T_h, P_l, P_h)$, describes all possible values of e that the government can induce through its choice of T_l, T_h, P_l and P_h. One can show that $\partial\widetilde{e}/\partial T_l \lesseqgtr 0, \partial\widetilde{e}/\partial T_h < 0, \partial\widetilde{e}/\partial P_l < 0$, and $\partial\widetilde{e}/\partial P_h > 0$.

[9] Naturally, the second-order condition $\Delta \equiv (d^2U/de^2)|_{e=\widetilde{e}} < 0$, where $\widetilde{e}$ denotes the solution to the individual's problem, must also be satisfied; I assume that this is the case. Observe also that if the left-hand side of (49) is non-positive at $e = 0$, one has a corner solution and the individual does not invest in e. This occurs for instance when $c_h = c_l$ (i.e., when $T_l - T_h = a(n_h - n_l)$) and $d_h = d_l$.

In the first stage, the government optimizes over the instruments (anticipating the individuals' second-stage optimization problem). Cremer *et al.* (2006), concentrating on the "normal" case which occurs if $\partial \widetilde{e} \;/ \partial T_l > 0$, show that the second-best allocation requires: Pension benefits that increase with the number of children, $P_h > P_l$ so that $d_h > d_l$, and contributions that decrease with the number of children, $T_h < T_l$. They also show that there is a tradeoff between externality-correcting and insurance considerations. Parents are more than compensated for the extra cost of children: $T_l - T_h > a(n_h - n_l)$ and $c_h > c_l$. The "over-insurance" occurs in order to induce a greater level of investment in fertility. However, the second-best level of investment in fertility, and the resulting fertility rate, remain below their corresponding first-best levels (because the associated positive externality is not fully corrected.)

8 Conclusion

This paper has studied the link between pensions and fertility. The literature has long recognized the existence of two types of externalities associated with fertility decisions. One is a positive externality called "intergenerational transfer" effect that arises because a higher fertility rate increases the number of future working people who support a retired person. The other is a negative externality called "capital dilution effect" that arises because increasing the number of future working people also increases the required capital to maintain the capital labor ratio. The two effects are present in overlapping generations models wherein the retired do not contribute to production in terms of labor but partake of consumption. The literature has in the main paid more attention to the positive externality and has called for its internalization through a policy of giving parents a child allowance, or a policy of positively linking pension benefits of the retired to their number of children, as a means of internalizing this externality.

The paper has argued that the case for a child allowance, or a positive linkage between pension benefits and the number of children, rests crucially on the assumption that no parents are better than others in raising their children and that fertility can be perfectly controlled. When either of these two assumptions are violated, the case for such policy recommendations are greatly weakened.

With parents' heterogeneity, the government has redistributive aims as well. If parents' types can be identified, this poses no problem and the two social objectives of externality correction and redistribution can be achieved. Public unobservability of parents' effort in raising children, on the other hand, gives rise to an adverse selection problem. Under this circumstance, the government will be unable to distinguish between those individuals who have a small number of children due to high costs and those with low costs who try to free ride on the system. The externality-correcting property of child allowances, and relating pension benefits to the number of children, will then have to be balanced against its adverse redistributive potential. Hence a positive link between fertility and pension benefits is not always desirable.

Similarly, when fertility is partly stochastic and cannot be fully controlled, the government may want to insure parents against the risks associated with becoming poor because of ending up with too many children. If parents' investment in fertility (or education of their children) is publicly unobservable, a moral hazard problem arises. This creates a tradeoff between achieving the correct level of fertility investment, through a subsidy on the number of children, and achieving full insurance. The balance of the tradeoff calls for too low an investment level and over-insurance.

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