Environmental tax design with endogenous earning abilities (with applications to France)*

Helmuth Cremer

Toulouse School of Economics (GREMAQ, IDEI and IuF)
Toulouse, France

Firouz Gahvari

Department of Economics
University of Illinois at Urbana-Champaign
Urbana, IL 61801, USA

Norbert Ladoux

Toulouse School of Economics (GREMAQ, IDEI)
Toulouse, France

February 2004
This revision, September 2009

*We thank two anonymous referees and the editor, Larry Goulder, for very helpful comments.

Abstract

This paper studies environmental taxation in a Mirrlees setting with two novel features. First, energy, a polluting good, is used both as a factor of production and a final consumption good; second, the wage is determined endogenously while labor of different individual types remain homogeneous. The model is calibrated for the French economy. We show that: (i) The optimal tax is less than the marginal social damage of emissions and turns into an outright subsidy when the inequality aversion index is high; (ii) the optimal tax on energy as an input is always equal to its marginal social damage; (iii) the social welfare gain due to lowering the current energy taxes to their optimal levels, with the general income tax being set optimally in both cases, is between 17 to 32 euro per household. This hurts the rich and benefits the poor.

JEL classification: H21; H23; D62

Keywords: The general income tax; endogenous earning abilities; emission taxes, consumption and intermediate goods; welfare gains.

1 Introduction

The existing empirical studies of environmental taxes are almost exclusively in the (homogeneous households) Ramsey tradition. They typically allow only for linear tax instruments and adhere to a "representative consumer" framework.[1] Some papers, starting with Sandmo (1975) and followed up more recently by Mayeres and Proost (2001) and Schöb (2003) have introduced consumer heterogeneity and distributional aims. However, these papers remains within the Ramsey tradition considering only *linear* tax instruments. The current paper attempts to break loose from this tradition and examine the efficiency and redistributive properties of optimal environmental taxes for the French economy within the context of the modern optimal tax theory à la Mirrlees (1971). This theory allows for heterogeneity among individuals, and includes nonlinear tax instruments. In this, we follow Cremer *et al.* (2003) while addressing two major shortcomings of our earlier work (as explained below). Additionally, we calculate the optimal environmental taxes, and their associated welfare gains, for a one-consumer representation of the French economy. This allows us to compare the implications of Mirrlees and Ramsey approaches to optimal taxation.

The first shortcoming of Cremer *et al.* (2003) is that it considered only polluting goods, ignoring polluting inputs. This is a serious omission; lumping final goods and inputs together may lead to incorrect policy recommendations. On the one hand, it is not difficult to find intermediate goods that are polluting;

energy being an obvious example. On the other, we know from Diamond and Mirrlees (1971) that the tax treatment of intermediate and final goods should in general be different. Applying their production efficiency result to economies with a consumption externality, leads to the conclusion that polluting intermediate goods should be taxed only in so far as they correct externalities. In contrast, Cremer and Gahvari (2001) have shown that polluting final goods are taxed for Pigouvian considerations as well as for redistributive concerns.

Second, Cremer *et al.* (2003) assumed that earning abilities are exogenously fixed. This is, with the notable exception of Naito (1999), the common assumption of the literature on optimal general income taxation. However, when there are other factors of production besides labor in the economy, the workers' pre-tax earning abilities (wages) change. We consider this problem at a theoretical level, we also compute an optimal general income tax schedule numerically while allowing for this endogeneity.

We model an open economy with three factors of production and two categories of consumption goods. The factors of production are labor, capital, and energy. Labor is homogeneous in efficiency units with different individual types having different endowments of labor in efficiency units . The assumption of homogeneity of labor in efficiency units rules out the possibility that energy used as an intermediate good can be a substitute to some type of labor and a complement to another. As will be demonstrated later, this assumption ensures that the

Diamond and Mirrlees production efficiency result continues to hold despite the endogeneity of wage.[2] All workers are immobile and no labor is either exported or imported. Capital inputs are rented from outside so that all capital incomes go to “foreigners”; energy inputs are also imported.[3] Emissions come from two sources: the use of energy input on the production side, and the consumption of one category of final goods on the consumption side (designated as polluting goods). The specific emissions we are concerned with are carbon dioxide emissions. The production process consists of two stages. First, a constant returns to scale production technology uses the three inputs to produce a “general-purpose” output. Second, a linear technology transforms the output into the two categories of consumption goods at constant marginal (equal to average) costs. The first-stage production function is “nested CES”. Consumers’ preferences are also nested CES, being a function of labor and the two final goods.

The model is calibrated for the French economy on the basis of the data from the “Institut National de la Statistique et des Etudes Economiques” (INSEE), France. We identify four groups of individuals who differ not only in earning abilities but also in tastes. They are identified as “managerial staff”, “intermediate-salaried employees”, “white-collar workers” and “blue-collar workers”. The polluting and non-polluting goods are constructed from 117 consumption goods according to whether they are energy related or not.[4] We use two values for the marginal social damage of emissions. A French Government Commission (Groupe

Interministériel sur l'effet de Serre) recommended a figure of 229 euro as the cost per ton of carbon. This is the basis for the benchmark figure we use. For the purpose of comparisons, we also use a second value for the social damage of emissions based on a cost of 92 euro per ton of carbon.[5]

We model the behavior of the government as one of setting *optimal* tax policies in light of the constraints that it faces. We use an iso-elastic social welfare function for this purpose. Moreover, the value of the inequality aversion index is chosen according to the observed degree of redistribution in the existing French tax system. Specifically, based on a recent study by Bourguignon and Spadaro (2000), we shall use 0.1 and 1.9 to be the limiting values for the inequality aversion index.

2 The private sector

Consider an open economy which uses three factors of production to produce two categories of consumption goods. The factors of production are labor, capital and energy. Labor is heterogeneous with different groups of individuals having different productivity levels. All types of workers are immobile so that labor is neither exported nor imported. All capital and energy inputs are rented from outside. There are two sources of emissions in the economy. On the production side, the use of labor and capital entail no emissions but the use of energy inputs does. On the consumption side, consuming one category of goods is non-polluting, but consuming the other category (energy) generates emissions.

Specifically, we assume that there are four groups of individuals with differing productivity levels and tastes. All persons, regardless of their type, are endowed with one unit of time. Denote a person's type by j ($j = 1, 2, 3, 4$), his productivity factor by n^j, and the proportion of people of type j in the economy by π^j. Normalize the population size at one, and define the Preferences of j-type person over his labor supply, L^j, consumption of a "non-polluting" good, x^j, a "polluting good", y^j, *and* total level of emissions in the atmosphere, E.

2.1 Production

The production process consists of two stages. First, a constant returns to scale production technology uses three inputs to produce a "general-purpose" output, O. Second, a linear technology transforms the output into the two categories of consumption goods, x and y, at constant marginal (equal to average) costs. The inputs to the first stage of production are: labor, L, capital, K, and energy, D. The production function $\boldsymbol{F}(L, K, D)$ is assumed to be "nested CES". It is written as

$$O = \boldsymbol{F}(L, K, D) = B\left[(1-\beta) L^{\frac{\sigma-1}{\sigma}} + \beta \Gamma^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \qquad (1)$$

with

$$\Gamma = A\left[\alpha K^{\frac{\delta-1}{\delta}} + (1-\alpha) D^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta}{\delta-1}}, \qquad (2)$$

where B and A are constants, and σ and δ are the (Allen) elasticities of substitution between L and Γ, and between K and D. Substituting for Γ from (2) into

(1), we have

$$O = B\left[(1-\beta)\,L^{\frac{\sigma-1}{\sigma}} + \beta A^{\frac{\sigma-1}{\sigma}}\left[\alpha K^{\frac{\delta-1}{\delta}} + (1-\alpha)\,D^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta(\sigma-1)}{\sigma(\delta-1)}}\right]^{\frac{\sigma}{\sigma-1}}. \quad (3)$$

Aggregate output, O, is the numeraire and the units of x and y are chosen such that their producer prices are equal to one.

Capital services and energy inputs are imported at constant world prices of r and p_D where the units of D is chosen such that $p_D = 1$. Let w denotes the price of one unit of *effective labor*, τ_D denotes the tax on energy input, and assume that there are no *producer taxes* on labor and capital.[6] The first-order conditions for the firms' input-hiring decisions are, assuming competitive markets,

$$\boldsymbol{O}_L(L,K,D) = w, \quad (4)$$

$$\boldsymbol{O}_K(L,K,D) = r, \quad (5)$$

$$\boldsymbol{O}_D(L,K,D) = p_D(1+\tau_D). \quad (6)$$

Equations (3)–(6) determine the equilibrium values of O, L, K and D as functions of w, r and $p_D(1+\tau_D)$ [where r and p_D are determined according to world prices].

2.2 Consumption

The consumer side is modeled à la Cremer *et al.* (2003). Consumers' preferences are nested CES, first in goods and labor supply and then in the two categories of consumer goods. All consumer types have identical elasticities of substitution between leisure and non-leisure goods, ρ, and between polluting and non-polluting goods, ω. Differences in tastes are captured by differences in other parameter values of the posited utility function (a^j and b^j in equations (8)–(9) below). Assume further that emissions enter the utility function linearly. The preferences for a person of type j can then be represented by

$$\mho^j = \boldsymbol{U}(x, y, L^j; \theta^j) - \phi E, \quad j = 1, 2, 3, 4, \tag{7}$$

where θ^j reflects the "taste parameter" and

$$\boldsymbol{U}(x, y, L^j, \theta^j) = \left(b^j {Q^j}^{\frac{\rho-1}{\rho}} + (1 - b^j)(1 - L^j)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}, \tag{8}$$

$$Q^j = \left(a^j x^{\frac{\omega-1}{\omega}} + (1 - a^j) y^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}. \tag{9}$$

With emissions emanating from production as well as consumption, total level of emissions is given by

$$E = \sum_{j=1}^{4} \pi^j y^j + D. \tag{10}$$

Consumers choose their consumption bundles by maximizing (7)–(9) subject to their budget constraints. These will be nonlinear functions when the income tax schedule is nonlinear. However, for the purpose of uniformity in exposition, we characterize the consumers' choices, as the solution to an optimization problem in which each person faces a (type-specific) *linearized and possibly truncated* budget constraint. To do this, introduce a "virtual income," G^j, into each type's budget constraint. Denote the j-type's net of tax wage by w_n^j. We can then write j's budget constraint as

$$px^j + qy^j = G^j + M^j + w_n^j L^j, \tag{11}$$

where p and q are the consumer prices of x and y, G^j is the income adjustment term (virtual income) needed for linearizing the budget constraint (or the lump-sum rebate if the tax function is linear), and M^j is the individual's exogenous income. The first-order conditions for a j-type's optimization problem are

$$\frac{1-a^j}{a^j}\left(\frac{x^j}{y^j}\right)^{\frac{1}{\omega}} = \frac{q}{p}, \tag{12}$$

$$\frac{(1-b^j)\left(x^j/(1-L^j)\right)^{\frac{1}{\rho}}}{a^j b^j \left[a^j + (1-a^j)(x^j/y^j)^{\frac{1-\omega}{\omega}}\right]^{\frac{\omega-\rho}{\rho(1-\omega)}}} = \frac{w_n^j}{p}. \tag{13}$$

Equations (11)–(13) determine x^j, y^j and L^j as functions of p, q, w_n^j and $G^j + M^j$.

Observe that our demand specification combines homothetic preferences with heterogeneous tastes. As it will become clear below, heterogeneous tastes play an important role in our analysis. As far as the analytical modeling is concerned, allowing for heterogeneity provides an additional measure of generality. However, when it comes to the calibration, our setting is restrictive. Because preferences are homothetic, we are effectively assuming that all differences in the mix of final goods are due to taste differences. Thus differences in consumption patterns, particularly the observed fact that low income people consume a higher proportion of their income on energy goods, are treated as if they have been caused by taste differentials. Although these differences may very well be due to differences in incomes. This has to be kept in mind in interpreting our results.

Finally, observe that w (the price of one unit of effective labor) from the production side, and w_n^j (the net of tax wage of a j-type person) from the consumption side, are related. Denote the productivity of a j-type worker by n^j. Then, j-type's gross of tax wage will be $w^j = wn^j$. Denoting the j-type's marginal income tax rate by t^j, his net of tax wage is $w_n^j = w^j(1-t^j)$. Determining w thus determines the general equilibrium solution for the economy as whole [equations (4)–(13)]. This is done by equating aggregate demand for, and aggregate supply of effective labor. Now, when j works for L^j hours, his effective labor is $n^j L^j$ resulting in aggregate supply of $\sum_{j=1}^{4} \pi^j n^j L^j$. This then should be equated with aggregate demand, L, as given by equation (4):

$$L = \sum_{j=1}^{4} \pi^j n^j L^j. \tag{14}$$

2.3 Data and the calibration

To determine the general equilibrium solution of the economy numerically, one must know different workers' productivity rates and their respective shares in total labor force (n^j, π^j), the parameters of the production function $(\sigma, \delta, \alpha, \beta, A, B)$, the parameters of the utility function $(\rho, \omega, a^j, b^j, \phi)$, world prices (r, p_D), the values of the tax parameters $(p-1, q-1, \tau_D, t^j, G^j)$, and exogenous income M^j. We calibrate the values of all non-tax parameters based on the available statistics for France. In doing this, we use the values of the tax parameters as they currently are in France. Later, we calculate the tax values endogenously as the solution to various optimal tax problems. All the data come from the "Institut National de la Statistique et des Etudes Economiques" (INSEE), France. We use 1989 as our base year.[7]

On the production side, σ and δ are calculated on the basis of current estimates in the literature.[8] We set $r = 0.08$. This is the commonly rate used in France for public investment decisions.[9] Given r, σ, δ, and the data for L, K, D and w, we calibrate α, β, A and B.

We identify *four* types of households: "managerial staff" (Type 1), "intermediate-salaried employees" (Type 2), "white-collar workers" (Type 3) and "blue-collar workers" (Type 4). The data also give the number of households in each type. Their productivity rates are determined from their hourly wages in relation to the hourly wage for all workers: $hw^j = n^j hw, (j = 1, 2, 3, 4)$. The marginal tax rates faced by the four types, t^j's, and the corresponding virtual incomes, G^j's, are reported in the French official tax publications for 1989 (Ministere de l'Economie et des Finances, 1989). Turning to consumption taxes, we note that the consumption of non-polluting goods are taxed at an average rate of 9.6% ($p - 1 = .096$), and consumption of polluting goods at 53.8% ($q - 1 = .538$); see INSEE Résultats (1998). There does not exist a reliable estimate of τ_D, the energy input tax averaged over all consumption goods. In lieu of this, we set $\tau_D = 0$.

On the consumption side, the values of ρ and ω are from Cremer *et al.* (2003).[10] We calculate a^j's and b^j's such that the observed data for households' expenditures and labor incomes are reconciled. This procedure is also used to calculate M^j's; we do not have direct estimates for them.[11]

Finally, given the disagreement in the literature over the size of the marginal social damage of carbon emissions, we use a value for ϕ based on a 1990 recommendation of the "Groupe Interministériel sur l'effet de Serre". This was a French Government Commission set up to undertake an economics study of the greenhouse effect. The recommendation called for a carbon tax of 229 euro (1,500 French Francs) per ton of carbon. This translates into approximately a 10% tax on the cost of producing one "unit" of polluting good.[12] Assuming that 229 euro measures the social damage of a ton of carbon, and because the optimal tax on the polluting good at the first best is its marginal social damage, we calibrate the value of ϕ in such a way as it would give rise to a first-best tax of 10%.[13] We then fix the value of ϕ at this estimated value for all the second-best tax calculations.

The 229 euro figure translates into 62 euro per ton of CO_2 emissions. Thus, if one considers CO_2 emissions only, this figure appears on the high side given the published values for the social damage of a ton of CO_2 emissions. These vary between 1.50 to 51 dollars.[14] Another argument in favor of using a lower figure for the social damage is that the domestic damage is lower than the global damage. It is not at all clear though that a country's government should consider only the domestic social damage in its welfare calculations. If all countries set their taxes optimally and act cooperatively, the global social damage appears to be the appropriate measure to use.[15] Given these considerations, and for the purpose of comparisons, we also consider a lower value for the social damage corresponding to a cost of 92 euro per ton of carbon. This, being 40% of the 229 euro figure, necessitates the calculation and use of a second value for ϕ such that the first-best tax will be 4% instead of 10%.

2.4 The French benchmark tax system

In order to compare the current French tax system with the alternative tax policies we study, the current system must be simplified so that it satisfies the assumptions of our model. We thus construct a simplified version of the French economy which we call "the French benchmark tax system". This differs from the "real" French tax system in three key respects. First, the population is comprised of only four types of households; second all households work; and third all capital is imported so that labor is the only source of domestic income.[16] Specifically, we solve the model of Section 2 using the observed values for the tax rates in France previously mentioned and the calibrated parameter values of subsection 2.3. This differs from the specifications of the French economy in that it sets all calculated exogenous incomes (M^j's) equal to zero. Table 2 reports the solution values whenever they differ from the actual system. Note that the macro (type-independent) variables are extremely close to the actual observed values given in Table 1. The solutions for the household types differ in two important respects. First, the figures for the four types' labor supplies are somewhat higher than their actual observed values. This is easy to explain. Given that the *aggregate* labor supply in the benchmark system is close to its actual observed value, the benchmark attributes the excluded groups' labor supplies to the included four. The second difference appears in consumption levels. The assumption of no domestic capital income results in expenditure levels for the benchmark system that are lower than the observed ones.

3 The government

The government is interested in designing an optimal tax system consisting of a general income tax, and taxes on energy as a consumption good and as an intermediate good. The design of an optimal tax structures must be based on some underlying social welfare function. For this purpose, we will use an iso-elastic social welfare function of the form

$$W = \frac{1}{1-\eta} \sum_{j=1}^{4} \pi^j (\mho^j)^{1-\eta} \quad \eta \neq 1 \quad \text{and} \quad 0 \leq \eta < \infty, \tag{15}$$

where η is the "inequality aversion index". The higher is η the more the society values equality.[17]

In choosing a value for η (the inequality aversion index) for our optimal tax calculations, we will be guided by the observed degree of redistribution in the existing French tax system. Bourguignon and Spadaro (2000) have recently studied France's social preferences as revealed through its tax system. They find that, if the uncompensated wage elasticity of labor supply is 0.1, the marginal social welfare falls from 110 to 90 percent of the mean as income increases from the lowest to the highest level. The fall would be from 150 percent to 50 percent if the uncompensated labor elasticity is 0.5.

With the social welfare function (15), the marginal social utility of income for a j-type person is given by

$$\frac{\partial \mho^j}{\partial M^j}(\mho^j)^{-\eta}.$$

This implies that the ratio of the marginal social utility of the Managerial Staff's (type 1)income to Blue Collars's (Type 4) income is

$$\frac{\partial \mho^1/\partial M^1}{\partial \mho^4/\partial M^4}\left(\frac{\mho^4}{\mho^1}\right)^{\eta}.$$

Calculating the values for $\partial \mho^j/\partial M^j$ and $\mho^j$ $(j = 1, 4)$ based on the French data summarized in Table 1, setting the above expression equal to 9/11, we derive a value for η equal to 0.1. This is the implied value for the inequality aversion index in France (if the uncompensated wage elasticity of labor supply is 0.1). Similarly, setting the above expression equal to 5/15, we derive a value for η equal to 1.9 for the implied value of the inequality aversion index in France (if the uncompensated wage elasticity of labor supply is 0.5).

3.1 Measuring welfare changes

A change in the government's tax policy, environmental or otherwise, changes the welfare different households differently. We shall measure these using the Hicksian "equivalent variation" concept of a welfare change, EV.[18] Similarly, we can associate a measure of "aggregate welfare change" to any tax policy by calculating how much one has to *uniformly* compensate each individual under the benchmark system, to bring social welfare under the benchmark to parity with that under the considered tax policy.[19]

4 The optimal general income tax with linear energy taxes

The standard assumption in the optimal income tax literature à la Mirrlees is that incomes are publicly observable so that the tax administration can levy an optimal general income tax. We adopt this, and the accompanying assumption on the unobservability of wages and labor supplies, in order to rule out differential lump-sum taxation. Regarding consumption taxes, we shall assume that the available information is on anonymous transactions (and not on personal consumption levels which would be difficult to justify). This rules out nonlinear consumption taxes. In determining the mix of optimal general income and linear consumption taxes, we follow Cremer and Gahvari (1997)'s method and begin with the characterization of Pareto-efficient allocations that are constrained not only by the standard self-selection constraints and the resource balance, but also by the linearity of commodity taxes. However, the current problem is fundamentally more complex than Cremer and Gahvari (1997)'s (as well as the traditional optimal income tax problems à la Mirrlees). The endogeneity of w adds another dimension to the problem which is generally missing in the formulation of optimal income tax problems.[20] This requires us to extend Cremer and Gahvari's method, as explained below.

We start by deriving an optimal revelation mechanism that consists of a set of type-specific before-tax incomes, I^j's, aggregate expenditures on private goods, c^j's, and a fixed tax rate (same for everyone) on the polluting good, $q-1$. Observe that, with one extra degree in setting the commodity tax rates, due to demand and labor supply functions being homogeneous of degree zero in prices and income, we have set the tax rate on the non-polluting goods equal to zero so that $p = 1$. This procedure determines the polluting tax rate right from the outset. A complete solution to the optimal tax problem per-se then requires only the design of a general income tax function.

The mechanism assigns (I^j, c^j, q) to an individual who reports type j; the consumer then allocates c^j between the produced goods, x and y.[21] Denote $G^j + w_n^j L^j \equiv c^j$. One can then determine, by setting $M^j = 0$ in equations (11) and (12), the "conditional" demand functions for x^j and y^j as $x^j = \boldsymbol{x}(p, q, c^j; \theta^j)$ and $y^j = \boldsymbol{y}(p, q, c^j; \theta^j)$. These functions are independent of individual j's labor supply because of the weak separability of his preferences.[22] Substituting these equations in the j-type's utility function, we have

$$\boldsymbol{V}\left(p, q, c^j, \frac{I^j}{wn^j}; \theta^j\right) \equiv \boldsymbol{U}\left(\boldsymbol{x}(p, q, c^j; \theta^j), \boldsymbol{y}(p, q, c^j; \theta^j), \frac{I^j}{wn^j}; \theta^j\right). \qquad (16)$$

4.1 Analytical results

The problem is solved analytically in Section A1 of the Appendix.[23] A number of analytical results are obtained. First, the optimal tax on energy as a consumption goods is non-Pigouvian (Proposition A1). This generalizes our earlier result in Cremer *et al.* (1998) derived for exogenously given wage rates. As in that paper, the "Pigouvian tax" is defined as the marginal social damage of pollution when each type's private damage in utility terms is transformed into "social" dollars using the government's shadow cost of public funds, μ. It is measured, given our specification of the social welfare function and preferences, by $\left[\sum_j \pi^j (\mho^j)^{-\eta}\right] \phi/\mu$.[24] Second, condition $w - \boldsymbol{O}_L\left(L, K, D\right) = 0$ imposes no constraint on the problem, i.e., the Lagrangian multiplier associated with it is zero when $w = \boldsymbol{O}_L(.)$ (Lemma A1). This tells us that production efficiency holds despite the endogeneity of wages *and* the breakdown of Atkinson and Stiglitz (1976) theorem (due to heterogeneity of tastes). Third, the optimal tax on the polluting input is always equal to the Pigouvian tax (Proposition A2). This result also extends the earlier property obtained by Cremer *et al.*'s (1998) to a setting with a general production technology and endogenous wages.

It is important to note that our result on the Pigouvian taxation of polluting inputs is due to the assumption that labor is homogeneous. Measured in efficiency units, labor appears as a single input in production with individuals of different types being endowed with varying efficiency units of labor. As a consequence, while the wage level is endogenous, relative wages are given in our setup and are not affected by emission levels. If, on the other hand, skilled and unskilled labor inputs (high- and low-ability workers) have different degrees of complementarity or substitutability with the polluting input, the relative wages will change with emission levels. This leads to redistribution among different labor types. Naito's (1999) has shown, in a model without externality, that manipulating relative wage changes to increase the wage of the low-ability types can serve as a useful redistributive mechanism.[25] Put differently, differential taxation of labor inputs, which violates the Diamond and Mirrlees (1971) production efficiency result, becomes optimal. Recast in a setup with externalities, this result means that different labor types should be taxed differently vis-a-vis energy inputs. This also implies that energy inputs should be taxed not just for Pigouvian considerations but for redistributive purposes as well.[26]

4.2 Numerical results

The numerical solutions are derived using GAUSS's constrained optimization program.[27]

4.2.1 Basic scenarios

The first row of Table 3 reports four values for the optimal tax on the polluting good based on two values for the marginal social damage of emissions and two values for the inequality aversion index. The second row reports the values for the Pigouvian tax which, on the basis of the results presented in the previous subsection, is equal to the optimal tax on the polluting input.

The interesting feature of our result is that while the optimal tax on the polluting input is always equal to the Pigouvian tax, the optimal tax on the polluting good is always *less* than the Pigouvian tax. Indeed, in three out of four cases, the polluting good must be subsidized rather than taxed. The reason for this is in the roles that input and output taxes play. The tax on the polluting input serves one purpose only: It is imposed to correct the social damage of emissions. The tax on the polluting good, on the other hand, serves two purposes: One is, as with the polluting inputs, externality correcting; the second is redistributive. Whereas the externality correction calls for the taxation of the polluting good, the redistributive objective calls for its *subsidization* (relative to non-polluting goods). This is because the poor spend proportionally more of their incomes on the polluting goods. The optimal tax on polluting goods balances these two objectives.

In interpreting our numerical results, however, a particular shortcoming of our calibrations must be borne in mind. Our demand specification combines homothetic preferences with heterogeneous tastes. This implies that we are effectively attributing all differences in the mix of final goods, in particular the observed fact that low income people consume a higher proportion of their income on energy goods, to taste differentials. Although these differences may very well be due to differences in incomes.

Note also that the higher is the inequality aversion index, the higher will be the deviation of the optimal tax relative to the Pigouvian tax. The intuition is found in the two roles that output taxes embody. The Pigouvian element of output taxes is invariant to redistributive ends. This is obviously not the case for their redistributive element. The more we care about the poor, the higher we want to subsidize their consumption of polluting goods (relative to non-polluting goods). Thus, with a low value for ϕ, when η increases from 0.1 to 1.9, the optimal subsidy on energy consumption increases from 1.71% to 12.40%; but the Pigouvian tax remains very much unchanged at 4%. Similarly, with a high value for ϕ, when η increases from 0.1 to 1.9, the optimal tax on the consumption decreases from 3.90% to an outright subsidy of 7.32%; with the Pigouvian tax remaining unchanged at 10%.[28]

The first row in Table 4 reports the changes in aggregate emissions. It indicates that the optimal policy entails an *increase* in aggregate emissions. This occurs for two reasons. First, the optimal policy entails a cut in taxes on energy-related consumption goods thus boosting their demand. Secondly, the increased efficiency of the tax system as a whole encourages production and with it energy use and consumption. Note that the introduction of taxes on energy inputs, on the other hand, has a dampening effect on the use of energy and thus works to mitigate the increase in emissions.

On the redistributive front, the tax system becomes much more progressive. The implied EV figures, and the associated social welfare changes, are reported in Table 4. The magnitude of the changes are extremely large. When $\phi = 0.016$ and

$\eta = 0.1$, the loss in Type 1's welfare amounts to as much as 6,528 euro. Type 2 loses by 379 euro and Types 3 and 4 gain by 2,458 and 2,066 euro. These translate into a social welfare gain of 405 euro. Similar results hold when When $\phi = 0.040$ and $\eta = 0.1$. Moreover, as one might expect, the gains to the poor and the losses of the rich increase with η. The increase in social welfare is also more pronounced for the higher value of η.

4.2.2 Optimal system with the current energy tax

These changes come about as a result of the change in the whole structure of the tax system, and particularly the change in the income tax structure. To isolate the effects of environmental taxes per se, we also find the tax equilibrium of the economy while only setting the income tax optimally keeping the energy taxes fixed at their current values. Then we compare the resulting equilibrium with the previously calculated optimal system. Any difference must be due solely to the change in environmental taxes. With some adjustments, the procedure for determining the new equilibrium is the same as that of solving the initial unconstrained tax problem. A formal solution is presented in Section A2 of the Appendix. It requires one to impose two additional constraints on the original problem. They are:

$$q = 1.4032, \tag{17}$$

$$\boldsymbol{O}_D (L, K, D) = 1, \tag{18}$$

reflecting the current 40.32% average energy consumption tax relative to other goods and zero energy input taxes. The first constraint implies that we no longer optimize with respect to q; the second constraint enters as an additional term in the Lagrangian expression. The interesting implication of this latter constraint is that it implies $\boldsymbol{O}_K(L, K, D)$ should no longer be set equal to r. Put differently, it calls for a producer tax on r. On the other hand, the condition $w = \boldsymbol{O}_L\,(L, K, D)$ is not affected and continues to be optimal.

With the exception of the values for polluting input and aggregate emissions, the equilibrium of this economy looks very much the same as when the environmental taxes were unrestricted. This suggests that the drastic changes to the benchmark system are essentially due to a switch to an optimal general income tax system. Table 5 reports the redistributive effects of environmental taxes per se by comparing the equilibria of the general income tax structure with and without emission taxes. It also reports the resulting changes in aggregate emissions. Observe that cutting the energy consumption taxes from their current values (as the optimum requires), and the introduction of energy input taxes, hurts the rich (Type 1) and benefits the poor (Type 4) substantially. Types 2 and 3 also benefit, although not by as much. Observe also that, for the same marginal social damage of emissions ϕ, a higher value for the inequality aversion index η translates into more losses for Type 1 and more gains for type 4. The implied gains to social welfare, while not very huge, are nevertheless more than modest.

4.2.3 Varying relative wages

Our specification allows for endogenous wages but in such a way as to keep relative wages of different labor types constant. Specifically, we have $w^j = wn^j$, where the n^j's are given parameters while w (the overall wages level) is endogenously determined according to (4). This is done to keep the numerical simulations tractable. It thus leaves open the question of how different complementarity/substitutability relationships between energy input and different types of labor inputs might affect the output tax. The point is that when energy input interacts differently with different labor inputs, individual types' relative wages change as the economy goes from one equilibrium to another. The resulting change in the relative (gross of tax) incomes necessitates a further change in redistributive taxes, including the output tax. A full-fledged analysis of this conjecture, in terms of numerical calculations, is too complex. Thus, to get an insight into this, we run a number of simulations that indicate how different (but exogenously determined) changes in relative wages of the four individual types, affect the output tax.

Recall that in our setting, Type 1 agents are the high-productivity workers. At the other end of the spectrum, Types 3 and 4 have almost identical wages (n^3 and n^4 are close). We thus consider two alternative scenarios. The first increases n^3 and n^4 by 5%, leaves n^2 unchanged, and adjusts n^1 such that the average level of n does not change. The constancy of average n is imposed so that we are able to separate the effect of a change in relative productivities from a change in the overall productivity level. The second scenario is the mirror image of the first in which we reduce n^3 and n^4 by 5%, again leave n^2 unchanged, and adjust n^1 to keep the average level of n intact.

Table 6 presents the values of the output tax obtained under these two scenarios. They are very much in line with what one might expect. In all cases but $\eta = 0.1$ and $\phi = 0.04$, we have outright subsidies. In these cases, an increase in the relative wage of low-productivity individuals causes the subsidy rate to decrease (as compared to the subsidy rate under the original set of productivities). The reason is that the narrowing of the wage gap lowers the need for a redistributive subsidy on the polluting good. Conversely, a decrease in the relative wage of low-productivity individuals causes the subsidy rate to increase. This time, the widening of the wage gap increases the need for a redistributive subsidy on the polluting good. The same explanation applies to the case where $\eta = 0.1$ and $\phi = 0.04$ in which polluting goods are taxed. When relative wages of low-productivity individuals increase, and the wage gap narrows, the tax on the polluting good increases. Thus the *implicit* subsidy relative to a Pigouvian output tax is reduced. And when relative wages of low-productivity individuals decrease, and the wage gap widens, the tax on the polluting good decreases. That is, the *implicit* subsidy relative to a Pigouvian output tax increases. Observe also that, as with the reference case, the higher is the inequality aversion index, the higher is the deviation of the optimal tax relative to the Pigouvian tax. For the rest, the changes that occur are not dramatic.

5 Summary and conclusion

This paper has explored the design of an optimal general income tax system when the wage rate is endogenous, labor is homogeneous, and energy is used both as a polluting consumption good and a polluting input. The paper has shown that the optimal tax on energy inputs is Pigouvian and equal to its marginal social damage. The optimal tax on the consumption of energy, on the other hand, is less than its marginal social damage. In fact, in three out of four cases, energy consumption should be subsidized. This is in marked contrast to the current tax system in France which taxes energy consumption over 40% relative to non-energy related goods. The reason for this is the fact that the poor spend proportionally more of their income on energy consumption than the rich.

To gauge the welfare implications of environmental taxes per se, we have compared the optimal general income tax equilibria at the current environmental taxes and at their optimal values. The results indicated substantial losses for the rich (Type 1) and substantial gains for the poor (type 4). In comparison, the effects on Types 2 and 3 were rather marginal. The redistributive aspects of such taxes, which are quite important, are naturally masked in a one-consumer representation of the economy. This tells us that in calculating optimal income and consumption taxes, including environmental taxes, one should go beyond the traditional

Ramsey tax framework.

The methods of this paper can be used to compute optimal tax structures for other countries. Better data may allow for a greater number of types. It should also allow for a more disaggregated set of goods and better parameter estimates. Another extension would be to consider non-homothetic preferences directly. Most importantly, an extension that considers non-homogeneous labor, and thus allows for energy input to be a substitute for low-skilled labor (low-ability types) and a complement to high-skilled labor (high-ability types) will throw light on the extent that the energy input tax should deviate from the Pigouvian tax for redistribution. The current paper should be viewed more as a first step contribution to this endeavor, rather than the exactness of its reported numbers. Nevertheless the numbers are interesting even if only indicative.

Notes

[1]See, e.g., Bovenberg and Goulder (1996) and the references contained therein.

[2]The implications of this assumption is discussed in subsection 4.1 below.

[3]There are two reasons for assuming capital is rented from outside. One, we do not have data on holdings of capital by different types of workers. Second, taxation of capital in a static setting is not an interesting question. The similar assumption on energy inputs is for simplicity in exposition and of no relevant consequence.

[4]This treats energy from all sources (fossil fuel and nuclear) symmetrically. While not quite satisfactory, this is inevitable at the level of aggregation we are working. The purpose of this paper is not to compute precise tax rates at a disaggregate level; this will not be possible with two types of goods and four groups of individuals. To be sure, there are more sophisticated "computable general equilibrium" models at more disaggregate levels in the literature which attempt to do this. However, unlike ours, these papers assume a *linear* income tax structure. Observe also that more recently a theoretical body of literature has appeared, under the heading of "New dynamic optimal taxation," that applies Mirrlees' approach to optimal taxation in dynamic settings. The focus of this literature, however, is on the stochastic evolution of skills. See, e.g., Kocherlakota (2005a,

2005b) and references therein. In our framework, although wages are endogenous, types are given. Our aim is to present a relatively simple framework for the numerical calculations of environmental taxes simultaneously with an optimal general income tax, in a setting with two dimensions of heterogeneity particularly when the wage is endogenous. It is these features that distinguishes our study from the other papers in the literature.

[5]See our discussion on calibrations below in subsection 2.3.

[6]Taxation of capital in a setting like ours will serve no purpose except to violate production efficiency.

[7]This is the most recent year for which there exist comprehensive consumption surveys for eight different household types ("Budget des familles") as well as surveys on employment and wages classified by household types ("Enquête sur l'emploi"). The data covers 117 consumption goods which we aggregate into: (i) non-energy consumption representing non-polluting goods (x), and (ii) energy consumption representing polluting goods (y).

Observe also that all published data are in French francs. We convert these into euro using the official conversion rate of 1 euro = 6.55957 French francs.

[8]These are based on the estimates of elasticities of substitution between various factors of production in Berndt and Wood (1975, 1985), Griffin and Gregory

(1976), and Devezeaux de Lavergne, Ivaldi and Ladoux (1990).

[9]See, e.g., DGEMP-DIGEC, Ministère de l'Economie et des Finances, 1997.

[10]Cruz and Goulder (1992) and Goulder *et al.* (1999) use a higher value for ω (0.85), and Bourguignon (1999) reports a range of 0.1 to 0.5 for the existing estimates of *wage elasticity of labor supply* which translate into a range of estimates for ρ from to 0.61 to 1.39. We thus also set $\omega = 0.99$, $\rho = 0.61$ and $\rho = 1.39$ in our optimal tax calculations for sensitivity analysis.

[11]Details of the data and calibration can be obtained from the authors directly.

[12]We assume that energy consists of coal, natural gas, electricity, gasoline, and oil. We have data on the consumption levels of each of these components in their physical units and the carbon content of each physical unit. This gives us the total carbon content of energy consumption. We also know the euro cost of producing each component and thus the production costs of the total energy used. (The component shares are 0.94% coal, 8.79% natural gas, 28.01% electricity, and 62.26% gasoline and oil). Dividing the number for total carbon content of energy consumption by the number for the production cost gives us the carbon content of producing one euro of energy. Assuming the component shares do not change, we can compute the cost of producing that amount of energy with a one ton carbon content. This comes to about 2,290 euro. Consequently, a 229 euro tax on one ton of carbon translates into a 10% tax.

[13]Specifying the social welfare function as $\sum_j \pi^j W(\mho^j)$, the marginal social damage of emissions is defined as $\left[\sum_j \pi^j W'^j)\right] \phi/\mu$, where μ is the shadow cost of public funds (the Lagrange multiplier associated with the government's budget constraint). This is the formula for the first-best Pigouvian tax.

Observe also that we will have a different value for ϕ for each set of parameters ρ, ω, a^j, b^j.

[14]See, "Marginal damage estimates for air pollutants", U.S. Environmental Protection Agency, http://www.epa.gov/oppt/epp/guidance/top20faqexterchart.htm.

[15]There is also the costs associated with nitrogen oxides due to use of some types of energy. Presumably, though, one should leave this out if one is concerned with a carbon tax only.

[16]As observed earlier, these simplifications are necessitated by the limitation of the existing data and the fact that we are interested only in labor income taxes. Optimal taxation of capital in a static model is not an interesting question.

[17]As is well-known, $\eta = 0$ implies a utilitarian social welfare function and $\eta \to \infty$ a Rawlsian. When $\eta = 1$, the social welfare function is given by $W = \sum_{j=1}^{4} \pi^j \ln(\mho^j)$.

[18]Let $\mathbf{V}(\cdot; \boldsymbol{\theta}^{\mathbf{j}})$ denote a j-type's "indirect utility" function as defined by equation (16). The equivalent variation in the j-type's going from the benchmark B to the

alternative tax system i, EV_i^j, is defined from

$$\mathbf{V}\big(p_B, q_B, w_{n,B}^j, G_B^j + M_B^j + EV_i^j; \theta^j\big) - \phi\big(\sum_{j=1}^{4} \pi^j y_B^j + D_B\big) =$$

$$\mathbf{V}\big(p_i, q_i, w_{n,i}^j, G_i^j + M_i^j; \theta^j\big) - \phi\big(\sum_{j=1}^{4} \pi^j y_i^j + D_i\big).$$

We will then measure the "welfare change" in going from policy i to k for individual j by $EV_k^j - EV_i^j$.

[19]The aggregate welfare measure associated with going from the benchmark B to the alternative tax system i, EV_i^S, is formally defined by

$$\sum_{j=1}^{4} \pi^j \Big[\mathbf{V}\big(p_B, q_B, w_{n,B}^j, G_B^j + M_B^j + EV_i^S; \theta^j\big) \quad - \quad \phi\big(\sum_{j=1}^{4} \pi^j y_B^j + D_B\big)\Big]^{1-\eta} =$$

$$\sum_{j=1}^{4} \pi^j \Big[\mathbf{V}\big(p_i, q_i, w_{n,i}^j, G_i^j + M_i^j; \theta^j\big) \quad - \quad \phi\big(\sum_{j=1}^{4} \pi^j y_i^j + D_i\big)\Big]^{1-\eta}.$$

[20]Naito (1999) is an exception.

[21]Strictly speaking, this procedure does not characterize "allocations" as such; the optimization is over a mix of quantities *and* prices. However, given the commodity prices, utility maximizing individuals would choose the quantities themselves. We can thus think of the procedure as indirectly determining the final allocations.

[22]The functions, and the corresponding indirect utility function $\boldsymbol{V}(\cdot;\theta^j)$, are *conditional* on c^j; they differ from the customary Marshallian demand and indirect utility functions.

[23]The appendix is available through *JEEM*'s online archive of supplementary material, which can be accessed through a link at http://www.aere.org/journals/.

[24]This is Cremer *et al.* (1998) definition of the Pigouvian tax. Bovenberg and van der Ploeg (1994), Bovenberg and de Mooij (1994) and others use the Samuelson's rule for optimal provision of public goods to define the "Pigouvian tax". They term a tax Pigouvian if it is equal to the sum of the private dollar costs of the environmental damage per unit of the polluting good across all households. In our notation, their Pigouvian tax is $\sum_j \pi^j \phi/\alpha^j$, where α^j is the j-type's private marginal utility of income.

[25]Intuitively, the reason that one may do better with an input tax in Naito is that pushing the wage of the unskilled workers up weakens an already binding incentive compatibility constraint.

[26]We are grateful to a careful referee who brought this point to our attention.

[27]This program, with a number of different iterative algorithms, is particularly suitable for optimization of nonlinear functions subject to nonlinear inequality constraints. One such routine, which we have used, is the Boyden-Fletcher-Goldfarb-Shanno (BFGS) method known for its excellent convergence properties even for ill-behaved problems.

[28]The alternative definition of the Pigouvian tax yields the following values

for $\phi \sum_j \pi^j / \alpha^j$: 3.88% ($\phi = 0.016, \eta = 0.1$), 3.68% ($\phi = 0.016, \eta = 1.9$), 9.68% ($\phi = 0.40, \eta = 0.1$), and 9.19% ($\phi = 0.040, \eta = 1.9$). Note that these values also exceed the values for the optimal polluting good taxes. However, they are smaller than the optimal input taxes.

References

[1] Berndt, E.R, Wood, D.O., 1975. Technology, prices, and the derived demand for energy. Review of Economics and Statistics 57, 259-268.

[2] Berndt, E.R, Wood, D.O., 1985. Energy Price Shocks and Productivity Growth in US Manufacturing. MIT Press, Cambridge, Mass.

[3] Bourguignon, F., 1999. Redistribution and labor-supply incentives. mimeo.

[4] Bourguignon, F., Spadaro A. 2000. Social preferences revealed through effective marginal tax rates. mimeo.

[5] Bovenberg, A.L., van der Ploeg, F., 1994 Environmental policy, public finance and the labor market in a second-best world. Journal of Public Economics 55, 349-390.

[6] Bovenberg, A.L., de Mooij, R.A., 1994. Environmental Levies and Distortionary Taxation. American Economic Review 84, 1085-1089.

[7] Bovenberg, A.L., Goulder, L. 1996 Optimal environmental taxation in the presence of other taxes: general equilibrium analyses. American Economic Review 86, 985-1000.

[8] Bovenberg, A.L., Goulder, L. 2002 Environmental taxation and regulation, In: Auerbach, A., Feldstein, M. (Eds.), Handbook of Public Economics, Vol 3, Amsterdam: North-Holland, Elsevier, 1471-1545.

[9] Cremer, H., Gahvari, F., Ladoux, N., 1998. Externalities and optimal taxation. Journal of Public Economics 70, 343-364.

[10] Cremer, H., Gahvari, F., Ladoux, N., 2003 Environmental taxes with heterogeneous consumers: an application to energy consumption in France. Journal of Public Economics 87, 2791-2815.

[11] Devezeaux de Lavergne J.G., Ivaldi M., Ladoux, N., 1990. La forme flexible de Fourier: une evaluation sur donnees macroeconomiques. Annales dEconomie et de Statistique 17. 1997.

[12] Les couts de reference de la production delectricite. DGEMP-DIGEC, Ministere de lEconomie et des Finances.

[13] Gahvari, F., 2002. Review of Ruud A. de Mooij: Environmental Taxation and the Double Dividend. Journal of Economic Literature 40, 221-223.

[14] Goulder, L.H., Parry, I.W.H., Williams III, R.C., Burtraw, D., 1999. The cost effectiveness of alternative instruments for environmental protection in a second-best setting. Journal of Public Economics 72, 329-360.

[15] Griffin, J.M., Gregory, P.R., 1976. An intercountry translog model of energy substitution responses. American Economic Review 66, 845-857.

[16] INSEE, 1998. Comptes et Indicateurs Economiques, Rapport sur les Comptes de la Nation 1997. Serie INSEE Resultats, 165-167. ISBN-2-11-0667748-6.

[17] INSEE, 1989. Emplois-revenus: enquete sur l'emploi de 1989, resultats detailles. Serie INSEE Resultats, 28-29, ISBN 2-11-065325-6.

[18] INSEE, 1991a. Consommation modes de vie: Ie budget des menages en 1989. Serie Consommation Modes de Vie, 116-117, ISBN 2-11-065922-X.

[19] INSEE, 1991b. Emplois-revenus: les salaires dans l'industrie, Ie commerce et les services en 1987-1989. Serie INSEE Resultats, 367-369, ISBN 2-11-06-244-1.

[20] INSEE, 1998. Comptes et indicateurs economiques: rapport sur les comptes de la nation 1997. Serie INSEE Resultats, 165-167, ISBN 2-11-0667748-6.

[21] Kocherlakota, N., 2005a. Zero expected wealth taxes: A Mirrlees approach to dynamic optimal taxation. Econometrica 73, 1587-1621.

[22] Kocherlakota, N., 2005b. Advances in dynamic optimal taxation. Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress, vol 1, 269-299.

[23] Mayeres, I., Proost, S., 2001. Marginal tax reform, externalities and income distribution. Journal of Public Economics 79, 343-363.

[24] Ministere de l'Economie et des Finances, 1989. Publication N 2041 S. France.

[25] Naito, H., 1999. Re-examination of uniform commodity taxes under a non-linear income tax system and its implication for production efficiency. Journal of Public Economics 71, 165-188.

[26] Poterba, J.M., 1991. Tax policy to combat global warming: on designing a carbon tax. In: Dornbusch, R., Poterba, J.M. (Eds.), Global Warming: Economic Policy Responses, The MIT Press, Cambridge, Mass.

[27] Schöb, R., 2003. The double dividend hypothesis of environmental taxes: a survey. CESifo Working Paper No. 946.

[28] U.S. Environmental Protection Agency, 1998. Marginal damage estimates for air pollutants. http://www.epa.gov/oppt/epp/guidance/top20faqexterchart.htm.

Table 1. Data Summary:1989
(monetary figures in euro)

	Managerial Staff (Type 1)	Intermediary Level ((Type 2)	White Collars (Type 3)	Blue Collars (Type 4)
π	15.41 %	24.77 %	20.00 %	39.82 %
I	32,753	18,071	11,795	11,650
px	38,739	26,558	19,510	19,571
qy	2,378	2,056	1,495	1,796
n	1.18735	0.72134	0.49174	0.45180
L	0.51750	0.47000	0.45000	0.48375
t	28.8 %	19.2 %	14.4 %	9.6 %
G	3,468	1,617	977	702
M	14,329	12,394	9,931	10,134
a	0.99999	0.99997	0.99997	0.99994
b	0.81206	0.75740	0.71987	0.74749
	Type-independent figures			
	$\sum_j \pi^j n^j L^j = 0.30996$	$K = 220{,}664$	$D = 2{,}388$	$O = 42{,}055$
	$p_O = 1.0$	$w = 53{,}304$	$r = 8.0$ %	$p_D = 1.0$
	$p = 1.00000$	$q = 1.40322$	$\sigma = 0.8$	$\delta = 0.32345$
	$\rho = 0.66490$	$\omega = 0.26892$	$\alpha = 0.99999$	$\beta = 0.68955$
	$A = 1.07130$	$B = 0.82647$		

* Aggregate labor includes labor supplied by the four types and other residual groups.

Table 2. The benchmark system
(monetary figures in euro)

	Managerial Staff (Type 1)	Intermediary Level (Type 2)	White Collars (Type 3)	Blue Collars (Type 4)
I	39,714	23,674	16,089	15,514
px	29,908	19,256	13,700	13,488
qy	1,836	1,490	1,050	1,238
L	0.62749	0.61572	0.61380	0.64419
M	0.0	0.0	0.0	0.0
	Type-independent figures			
	$\sum_j \pi^j n^j L^j = 0.40110$	$K = 214{,}688$	$D = 2{,}290$	$O = 40{,}845$

Table 3. Optimal linear environmental and "Pigouvian" taxes

	$\phi = 0.016$		$\phi = 0.040$	
	$\eta = 0.1$	$\eta = 1.9$	$\eta = 0.1$	$\eta = 1.9$
Optimal polluting good tax	-1.79 %	-12.40 %	3.90 %	-7.32 %
Pigouvian tax	4.00 %	4.01 %	10.00 %	10.03 %

Table 4. Emission and welfare changes in going to an optimal environmental-cum-general-income-tax system

(monetary figures in euro)

	$\phi = 0.016$		$\phi = 0.040$	
	$\eta = 0.1$	$\eta = 1.9$	$\eta = 0.1$	$\eta = 1.9$
E	5.48 %	3.34 %	2.93 %	0.82 %
EV^1	-6,528	-8,068	-6,585	-8,120
EV^2	-379	-547	-399	-566
EV^3	2,458	2,715	2,469	2,725
EV^4	2,066	2,405	2,072	2,409
EV^S	405	1,202	398	1,202

Table 5. Effects due to changes in environmental taxes per se

(monetary figures in euro)

	$\phi = 0.016$		$\phi = 0.040$	
	$\eta = 0.1$	$\eta = 1.9$	$\eta = 0.1$	$\eta = 1.9$
E	1.87 %	2.87 %	0.09 %	1.05 %
EV^1	-129	-148	-109	-132
EV^2	6	3	5	2
EV^3	11	3	10	3
EV^4	78	91	68	81
EV^S	19	32	17	28

Table 6. Optimal polluting good taxes when relative, but not average, productivities change

(n_2 is unchanged and n_1 is adjusted)

	$\phi = 0.016$		$\phi = 0.040$	
	$\eta = 0.1$	$\eta = 1.9$	$\eta = 0.1$	$\eta = 1.9$
Reference productivities	-1.79%	-12.40%	3.90%	-7.32%
n_3 and n_4 $\uparrow$ by 5%	-0.61%	-11.17%	5.17%	-6.00%
n_3 and n_4 $\downarrow$ by 5%	-2.96%	-13.50%	2.65%	-8.51%

Appendix

A1 General income plus linear commodity taxes

The Lagrangian for the second-best problem is (where p is set equal to 1),

$$\pounds = \frac{1}{1-\eta}\sum_{j=1}^{4}\pi^j\left[\mathbf{V}\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)-\phi\sum_{j=1}^{4}\pi^j\mathbf{y}\left(q,c^j;\theta^j\right)-\phi D\right]^{1-\eta}+$$
$$\mu\left\{\mathbf{O}\left(K,L,D\right)-\sum_{j=1}^{4}\pi^j\left[\mathbf{x}\left(q,c^j;\theta^j\right)+\mathbf{y}\left(q,c^j;\theta^j\right)\right]-rK-D-\overline{R}\right\}+$$
$$\sum_j\sum_{k\neq j}\lambda^{jk}\left[\mathbf{V}\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)-\mathbf{V}\left(q,c^k,\frac{I^k}{wn^j};\theta^j\right)\right]+\gamma\left[w-\mathbf{O}_L\left(K,D,L\right)\right]. \tag{A1}$$

where μ, λ^{jk} and γ are the multipliers associated respectively with the resource constraints, the incentive constraint and the endogenous wage condition. The first-order conditions are, for $j = 1, 2, 3, 4$,

$$\begin{aligned}
\frac{\partial \pounds}{\partial q} &= \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \times \\
&\left[\mathbf{V}_q\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}_q\left(q, c^j; \theta^j\right) \right] - \mu \sum_{j=1}^{4} \pi^j \left[\mathbf{x}_q\left(q, c^j; \theta^j\right) + \mathbf{y}_q\left(q, c^j; \theta^j\right) \right] + \\
&\sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left[\mathbf{V}_q\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \mathbf{V}_q\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right) \right] = 0, \qquad \text{(A2)} \\
\frac{\partial \pounds}{\partial c^j} &= \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]_c^{-\eta} \mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \\
&\phi \pi^j \mathbf{y}_c\left(q, c^j; \theta^j\right) \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} - \\
&\mu \pi^j \left[\mathbf{x}_c\left(q, c^j; \theta^j\right) + \mathbf{y}_c\left(q, c^j; \theta^j\right) \right] + \sum_{k \neq j} \lambda^{jk} \mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \\
&\sum_{k \neq j} \lambda^{kj} \mathbf{V}_c\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) = 0, \qquad \text{(A3)}
\end{aligned}$$

$$\frac{\partial \pounds}{\partial I^j} = \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \frac{1}{wn^j} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) +$$
$$\mu \mathbf{O}_L(L, K, D) \frac{\pi^j}{w} + \sum_{k \neq j} \lambda^{jk} \frac{1}{wn^j} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) -$$
$$\sum_{k \neq j} \lambda^{kj} \frac{1}{wn^k} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) - \gamma \frac{\pi^j}{w} \mathbf{O}_{LL}(L, K, D) = 0, \tag{A4}$$
$$\frac{\partial \pounds}{\partial D} = -\sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \phi +$$
$$\mu \left[\mathbf{O}_D(L, K, D) - 1 \right] - \gamma \mathbf{O}_{LD}(L, K, D) = 0, \tag{A5}$$
$$\frac{\partial \pounds}{\partial K} = \mu \left[\mathbf{O}_K(L, K, D) - r \right] - \gamma \mathbf{O}_{LK}(L, K, D) = 0, \tag{A6}$$
$$\frac{\partial \pounds}{\partial w} = \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \left(\frac{-I^j}{n^j w^2} \right) \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) +$$
$$\mu \mathbf{O}_L(L, K, D) \frac{-1}{w^2} \sum_{j=1}^{4} \pi^j I^j + \sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left(\frac{-I^j}{n^j w^2} \right) \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) +$$
$$\sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left(\frac{I^k}{n^j w^2} \right) \mathbf{V}_L\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right) + \gamma \left[1 + \frac{1}{w^2} \sum_{j=1}^{4} \pi^j I^j \mathbf{O}_{LL}(L, K, D) \right] = 0. \tag{A7}$$

We now show that whereas the optimal tax on the polluting good is non-Pigouvian, the optimal tax on polluting input is Pigouvian. Consider first the polluting good tax. We have:

Proposition A1 *The optimal tax on the polluting good is non-Pigouvian.*

Proof. Multiply equation (A3) by $\mathbf{y}\left(q, c^j; \theta^j\right)$, sum over j, and add the resulting equation to (A2). Simplifying, using Roy's identity, results in

$$
\begin{aligned}
&-\sum_{j=1}^{4}\pi^{j}\left[\mathbf{y}_{q}\left(q,c^{j};\theta^{j}\right)+\mathbf{y}\left(q,c^{j};\theta^{j}\right)\mathbf{y}_{c}\left(q,c^{j};\theta^{j}\right)\right]\times\\
&\left\{\mu+\phi\sum_{j=1}^{4}\pi^{j}\left[\mathbf{V}\left(q,c^{j},\frac{I^{j}}{wn^{j}};\theta^{j}\right)-\phi\sum_{j=1}^{4}\pi^{j}\mathbf{y}\left(q,c^{j};\theta^{j}\right)-\phi D\right]^{-\eta}\right\}\\
&-\mu\sum_{j=1}^{4}\pi^{j}\left[\mathbf{x}_{q}\left(q,c^{j};\theta^{j}\right)+\mathbf{y}\left(q,c^{j};\theta^{j}\right)\mathbf{x}_{c}\left(q,c^{j};\theta^{j}\right)\right]\\
&-\sum_{j=1}^{4}\sum_{k\neq j}\left[\lambda^{kj}\mathbf{V}_{c}\left(q,c^{j},\frac{I^{j}}{wn^{k}};\theta^{k}\right)\mathbf{y}\left(q,c^{j};\theta^{j}\right)+\lambda^{jk}\mathbf{V}_{q}\left(q,c^{k},\frac{I^{k}}{wn^{j}};\theta^{j}\right)\right]=0. \quad \text{(A8)}
\end{aligned}
$$

To simplify equation (A8), partially differentiate the j-type individual's budget constraint, $\mathbf{x}\left(q,c^{j};\theta^{j}\right)+q\mathbf{y}\left(q,c^{j};\theta^{j}\right)=c^{j}$, once with respect to c^{j} and once with respect to q. This yields

$$
\mathbf{x}_{c}\left(q,c^{j};\theta^{j}\right)+q\mathbf{y}_{c}\left(q,c^{j};\theta^{j}\right) = 1, \quad \text{(A9)}
$$

$$
\mathbf{x}_{q}\left(q,c^{j};\theta^{j}\right)+q\mathbf{y}_{q}\left(q,c^{j};\theta^{j}\right) = -\mathbf{y}\left(q,c^{j};\theta^{j}\right). \quad \text{(A10)}
$$

Multiply equation (A9) by $\mathbf{y}\left(q,c^{j};\theta^{j}\right)$ and add the resulting equation to equation (A10). We get

$$
\begin{aligned}
&\mathbf{x}_{q}\left(q,c^{j};\theta^{j}\right)+\mathbf{y}\left(q,c^{j};\theta^{j}\right)\mathbf{x}_{c}\left(q,c^{j};\theta^{j}\right)=\\
&-q\left[\mathbf{y}_{q}\left(q,c^{j};\theta^{j}\right)+\mathbf{y}\left(q,c^{j};\theta^{j}\right)\mathbf{y}_{c}\left(q,c^{j};\theta^{j}\right)\right]. \quad \text{(A11)}
\end{aligned}
$$

Next, rewrite the last term on the left-hand side of equation (A8) as

$$
\begin{aligned}
\sum_{j=1}^{4}\sum_{k\neq j}\lambda^{jk}\mathbf{V}_{q}\left(q,c^{k},\frac{I^{k}}{wn^{j}};\theta^{j}\right) &= \sum_{j=1}^{4}\sum_{k\neq j}\lambda^{kj}\mathbf{V}_{q}\left(q,c^{j},\frac{I^{j}}{wn^{k}};\theta^{k}\right)\\
&= -\sum_{j=1}^{4}\sum_{k\neq j}\lambda^{kj}\mathbf{V}_{c}\left(q,c^{j},\frac{I^{j}}{wn^{k}};\theta^{k}\right)\mathbf{y}\left(q,c^{j};\theta^{k}\right), \quad \text{(A12)}
\end{aligned}
$$

where in going from the second to the last expression, we have made use of Roy's identity. Now substituting from (A11)–(A12) into (A8) and simplifying results in

$$-\sum_{j=1}^{4}\pi^{j}\left[\mathbf{y}_{q}\left(q,c^{j};\theta^{j}\right)+\mathbf{y}\left(q,c^{j};\theta^{j}\right)\mathbf{y}_{c}\left(q,c^{j};\theta^{j}\right)\right]\times$$
$$\left\{\mu+\phi\sum_{j=1}^{4}\pi^{j}\left[\mathbf{V}\left(q,c^{j},\frac{I^{j}}{wn^{j}};\theta^{j}\right)-\phi\sum_{j=1}^{4}\pi^{j}\mathbf{y}\left(q,c^{j};\theta^{j}\right)-\phi D\right]^{-\eta}-\mu q\right\}-$$
$$\sum_{j=1}^{4}\sum_{k\neq j}\lambda^{kj}\left\{\mathbf{V}_{c}\left(q,c^{j},\frac{I^{j}}{wn^{k}};\theta^{k}\right)\left[\mathbf{y}\left(q,c^{j};\theta^{j}\right)-\mathbf{y}\left(q,c^{j};\theta^{k}\right)\right]\right\}=0.$$

Denote the compensated demand function for y by $\widetilde{\mathbf{y}}\left(q,c^{j};\theta^{j}\right)$. Substituting $\widetilde{\mathbf{y}}_{q}\left(q,c^{j};\theta^{j}\right)$ for $\mathbf{y}_{q}\left(q,c^{j};\theta^{j}\right)+\mathbf{y}\left(q,c^{j};\theta^{j}\right)\mathbf{y}_{c}\left(q,c^{j};\theta^{j}\right)$ in above from the Slutsky equation, dividing the resulting equation by $\mu\sum_{j=1}^{4}\pi^{j}\widetilde{\mathbf{y}}_{q}\left(q,c^{j};\theta^{j}\right)$ and rearranging, we have

$$q-1=\frac{\phi}{\mu}\sum_{j=1}^{4}\pi^{j}\left[\mathbf{V}\left(q,c^{j},\frac{I^{j}}{wn^{j}};\theta^{j}\right)-\phi\sum_{j=1}^{4}\pi^{j}\mathbf{y}\left(q,c^{j};\theta^{j}\right)-\phi D\right]^{-\eta}+\frac{\sum_{j=1}^{4}\sum_{k\neq j}\lambda^{kj}\mathbf{V}_{c}\left(q,c^{j},\frac{I^{j}}{wn^{k}};\theta^{k}\right)\left[\mathbf{y}\left(q,c^{j};\theta^{j}\right)-\mathbf{y}\left(q,c^{j};\theta^{k}\right)\right]}{\mu\sum_{j=1}^{4}\pi^{j}\widetilde{\mathbf{y}}_{q}\left(q,c^{j};\theta^{j}\right)}. \tag{A13}$$

This proves that $q-1$ is non-Pigouvian unless the polluting good demand depends only on one's income but not on his taste so that the second expression on the right-hand side of (A13) will be zero. ■

Second, we prove that the input tax is Pigouvian regardless of individuals' tastes. The proof is facilitated through the following lemma.

Lemma A1 *In the optimal income tax problem (A1), and characterized by the* *first-order conditions (A2)–(A7), the Lagrange multiplier associated with the constraint $w = O_L(K, D, L)$, γ, is equal to zero.*

Proof. Multiply equation (A4) through by I^j/w, sum over j, and simplify to get

$$\frac{1}{w^2}\sum_{j=1}^{4}\frac{\pi^j I^j}{n^j}\left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi\sum_{j=1}^{4}\pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D\right]^{-\eta}\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) +$$
$$\mu L + \frac{1}{w^2}\sum_j\sum_{k\neq j}\left[(\frac{I^j}{n^j})\lambda^{jk}\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - (\frac{I^j}{n^k})\lambda^{kj}\mathbf{V}\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right)\right] -$$
$$\frac{1}{w^2}\gamma\mathbf{O}_{LL}(L, K, D)(wL) = 0. \tag{A14}$$

Substituting (A14) into (A7) and simplifying, we get

$$\sum_j\sum_{k\neq j}(\frac{I^j}{n^k})\lambda^{kj}\mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) = \gamma w^2 + \sum_j\sum_{k\neq j}(\frac{I^k}{n^j})\lambda^{jk}\mathbf{V}_L\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right). \tag{A15}$$

Then rewrite the left-hand side of (A15) as

$$\sum_j\sum_{k\neq j}(\frac{I^j}{n^k})\lambda^{kj}\mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) = \sum_j\sum_{k\neq j}(\frac{I^k}{n^j})\lambda^{jk}\mathbf{V}_L\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right). \tag{A16}$$

Substituting from (A16) into (A15) implies

$$\gamma = 0.$$

■

Observe that Lemma A1 is in fact an application of the production efficiency result as it tells us that $w = \mathbf{O}_L(K, D, L)$ imposes no constraint on our second-best problem. Using this lemma, we can easily show:

Proposition A2 *The optimal tax on energy input is Pigouvian.*

Proof. Using the result that $\gamma = 0$ in the first-order conditions (A4)–(A7), simplifies them to

$$\frac{\partial \pounds}{\partial I^j} = \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \frac{1}{wn^j} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) + \mu \pi^j + \sum_{k \neq j} \lambda^{jk} \frac{1}{wn^j} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \sum_{k \neq j} \lambda^{kj} \frac{1}{wn^k} \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) = 0, \tag{A17}$$

$$\frac{\partial \pounds}{\partial D} = - \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \phi + \mu \left[\mathbf{O}_D(L, K, D) - 1\right] = 0, \tag{A18}$$

$$\frac{\partial \pounds}{\partial K} = \mu \left[\mathbf{O}_K(L, K, D) - r\right] = 0, \tag{A19}$$

$$\frac{\partial \pounds}{\partial w} = \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \left(\frac{-I^j}{n^j w^2}\right) \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) + \mu \mathbf{O}_L(L, K, D) \frac{-1}{w^2} \sum_{j=1}^{4} \pi^j I^j + \sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left(\frac{-I^j}{n^j w^2}\right) \mathbf{V}_L\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) + \sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left(\frac{I^k}{n^j w^2}\right) \mathbf{V}_L\left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right). \tag{A20}$$

That the input tax is Pigouvian follows immediately from equation (A18). ■

A2 General income tax problem when $q = 1.4032$ and $\mathbf{O}_D(L, K, D) = 1$

The implications of these constraints for the optimization problem of the government are twofold. First, we no longer optimize with respect to q. Consequently, the first-order conditions do not include equation (A2). Second, denote the Lagrange multiplier associated with the constraint $1 - \mathbf{O}_D(L,K,D) = 0$ by ζ. This brings about the following changes in the first-order conditions (A3)–(A7): The expression for $\partial\pounds/\partial c^j$ remain unaffected; the expression for $\partial\pounds/\partial I^j$ now includes an additional term $-\zeta\mathbf{O}_{DL}(\pi^j/w)$; the expression for $\partial\pounds/\partial D$ now includes an additional term $-\zeta\mathbf{O}_{DD}$; the expression for $\partial\pounds/\partial K$ now includes an additional term $-\zeta\mathbf{O}_{DK}$; and the expression for $\partial\pounds/\partial w$ now includes an additional term $-\zeta\mathbf{O}_{DL}(\sum \pi^j I^j/w^2)$.

Using the same method as previously, one can again show that the Lagrange multiplier associated with the constraint $\mathbf{O}_L(L,K,D) = w$ continues to be zero so that this condition imposes no restriction on the problem even in the presence of the additional constraint on $\mathbf{O}_D(L,K,D)$. On the other hand, setting $\mathbf{O}_D(L,K,D) = 1$ in (A5)–(A6), these will change to

$$-\sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \phi - \zeta \mathbf{O}_{DD}(L,K,D) = 0, \quad \text{(A21)}$$

$$\mu\left(\mathbf{O}_K(L,K,D) - r\right) - \zeta \mathbf{O}_{DK}(L,K,D) = 0. \quad \text{(A22)}$$

Conditions (A21)–(A22) then imply that

$$\mathbf{O}_K(L,K,D) = r - \frac{\mathbf{O}_{DK}(L,K,D)}{\mu\mathbf{O}_{DD}(L,K,D)} \times \phi \sum_{j=1}^{4} \pi^j \left[\mathbf{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \mathbf{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta}. \quad \text{(A23)}$$

Thus, the constraint $\mathbf{O}_D(L,K,D) = 1$ implies that $\mathbf{O}_K(L,K,D)$ should no longer be set equal to r. Put differently, a producer tax on r is now optimal.