Pensions with heterogenous individuals and endogenous fertility*

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Abstract

We study the design of pension schemes when fertility is endogenous and parents differ in ability to raise children. Pay-as-you-go schemes require, under perfect information, a marginal subsidy on fertility to correct for the externality they create, equal pensions, and contributions that increase or decrease with the number of children. Under asymmetric information, incentive-related distortions supplement the Pigouvian subsidy. These require an additional subsidy, or an offsetting tax, depending on whether the redistribution is towards people with more or with less children. In the former case, pensions are decreasing in the number of children; otherwise, they are increasing. [Corresponding author: Firouz Gahvari]

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1 Introduction

The recent fertility decline in the West is often cited as a major impediment to the fiscal solvency of pay-as-you-go social security systems. At the same time, the pay-as-you-go feature of the social security systems has partly been blamed for causing the observed fertility decline. The reason for this latter linkage is that in such systems, the size of a person's pension benefits depends on everybody else's fertility decisions leading to a decentralized equilibrium outcome with "too few" children. It is thus not surprising that some economists have recently advocated a policy of linking pension benefits (or contributions) to individuals' fertility choices.

Such a policy raises a number of objections which one can group into "moral hazard" and "adverse selection" problems. The moral hazard problem arises when individuals do not have full control over fertility. The actual number of children in a family entails a random component and does not necessarily coincide with the number the parents initially intended to have. Making pension benefits to be independent of the number of children can then be viewed as a mechanism to insure parents against such random shocks. We have studied this problem in an earlier paper.[1]

The adverse selection problem, which is the subject of the current paper, arises when individuals are heterogenous. Specifically, assume, as is often the case, that parents differ in the ability to raise children (of a certain quality). Such individual characteristics are seldom publicly observable so that there is asymmetric information between parents and policy makers. Under this circumstance, linking pension benefits to fertility penalizes high-cost families (the low-ability parents). This in turn may have an adverse redistributive impact. Put differently, it may not be possible to distinguish between those individuals who have a small number of children due to high costs, from those with low costs who try to free ride on the system. Consequently, the fertility-incentive effects of the pension system may have to be balanced against its redistributive impact, and a positive link between fertility and benefits is not always desirable.

This paper focuses on this potential trade-off between fertility incentives and redistribution. It studies the design of pension systems in a setting in which individuals differ in their cost of raising children (or alternatively in their preferences for their number of children). Specifically, we consider two mechanisms for financing pensions. The first relies on a storage technology which amounts to a fully funded system. The second is the pay-as-you-go formula wherein the rate of return depends on the rate of population growth. In this latter case, individuals' fertility decisions entail an externality that has to be taken into account in the design of the social security system. The paper's main message is that, in the absence of moral hazard problems, the case for a positive link between pension benefits and fertility is not as strong it may at first appear, and as it has been advocated in some recent work.[2]

2 The model

2.1 The basics

Consider a two-period overlapping generations model in the steady-state. Each generation consists of two types of individuals who differ in their "ability" to raise (productive) children.[3] Each type is characterized by its cost of raising a child, θ_j ($j = 1, 2$), with $0 < \theta_2 < \theta_1$. Type 2 is thus the more able parent. All individuals are endowed with the same level of exogenous income, y, and have identical preferences over the number of children they will have, and present and future consumption.

Denote the proportion of type j by π_j and define average fertility as

$$\bar{n} = \pi_1 n_1 + \pi_2 n_2. \tag{1}$$

Introduce

$$z_j = \theta_j n_j$$

to denote a j-type parent's expenditure on children (excluding any subsidy he may receive, or any tax that he may have to pay, for this purpose). It will become clear below that whether $z_2 > z_1$, or $z_1 > z_2$, plays an important role in the type of solutions that emerge.

To keep the model simple, we assume that preferences over present and future consumption (c_j, d_j) and the number of children, n_j, are represented by an additive utility function. The lifetime utility of an individual of type j is written as

$$U_j = u(c_j) + v(d_j) + h(n_j), \tag{2}$$

where $u(\cdot), v(\cdot)$ and $h(\cdot)$ are increasing and strictly concave functions. There are two potential mechanism for financing second-period consumption: storage or a pay-as-you-go (PAYGO) pension plan. Under the storage technology, part of the initial endowment y is invested in a fund yielding a fixed rate of return r. Under the PAYGO scheme, the government collects taxes from the current young and distributes the proceeds to the retired according to some rule to be designed. The rate of return of the PAYGO is $\bar{n} - 1$. This corresponds to what Samuelson (1958) called the biological rate of interest.

2.2 The *Laissez-faire*

Absent any government intervention, each individual maximizes his utility (2) subject to his budget constraint

$$y = c_j + n_j \theta_j + \frac{d_j}{1+r},$$

where the pension (or savings) technology is the storage. Using the superscript L for *laissez-faire*, the optimality conditions are

$$(1+r)\ v'\left(d_j^L\right) = u'\left(c_j^L\right),$$
$$h'\left(n_j^L\right) = \theta_j\ u'\left(c_j^L\right).$$

Given our assumptions on preferences, c_j, d_j and n_j are all normal goods. The normality of n_j and the fact that $\theta_2 < \theta_1$ then imply

$$n_2^L > n_1^L.$$

The comparisons between the c^L's, the d^L's and the z^L's are ambiguous. However, given the specification for preferences, it must be the case that $c_2^L - c_1^L$ and $d_2^L - d_1^L$ are always of the same sign and opposite the sign of $z_2^L - z_1^L$.

As an illustration, consider a logarithmic utility function,

$$U_j = \alpha \ln c_j + \beta\ \ln\ d_j + \gamma \ln\ n_j, \tag{3}$$

where $\alpha + \beta + \gamma = 1$. With this specification, the *laissez-faire* implies that consumption levels in both periods and expenditures on children are the same for all households:

$$\begin{aligned} c_1^L &= c_2^L = \alpha y, \\ d_1^L &= d_2^L = \beta (1+r) y, \\ z_1^L &= z_2^L = \gamma y. \end{aligned}$$

With a CES utility function, the comparison would depends on the elasticity of substitution. When this elasticity is small, the demand for n is price inelastic and

$$z_1^L > z_2^L.$$

In other words, in this special case, the less able family has less children but spends more on raising them than the more able family does. A large elasticity of substitution yields the opposite result. Between these cases lies the Cobb-Douglas (logarithmic) specification with a unitary elasticity of substitution and constant budget shares.

3 The utilitarian first-best

Assume that the social planner controls all relevant variables in the economy and has perfect information regarding every individual's ability to raise children. The planner determines which technology, storage or PAYGO, is used to finance old-age consumption and it sets c_j, d_j,n_j accordingly. We study the utilitarian solution which maximizes the sum of lifetime utility

$$W = \sum_j \pi_j \; U_j, \tag{4}$$

subject to the appropriate resource constraints, namely

$$\sum_j \pi_j \left(y - c_j - n_j \theta_j - \frac{d_j}{1+r} \right) = 0 \tag{5}$$

under storage, or

$$\sum_j \pi_j \left(y - c_j - n_j \theta_j - \frac{d_j}{\bar{n}} \right) = 0 \tag{6}$$

when the PAYGO technology is used.

3.1 Storage

The storage problem can be expressed by the following Lagrangian expression

$$\mathcal{L}_{FS} = \sum_j \pi_j \left[u(c_j) + v(d_j) + h(n_j) + \mu \left(y - c_j - \theta_j n_j - \frac{d_j}{1+r} \right) \right]$$

where FS stands for "first-best under storage" and μ is the Lagrange multiplier associated with the economy's resource constraint. The first order conditions yield

$$\begin{aligned} u'\left(c_1^{FS}\right) &= u'\left(c_2^{FS}\right) = \mu, \\ v'\left(d_1^{FS}\right) &= v'\left(d_2^{FS}\right) = \frac{\mu}{1+r}, \\ h'\left(n_j^{FS}\right) &= \mu\theta_j. \end{aligned}$$

The first two equations imply $c_1^{FS} = c_2^{FS} = c^{FS}$, $d_1^{FS} = d_2^{FS} = d^{FS}$, and the third equation $n_1^{FS} < n_2^{FS}$.[4]

Decentralization of the first-best solution is simple. It requires first-period lump-sum tax and transfers between the two types while allowing them to save for their retirement voluntarily. Whether a type j ($j = 1, 2$) person receives a transfer or will have to pay a tax depends on whether he spends more or less on child raising than a person of the other type. Specifically, if type 1 persons spend more than type 2, they should each receive a compensatory lump-sum transfer; if they spend less, they should pay a tax. Alternatively, decentralization can be achieved through a *fully funded* pension system where everyone receives the same pension but different types pay different contributions. Thus type j persons each pay $y - c_j^{FS} - \theta_j n_j^{FS})$ when they work, and receive d^{FS} when they retire.[5] Observe that contributions may depend on family size either positively or negatively. If $z_1^{FS} > z_2^{FS}$, more able type 2 (who have a greater number of children) pay more in contributions.[6] If $z_1^{FS} < z_2^{FS}$, the opposite holds and contributions decrease with family size.

One can easily check that *laissez-faire* and first-best solutions coincide if $z_1^L = z_2^L$.[7] Then pension contributions are the same for all individuals. Moreover, it is also the case that if $z_1^L > z_2^L$, the first-best solution will be characterized by $z_1^{FS} > z_2^{FS}$. Similarly, $z_1^L < z_2^L \Rightarrow z_1^{FS} < z_2^{FS}$. To see these, assume $z_1^L > z_2^L$. Under this circumstance, we have $c_1^L < c_2^L$ and $d_1^L < d_2^L$. To attain first-best, which requires the equality for consumption levels, one must then redistribute from type 2 to type 1. With n_j being a normal good, such a redistribution implies that $n_1^{FS} > n_1^L$ and $n_2^{FS} < n_2^L$. Consequently, $z_1^{FS} = \theta_1 n_1^{FS} > \theta_1 n_1^L > z_1^L$, and $z_2^{FS} = \theta_2 n_2^{FS} < \theta_2 n_2^L < z_2^L$. These inequalities then imply that $z_1^{FS} > z_1^L > z_2^L > z_2^{FS}$. A similar argument shows that if $z_1^L < z_2^L$, the corresponding first-best solution will be characterized by $z_1^{FS} < z_2^{FS}$.

3.2 Pay-as-you-go

With PAYGO, the maximization of utilitarian welfare can be expressed by the following Lagrangian

$$\mathcal{L}_{FP} = \sum_j \pi_j \left[u\left(c_j\right) + v\left(d_j\right) + h\left(n_j\right) + \mu \left(y - c_j - \theta_j n_j - \frac{d_j}{\bar{n}} \right) \right]$$

where FP stands for "first-best with PAYGO". The optimality conditions are given by:

$$u'\left(c_1^{FP}\right) = u'\left(c_2^{FP}\right) = \mu, \tag{7}$$

$$v'\left(d_1^{FP}\right) = v'\left(d_2^{FP}\right) = \frac{\mu}{\bar{n}^{FP}}, \tag{8}$$

$$h'\left(n_j^{FP}\right) = \mu \left[\theta_j - \frac{\pi_1 d_1^{FP} + \pi_2 d_2^{FP}}{(\bar{n}^{FP})^2} \right]. \tag{9}$$

Equations (7) and (8) are standard; they imply $c_1^{FP} = c_2^{FP} = c^{FP}$ and $d_1^{FP} = d_2^{FP} = d^{FP}$. Equation (9) has two interesting implications. The first is that $n_1^{FP} < n_2^{FP}$. That is, as with storage, the more productive individuals should have more children. Secondly, the equation shows that the existence of the PAYGO system affects the tradeoff between c and n. To make this more explicit, one can rewrite (9) as

$$\frac{h'\left(n_j^{FP}\right)}{u'\left(c_j^{FP}\right)} = \left[\theta_j - \frac{d^{FP}}{(\bar{n}^{FP})^2} \right].$$

The right-hand side of this expression represents the net marginal cost of n, accounting for the "externality" term $d^{FP}/(\bar{n}^{FP})^2$ which reflects the impact of one's fertility on the rate of return of the PAYGO system.[8]

To decentralize this solution, a Pigouvian subsidy at the rate of $s = d^{FP}/(\bar{n}^{FP})^2$ must supplement the pension system. Thus, with PAYGO, expenditures on children are subsidized *at the margin.* This was *not* the case under storage. The marginal subsidy implies that, under PAYGO, type 2 individuals who have a higher number of children will always receive a larger Pigouvian subsidy.[9] Nevertheless, this does not imply that they are necessarily the beneficiaries of the pension system. The direction of *net* transfers between the types depends, once again, on the expenditures on children. If $z_1^{FP} > z_2^{FP}$, there will be a net transfer from low-cost households (with many children) to high-cost households (with fewer children); the opposite is true if $z_1^{FP} < z_2^{FP}$.

While individuals of different types receive the same pensions, d^{FP}, they will generally pay different contributions. Let T_j denote the j-type's contribution. And, for ease in notation, drop the superscript FP on $n_j, \bar{n}, z_j$ and T_j. It then follows from the individuals' budget constraints that

$$T_1 - T_2 = \left(\theta_2 - \frac{d^{FP}}{\bar{n}^2}\right) n_2 - \left(\theta_1 - \frac{d^{FP}}{\bar{n}^2}\right) n_1,$$

where, Observe that T_j $(j = 1, 2)$ will be greater than T_k $(k \neq j)$ if and only if the j-type's expenditure on children *net* of subsidies received is smaller than k-type's. Moreover, from the government's budget constraint,

$$\sum_j \pi_j T_j - \frac{d^{FP}}{\bar{n}^2} \sum_j \pi_j n_j = \frac{d^{FP}}{\bar{n}}.$$

Solving these two equations yields

$$\begin{aligned} T_1 &= \frac{d^{FP}}{\bar{n}} + n_1 \frac{d^{FP}}{\bar{n}^2} - \pi_2 (z_1 - z_2), \\ T_2 &= \frac{d^{FP}}{\bar{n}} + n_2 \frac{d^{FP}}{\bar{n}^2} - \pi_1 (z_2 - z_1). \end{aligned}$$

Note that even in the Cobb-Douglas case, the contributions will be different. In this case, with $z_1 = z_2$, we have

$$\begin{aligned} T_1 &= \frac{d^{FP}}{\bar{n}} + n_1 \frac{d^{FP}}{\bar{n}^2}, \\ T_2 &= \frac{d^{FP}}{\bar{n}} + n_2 \frac{d^{FP}}{\bar{n}^2}. \end{aligned}$$

3.3 Storage versus PAYGO

The choice between storage and PAYGO depends on the respective levels of welfare achieved, W^{FS} and W^{FP}. This in turn will depend on the relationship between $1 + r$,

$\bar{n}^{FP}$, and $\bar{n}^{FS}$. However, the comparison is more complicated than in the standard Samuelsonian world in which there is a unique biological rate of interest. Specifically, if $1 + r > \bar{n}^{FP}$, storage dominates PAYGO. Under this circumstance, the optimal allocation under PAYGO is also feasible under storage with a non-binding resource constraint. Similarly, if $1 + r = \bar{n}^{FP}$, storage continues to dominate PAYGO. In this case, the return of the two systems is the same, but we know from the results presented above that the allocations differ. Moreover, whereas the return on storage, r, is fixed at an exogenously given rate, the (implicit) return on PAYGO is endogenous and imposes a restriction on the choice of n. Consequently, welfare must be strictly larger under storage.

The comparison is more involved if $1 + r < \bar{n}^{FP}$. However, while the question of the superiority of storage versus PAYGO is an interesting one, it is not what we are concerned with in this paper. It is, therefore, omitted here.[10]

4 Second-best solution

The first-best characterization rests on the assumption that the government observes θ_j and can use all instruments. If types are not publicly observable, one has to resort to a tax-transfer policy which induces type revelation and leads to the appropriate fertility rate. Thus assume that θ_j is not observable but the number of children, n_j, is. The unobservability of types requires that $z_j = n_j\theta_j$ and c_j not to be observable either. Otherwise, one could infer the value of θ_j. However, the second-period consumption level, d_j, can be observable—an assumption that we maintain throughout this section.[11]

To write the second-best problem in terms of observable variables, one must replace c_j by T_j (the first-period tax levied on the j-type). We can then determine the utilitarian allocation subject to the appropriate self-selection constraints. The solution can be decentralized through non-linear functions $T(n)$ and $d(n)$ which specify contributions and pensions as functions of the family size. Thus the j-type household chooses n_j to maximize $u\left(y - T(n_j) - n_j\theta_j\right) + v\left(d(n_j)\right) + h(n_j)$. This yields the following first-order condition

$$-u'\left(y - T_j - n_j\theta_j\right)\left(\theta_j + T_j'\right) + h'\left(n_j\right) + v'\left(d_j\right)d_j' = 0,$$

or

$$T_j' - d_j' \frac{v'(d_j)}{u'(c_j)} = -\theta_j + \frac{h'(n_j)}{u'(c_j)}, \tag{10}$$

where $T_j' = T'(n_j)$ and $d_j' = d'(n_j)$. The left-hand-side of this expression specifies the net marginal tax on n for household j. It has two components: a first-period tax, T_j', combined with a second-period transfer, d_j', weighted by the intertemporal marginal rate of substitution, $v'(d_j)/u'(c_j)$. In the discussion below, it is more convenient to speak of a subsidy rather than a tax; we thus define the marginal subsidy rate on n_j as

$$s_j \equiv -T_j' + d_j' \frac{v'(d_j)}{u'(c_j)}, \tag{11}$$

where, at the optimum, s_j is set equal to $\theta_j - h'(n_j)/u'(c_j)$.

4.1 Storage

Let c_{jk} and U_{jk} $(j \neq k = 1, 2)$ denote the consumption and the utility of a j-type who mimics a k-type. We have

$$c_{jk} = y - \theta_j n_k - T_k$$
$$U_{jk} = u(c_{jk}) + v(d_k) + h(n_k).$$

The optimal utilitarian allocation is obtained by maximizing the sum of individual utilities, subject to the resource constraint and the two potential self-selection constraints. The Lagrangian expression associated with this problem is given by

$$\mathcal{L} = \sum_j \pi_j \left[U_j + \mu \left(T_j - \frac{d_j}{1+r} \right) \right] + \lambda_2 (U_2 - U_{21}) + \lambda_1 (U_1 - U_{12}).$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial T_1} = -(\pi_1+\lambda_1)u'(c_1)+\pi_1\mu+\lambda_2 u'(c_{21})=0, \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial T_2} = -(\pi_2+\lambda_2)u'(c_2)+\pi_2\mu+\lambda_1 u'(c_{12})=0, \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial d_1} = (\pi_1+\lambda_1-\lambda_2)v'(d_1)-\frac{\pi_1\mu}{(1+r)}=0, \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial d_2} = (\pi_2+\lambda_2-\lambda_1)v'(d_2)-\frac{\pi_2\mu}{(1+r)}=0, \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial n_1} = -(\pi_1+\lambda_1)\left[u'(c_1)\theta_1-h'(n_1)\right]+\lambda_2\left[u'(c_{21})\theta_2-h'(n_1)\right]=0, \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial n_2} = -(\pi_2+\lambda_2)\left[u'(c_2)\theta_2-h'(n_2)\right]+\lambda_1\left[u'(c_{12})\theta_1-h'(n_2)\right]=0. \tag{17}$$

Combining the first-order conditions (12) and (14) yields[12]

$$v'(d_1)=\frac{u'(c_1)}{1+r}\left[\frac{\pi_1+\lambda_1-\lambda_2\dfrac{u'(c_{21})}{u'(c_1)}}{\pi_1+\lambda_1-\lambda_2}\right]\geq\frac{u'(c_1)}{1+r}. \tag{18}$$

Similarly, (13) and (15) lead to

$$v'(d_2)=\frac{u'(c_2)}{1+r}\left[\frac{\pi_2+\lambda_2-\lambda_1\dfrac{u'(c_{12})}{u'(c_2)}}{\pi_2+\lambda_2-\lambda_1}\right]\leq\frac{u'(c_2)}{1+r}. \tag{19}$$

Finally, combining (14) and (15) results in

$$\frac{v'(d_2)}{v'(d_1)}=\frac{\pi_2(\pi_1+\lambda_1-\lambda_2)}{\pi_1(\pi_2+\lambda_2-\lambda_1)}. \tag{20}$$

To discuss and interpret the results, we have to distinguish between three regimes: $\lambda_1=\lambda_2=0$, $\lambda_2>0$, $\lambda_1=0$ and $\lambda_2=0$, $\lambda_1>0$.[13]

Regime 1. $\lambda_1=\lambda_2=0$ This solution corresponds to the first-best where none of the self-selection constraints binds. It will necessarily hold if at the laissez-faire solution $z_1^L=z_2^L$ so that there is no need for redistribution. With the logarithmic utilities (3), we have precisely this case. More generally, this occurs whenever the first-best allocation satisfies the self-selection constraints. In turn, this arises when laissez-faire levels of expenditure on children are "not too different".

Regime 2. $\lambda_1 = 0$ **and** $\lambda_2 > 0$ In this regime, the prevailing self-selection constraint is that of type 2. Put differently, type 1 benefits from redistribution and type 2 is tempted to mimic him by having less children. To discourage type 2 from mimicking type 1, the social planner sets $d_2 > d_1$. This follows immediately from (20). One can also show that $c_2 > c_1$; see the Appendix.[14] Consequently, in this regime, despite the redistribution towards the less able parents, the more able parents end up with more consumption in both periods. Observe also that, with the redistribution being from type 2 to type 1, one must have $T_2 > T_1$. Otherwise, given that $d_2 > d_1$, type 2 will be the net beneficiary of redistribution. A formal proof is given in the Appendix.

Turning to n_j's, we have from (17)

$$\theta_2 - \frac{h'(n_2)}{u'(c_2)} = 0,$$

which implies there is no distortion in the choice of n_2. This is of course due to the fact that in this regime redistribution is from type 2 to type 1.

Next, from (16), one obtains

$$\pi_1 \left[u'(c_1)\theta_1 - h'(n_1) \right] = \lambda_2 \left[u'(c_{21})\theta_2 - h'(n_1) \right],$$

implying

$$\theta_1 - \frac{h'(n_1)}{u'(c_1)} = \frac{\lambda_2}{\pi_1 - \lambda_2} \frac{u'(c_{21})\theta_2 - u'(c_1)\theta_1}{u'(c_1)} < 0,$$

where $u'(c_{21})\theta_2 - u'(c_1)\theta_1 < 0$ because $\theta_2 < \theta_1$ and $c_{21} = c_1 + (\theta_1 - \theta_2) > c_1$.[15] Consequently, there is a downward distortion in n_1 (as compared to the first-best tradeoff wherein $u'(c_1)\theta_1 - h'(n_1) = 0$). In words, to discourage type 2 from mimicking type 1, the social planner "suggests" a low value of n_1 that is not attractive to type 2, given $h(\cdot)$ and θ_2. This means, as a matter of implementation, there will be a tax on n_1.[16]

These results imply, using (10)–(11), that while $s_1 < 0$, $s_2 = 0$ Consequently, *marginal* subsidy rates are non-positive for all households. Although this in itself is not surprising (at least not under storage), it may appear to be at odds with the property that households with many children (type 2) receive higher pensions than households with fewer children. However, under closer scrutiny, it becomes clear that similar properties arise in standard optimal tax models, where *marginal* and *average* tax rates do not always go hand in hand.[17] Finally, the pattern of marginal subsidies ensure that $n_2 > n_1$; see the Appendix. That is, parents who are more productive in raising children will end up having more children.

Regime 3: $\lambda_1 > 0$ and $\lambda_2 = 0$ In this regime, type 2 benefits from redistribution and type 1 is tempted to mimic him by having more children. To counter this, the planner sets $d_2 < d_1$. This result is easily established from (20). Moreover, making use of (18) and (19), it is now also the case that $c_2 < c_1$. In this regime too, redistribution does not change the initial consumption patterns so that the less able parents end up with more consumption in both periods. As far as the first-period taxes are concerned, we must now have $T_2 < T_1$. Otherwise, with $d_2 < d_1$, type 1 will be the net beneficiary of redistribution; see the Appendix for a formal proof.

Turning to n_j's, (17) can now be rearranged to yield

$$\theta_2 - \frac{h'(n_2)}{u'(c_2)} = \frac{\lambda_1}{\pi_2 - \lambda_1} \frac{u'(c_{12})\theta_1 - u'(c_2)\theta_2}{u'(c_2)} > 0,$$

so that n_2 is distorted upward. In this regime, the binding incentive constraint is to prevent type 1 households from mimicking type 2 households by having more children. To relax this constraint, the social planner induces type 2 to have even more children (than they would in the absence of distortion).

In the case of type 1 parents, one may easily show from (16) that no distortion is to be imposed on n_1. This is not surprising as, in this regime, redistribution is away from type 1 and towards type 2. Using (10)–(11), these results imply that, in terms of implementation, $s_1 = 0$ and $s_2 > 0$ so that *marginal* subsidy rates are non-negative for all types of households.

Which regime? We have already seen that if the two types spend equal amounts on raising children in the *laissez-faire*, i.e. if $z_1^L = z_2^L$, the *laissez-faire* and the optimal utilitarian solutions coincide and regime 1 necessarily prevails. We show in the Appendix that if $z_1^L < z_2^L$ the prevailing regime is either 1 or 2, while if $z_1^L > z_2^L$, we will have regime 1 or 3. Put differently, *if* the utilitarian solution calls for redistribution, it will be towards the parents who spend more on raising children. Thus the self-selection

Regime	Pension	s_1	s_2	When?
$1: \lambda_1 = 0,\ \lambda_2 = 0$	$d_1 = d_2$ $T_1 = T_2$	0	0	$\left\| z_1^L - z_2^L \right\|$ "small"
$2: \lambda_1 = 0,\ \lambda_2 > 0$	$d_2 > d_1$ $T_2 > T_1$	$-$	0	$z_1^L - z_2^L > 0$ and "large"
$3: \lambda_1 > 0,\ \lambda_2 = 0$	$d_2 < d_1$ $T_2 < T_1$	0	$+$	$z_2^L - z_1^L > 0$ and "large"

Table 1: Second-best under storage

constraint that may restrict the extent of redistribution will be that of the parent who spends the least on his children.

Recall that in the first best, there is full compensation for the differences in expenditures with the consumption levels being equalized across types. In a world of asymmetric information, this full equalization may or may not be achievable. When it is not, redistribution is limited by the binding incentive constraints of parents who lose from redistribution; namely, those who spend the least on raising their children. If these are the parents who have more children (type 2), then to make the alternative less appealing to them, n_1 is distorted downward and d_1 is set less than d_2. On the other hand, if the losers are the parents with less children, their alternative is made less appealing by distorting n_2 upward and setting $d_2 < d_1$.

The results obtained for the storage case are summarized in Table 1.

4.2 PAYGO

Assume now that the government controls d_j's through pensions, setting them at levels such that there will be no private savings. One can then write the individual's budget constraint as

$$c_j + \theta_j n_j + T^j = y,$$

and the resource constraint by

$$\sum_{j=1}^{2} \pi_j \left(T_j - \frac{d_j}{\overline{n}} \right) = 0.$$

As with the storage, let c_{jk} and U_{jk} $(j \neq k = 1, 2)$ denote the consumption and the utility of a j-type who mimics a $k-$type. We have

$$c_{jk} = y - \theta_j n_k - T_k$$
$$U_{jk} = u\left(c_{jk}\right) + v(d_k) + h(n_k).$$

The second-best problem is then summarized by the Lagrangian

$$\mathcal{L} = \sum_j \pi_j \left[U_j + \mu \left(T_j - \frac{d_j}{\overline{n}} \right) \right] + \lambda_2(U_2 - U_{21}) + \lambda_1(U_1 - U_{12}),$$

where $\overline{n}$ is defined by (1).

The first-order conditions for this problem are

$$\frac{\partial \mathcal{L}}{\partial T_1} = -(\pi_1 + \lambda_1)u'(c_1) + \pi_1\mu + \lambda_2 u'(c_{21}) = 0, \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial T_2} = -(\pi_2 + \lambda_2)u'(c_2) + \pi_2\mu + \lambda_1 u'(c_{12}) = 0, \tag{22}$$

$$\frac{\partial \mathcal{L}}{\partial d_1} = (\pi_1 + \lambda_1 - \lambda_2)v'(d_1) - \frac{\pi_1\mu}{\overline{n}} = 0, \tag{23}$$

$$\frac{\partial \mathcal{L}}{\partial d_2} = (\pi_2 + \lambda_2 - \lambda_1)v'(d_2) - \frac{\pi_2\mu}{\overline{n}} = 0, \tag{24}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n_1} &= -(\pi_1 + \lambda_1)\left[u'(c_1)\theta_1 - h'(n_1)\right] + \pi_1\mu\frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2} \\ &\quad + \lambda_2\left[u'(c_{21})\theta_2 - h'(n_1)\right] = 0,\end{aligned} \tag{25}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n_2} &= -(\pi_2 + \lambda_2)\left[u'(c_2)\theta_2 - h'(n_2)\right] + \pi_2\mu\frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2} \\ &\quad + \lambda_1\left[u'(c_{12})\theta_1 - h'(n_2)\right] = 0.\end{aligned} \tag{26}$$

Observe that the first-order conditions with respect to n_j, i.e. equations (25) and (26), differ from their storage counterparts. On the other hand, expressions (21)–(24) are equivalent to (12–(15) under the storage, except that $\overline{n}$ has replaced $(1 + r)$. Similar manipulations of these equations then yield

$$v'(d_1) \geq \frac{1}{\overline{n}}u'(c_1), \tag{27}$$

$$v'(d_2) \leq \frac{1}{\overline{n}}u'(c_2), \tag{28}$$

along with equation (20) which continues to hold. We again have three possible regimes.

Regime 1: $\lambda_1 = \lambda_2 = 0$ The solution corresponds to the first best. Pensions are set equally ($d_1 = d_2$) and lump-sum taxes (contributions) are used to ensure $c_1 = c_2$. Additionally, a Pigouvian subsidy is used to induce the optimal values of n_j's. In the Cobb-Douglas case, where $z_1^L = z_2^L$, there is no net redistribution between the two types. But with $n_2 > n_1$ in the first best, type-2 receives more subsidy for raising children. Consequently, they will have to be taxed in the first period to ensure there will be no net transfers.

Regime 2: $\lambda_1 = 0$ **and** $\lambda_2 > 0$ Comparisons between the consumption and pension levels of the two types are exactly the same as in the storage. Specifically, it follows from (21)–(24) that $d_2 > d_1$ and $c_2 > c_1$; see the Appendix. Consequently, as with storage, the more able parents end up with higher first- and second-period consumption levels.

Regarding n_j, for individuals of type 2, we have from (26)

$$-(\pi_2 + \lambda_2)\left[u'(c_2)\theta_2 - h'(n_2)\right] + \pi_2\mu\frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2} = 0$$

Rearranging and using (22) gives

$$\theta_2 - \frac{h'(n_2)}{u'(c_2)} = \frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2},$$

so that, from (10)–(11),

$$s_2 = \frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2}.$$

In words, there is a "Pigouvian" marginal subsidy on n for type 2 individuals. Observe that, as with storage, with the redistribution being away from type 2, incentive considerations require no distortion to be imposed on type 2.

Turning to type-1 individuals, we have from (25),

$$\pi_1\left[u'(c_1)\theta_1 - h'(n_1)\right] = \pi_1\mu\frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2} + \lambda_2\left[u'(c_{21})\theta_2 - h'(n_1)\right],$$

so that

$$(\pi_1 - \lambda_2)\left[u'(c_1)\theta_1 - h'(n_1)\right] = \pi_1\mu\frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2} + \lambda_2\left[u'(c_{21})\theta_2 - u'(c_1)\theta_1\right],$$

where $u'(c_{21})\theta_2 - u'(c_1)\theta_1 < 0$. Rearranging then yields

$$\theta_1 - \frac{h'(n_1)}{u'(c_1)} = \frac{\pi_1 \mu}{(\pi_1 - \lambda_2)u'(c_1)} \frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2} + \frac{\lambda_2}{\pi_1 - \lambda_2} \frac{u'(c_{21})\theta_2 - u'(c_1)\theta_1}{u'(c_1)}. \tag{29}$$

The first term in the right-hand side of (29) is the Pigouvian subsidy (adjusted by the fact that the "marginal cost of public fund" is no longer equal to one). The second term is the distortion aimed at relaxing the binding incentive constraint of type 2 households (who are hurt by redistribution). As with the storage, this term is negative thus inducing a downward distortion on n_1. Consequently, the sign of s_1 is ambiguous. There is a conflict between externality (requiring a subsidy to induce a higher value for n_1) and incentive (requiring a tax to induce a lower value for n_1) terms.[18]

Finally, observe that the ambiguity in the sign of s_1 results also in an ambiguity in the sign of $n_2 - n_1$. If $s_1 > 0$, and is large enough, it is possible that it may encourage the less productive parents end up with more children. On the other hand, if $s_1 \leq 0$, so that less productive parents are not subsidized at the margin for having more children, it will necessarily be the case that $n_2 > n_1$. See the Appendix.

Regime 3: $\lambda_1 > 0$ and $\lambda_2 = 0$ The comparisons between consumption and pension levels are, once again, exactly the same as with storage. Specifically, it follows from (20) that $d_2 < d_1$. Hence pensions decrease with ability, even though fertility entails a positive externality. Moreover, making use of (27) and (28) we obtain, as with storage, $c_2 < c_1$.

Turning to n_j's, we again consider type-2 households first. One obtains, from (26),

$$(\pi_2 - \lambda_1)\left[u'(c_2)\theta_2 - h'(n_2)\right] = \pi_2 \mu \frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2} + \lambda_1 \left[u'(c_{12})\theta_1 - u'(c_2)\theta_2\right],$$

where $u'(c_{12})\theta_1 - u'(c_2)\theta_2 > 0$. Rearranging yields

$$\theta_2 - \frac{h'(n_2)}{u'(c_2)} = \frac{\pi_2 \mu}{(\pi_2 - \lambda_1)u'(c_2)} \frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2} + \frac{\lambda_1}{\pi_2 - \lambda_1} \frac{u'(c_{12})\theta_1 - u'(c_2)\theta_2}{u'(c_2)}.$$

In this case, externality and incentive terms are of the same sign and we necessarily have a marginal subsidy on n_2 (a negative marginal tax). This implies that $s_2 > 0$.

Regime	Pension	s_1			s_2		
		IC	Pigou	Total	IC	Pigou	Total
$1 : \lambda_1 = 0,\ \lambda_2 = 0$	$d_1 = d_2$	0	+	+	0	+	+
$2 : \lambda_1 = 0,\ \lambda_2 > 0$	$d_2 > d_1$	−	+	?	0	+	+
$3 : \lambda_1 > 0,\ \lambda_2 = 0$	$d_2 < d_1$	0	+	+	+	+	+

Table 2: Second-best under PAYGO

In the case of type-1 parents, proceeding in the same manner as in regime 2, one obtains from (25) that they should face a Pigouvian marginal subsidy. Specifically,

$$s_1 = \frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2},$$

with no distortions due to incentive considerations (recall that redistribution is away from type 1).

The results for the PAYGO case are summarized in Table 2.

5 Conclusion

This paper has shown that the design of pension schemes, when fertility is endogenous and parents have different abilities in raising children, depends mainly on two factors. First, whether the system is based on storage or is PAYGO; second, whether it is the high-ability (type 2) or the low-ability (type 1) who incurs a higher expenditure on raising his children. The first factor is related to the inherent externality in a PAYGO social security system. Increasing the number of one's (productive) children bestows a positive externality on others by increasing the biological rate of return of the system. This is operative only in the PAYGO setting and its internalization requires a subsidy on having children. The second factor is related to the elasticity of substitution between consumption and fertility. In case of unitary elasticity (when the utility function is logarithmic) both types spend the same amount on raising children, and there is no need for redistribution. When one type spends more, that type should be "compensated".

We have shown that if redistribution is from type 1 (less able) to type 2 (more able) parents, the more-able parents are induced to have more children. This reinforces the externality correction, which requires a Pigouvian marginal subsidy on children, and the net effect is a subsidy on type 2's number of children. On the other hand, if the redistribution is towards type 1, these parents are induced to have a smaller number of children. This requires a tax on type 1's number of children, and works in opposite direction to the marginal Pigouvian subsidy required for externality correction. The final outcome in this case may be a net tax, or a net subsidy, depending on the relative size of the "distortion" required to align the incentives and the "distortion" needed to correct for the externality. The parent whom one redistributes away from must, in both cases, face a Pigouvian subsidy only with no incentive-related distortion. These results suggests a distinction between redistributive goals achieved by average taxation, and incentive considerations (changing the parents' behavior at the margin) achieved by marginal taxation.

Appendix

A.1. The prevailing regimes at the second-best optimum under storage

We show that if $z_1^L > z_2^L$, the incentive constraint of type 1 individuals cannot be binding at the second-best optimum. Consequently, in this case, regime 2 cannot prevail and one has either regime 1 or regime 3. Formally, we have

Lemma 1 *If $z_1^L > z_2^L$, then at the second-best optimum under storage, one cannot have both $\lambda_1 > 0$ and $\lambda_2 = 0$.*

Proof. The proof is by contradiction. Assume $\lambda_1 > 0$ and $\lambda_2 = 0$ so that the incentive constraint $U_1 \geq U_{12}$ is binding. Using superscript SS for the second-best optimum under storage, the binding incentive constraint implies

$$u(c_1^{SS}) + v(d_1^{SS}) + h(n_1^{SS}) = u(c_{12}^{SS}) + v(d_2^{SS}) + h(n_2^{SS}), \tag{A1}$$

where

$$\begin{aligned} c_1^{SS} &= y - \theta_1 n_1^{SS} - T_1^{SS}, \\ c_{12}^{SS} &= y - \theta_1 n_2^{SS} - T_2^{SS}. \end{aligned}$$

Next define

$$\Psi_i(\Delta) = \max_{c,d,n} \; u(c) + v(d) + h(n) \tag{A2}$$

$$\text{s.t. } c + \frac{d}{1+r} + \theta_i n = y + \Delta; \tag{A3}$$

so that $\Psi_i(\Delta)$ is household i's ($i = 1, 2$) maximal utility when facing the budget constraint (A3) and the (given) net transfer Δ. With $\lambda_1 > 0$ and $\lambda_2 = 0$, first-order conditions (12), (14) and (16) imply that $(c_1^{SS}, d_1^{SS}, n_1^{SS})$ solves problem (A2)–(A3) for household 1 given

$$\Delta = \Delta_1^{SS} = \frac{d_1^{SS}}{1+r} - T_1^{SS}.$$

In words, allocation $(c_1^{SS}, d_1^{SS}, n_1^{SS})$ yields the maximal utility for type 1 households given $\Delta = \Delta_1^{SS}$.[19] Clearly then, only if $\Delta > \Delta_1^{SS}$, it will be possible for type 1 to have same level of utility at another allocation. Consequently, to have (A1) satisfied, the Δ associated with $(c_{12}^{SS}, d_2^{SS}, n_2^{SS})$, Δ_2^{SS}, must be greater than Δ_1^{SS}. Making use of the resource constraint under storage, it follows that

$$\Delta_1^{SS} < 0 < \Delta_2^{SS} = \frac{d_2^{SS}}{1+r} - T_2^{SS},$$

i.e., there is a net transfer from type 1 to type 2.

Finally define

$$W(\Delta_1) = \pi_1 \Psi_1(\Delta_1) + \pi_2 \Psi_2 \left(-\frac{\pi_1 \Delta_1}{\pi_2} \right). \tag{A4}$$

Thus $W(.)$ represents the maximum utilitarian welfare, in the absence of the incentive constraints, as a function of the net transfers to type 1 households (with the transfers to type 2 being determined by the resource constraint). It thus follows from the definition of $W(.)$ that

$$\sum_i \pi_i U_i^{SS} \leq W(\Delta_1^{SS}). \tag{A5}$$

Now differentiating (A4), using the envelope theorem, yields

$$W'(\Delta_1) = \pi_1(u'(c_1) - u'(c_2)),$$

so that $W(.)$ is increasing in Δ_1 if and only if $c_1 < c_2$. But, with $z_1^L > z_2^L$, the normality of c implies $c_1 < c_2$ whenever $\Delta_1 < 0$. Consequently, $W(.)$ is increasing in Δ_1 whenever $\Delta_1 < 0$. Hence $\Delta_1^{SS} < 0$ together with (A5) yields

$$\sum_i \pi_i U_i^{SS} \leq W(\Delta_1^{SS}) < W(0).$$

Given that the *laissez-faire* solution corresponds to $\Delta_1 = 0$, the above inequality implies that welfare at the second best optimum is smaller than welfare at the *laissez-faire* solution. This is a contradiction because the *laissez-faire* is feasible in the second-best.

■

In exactly the same way, one can prove the following lemma.

Lemma 2 *If $z_1^L < z_2^L$, then at the second-best optimum under storage, one cannot have both $\lambda_1 = 0$ and $\lambda_2 > 0$.*

Thus, if $z_1^L < z_2^L$, regime 3 cannot prevail, and one has either regime 1 or regime 2. Observe that potentially one could have a fourth regime where both incentive constraints are binding. The following Lemma shows that this is not possible

Lemma 3 *At the second-best optimum under storage, one cannot have both $\lambda_1 > 0$ and $\lambda_2 > 0$.*

Proof. Using the same notation as in the proof of Lemma 1, welfare at the laissez-faire is given by

$$W(0) = \pi_1 \Psi_1(0) + \pi_2 \Psi_2(0).$$

Welfare at the second best must be at least as large that at laissez-faire (which remains feasible)

$$W^{SS} = \pi_1 U_1^{SS} + \pi_2 U_2^{SS} \geq W(0). \tag{A6}$$

The second best implies either $\Delta_1^{SS} > 0$ or $\Delta_1^{SS} < 0$ (we can neglect the case where $\Delta_1^{SS} = 0$ because then we are at laissez-faire at which the SS constraints are not binding).

Assume first that $\Delta_1^{SS} > 0$. Then we must have

$$U_2^{SS} \leq \Psi_2 \left(-\frac{\pi_1 \Delta_1^{SS}}{\pi_2} \right) < \Psi_2(0)$$

and thus to satisfy (A6) it is necessary that

$$U_1^{SS} > \Psi_1(0) > \Psi_1 \left(-\frac{\pi_1 \Delta_1^{SS}}{\pi_2} \right) \geq U_{12}^{SS},$$

so that the incentive constraint associated with λ_1 is not binding.

Finally, assume $\Delta_1^{SS} < 0$. Then we must have

$$U_1^{SS} \leq \Psi_1 \left(\Delta_1^{SS} \right) < \Psi_1(0),$$

and thus to satisfy (A6) it is necessary that

$$U_2^{SS} > \Psi_2(0) > \Psi_2 \left(\Delta_1^{SS} \right) \geq U_{21}^{SS},$$

so that the incentive constraint associated with λ_2 is not binding. ■

A.2. The relationship between T_2 and T_1 under storage

(i) **Regime 2:** In this regime, the incentive compatibility constraint is binding for type 2. Hence,

$$u(y - \theta_2 n_2 - T_2) + v(d_2) + h(n_2) = u(y - \theta_2 n_1 - T_1) + v(d_1) + h(n_1). \quad \text{(A7)}$$

Moreover, type 2 faces no distortion in his choice n in this regime. It thus follows that, given T_2 and d_2, his utility is maximized at $n = n_2$. Consequently,

$$u(y - \theta_2 n_2 - T_2) + v(d_2) + h(n_2) \geq u(y - \theta_2 n_1 - T_2) + v(d_2) + h(n_1). \quad \text{(A8)}$$

Subtracting (A8) from (A7) yields,

$$u(y - \theta_2 n_1 - T_1) \geq u(y - \theta_2 n_1 - T_2) + v(d_2) - v(d_1) > u(y - \theta_2 n_1 - T_2), \quad \text{(A9)}$$

where the last inequality sign follows from the fact that $d_2 > d_1$. It now follows from (A9) that $T_2 > T_1$.

(ii) **Regime 3:** In this regime, it is the incentive compatibility constraint of type 1 which is binding. Hence,

$$u(y - \theta_1 n_1 - T_1) + v(d_1) + h(n_1) = u(y - \theta_1 n_2 - T_2) + v(d_2) + h(n_2). \quad \text{(A10)}$$

Moreover, it is now type 1 who faces no distortion in his choice n. Thus, given T_1 and d_1, his utility is maximized at $n = n_1$. Consequently,

$$u(y - \theta_1 n_1 - T_1) + v(d_1) + h(n_1) \geq u(y - \theta_1 n_2 - T_1) + v(d_1) + h(n_2). \quad \text{(A11)}$$

Subtracting (A11) from (A10) yields,

$$u(y - \theta_1 n_2 - T_2) \geq u(y - \theta_1 n_2 - T_1) + v(d_1) - v(d_2) > u(y - \theta_1 n_2 - T_1), \quad \text{(A12)}$$

where it is now $d_1 > d_2$. From (A12) then, $T_1 > T_2$.

A.3. Proofs of the results under Regime 2

(i) **Proof of $c_2 > c_1$ under storage:** Substitute for $v'(d_1)$ from (18), and for $v'(d_2)$ from (19), into (20); set $\lambda_1 = 0$, and simplify to get

$$\frac{u'(c_2)}{u'(c_1)} = \frac{\pi_1\pi_2 - \pi_2\lambda_2\frac{u'(c_{21})}{u'(c_1)}}{\pi_1\pi_2 + \pi_1\lambda_2} < 1.$$

Concavity of $u(\cdot)$ then implies that $c_2 > c_1$.

(ii) **Proof of $n_2 > n_1$ under storage:** Set $\lambda_1 = 0$ in (16)–(17) and "solve" for $h'(n_1)$ and $h'(n_2)$:

$$h'(n_1) = \frac{\pi_1\theta_1 u'(c_1) - \lambda_2\theta_2 u'(c_{21})}{\pi_1 - \lambda_2}, \tag{A13}$$

$$h'(n_2) = \theta_2 u'(c_2). \tag{A14}$$

Subtract equation (A14) from equation (A13), add and subtract $\lambda_2\theta_1 u'(c_1)/(\pi_1 - \lambda_2)$ to the right-hand side. We have

$$h'(n_1) - h'(n_2) = \left[\theta_1 u'(c_1) - \theta_2 u'(c_2)\right] + \frac{\lambda_2}{\pi_1 - \lambda_2}\left[\theta_1 u'(c_1) - \theta_2 u'(c_{21})\right]. \tag{A15}$$

It then follows from $c_2 > c_1, c_{21} > c_1$, and concavity of $u(\cdot)$ that the bracketed expressions on the right-hand side of (A15) are positive so that $h'(n_1) - h'(n_2) > 0$. In turn, concavity of $h(\cdot)$ implies that $n_2 > n_1$.

(iii) **Proof of $d_2 > d_1$ and $c_2 > c_1$ under PAYGO:** Set $\lambda_1 = 0$ in (21)–(24). Combining equation (23) with equation then yields (24). This yields

$$v'(d_2) = \frac{\pi_2}{\pi_2 + \lambda_2}\frac{\pi_1 - \lambda_2}{\pi_1}v'(d_1) = \frac{\pi_1\pi_2 - \pi_2\lambda_2}{\pi_1\pi_2 + \pi_1\lambda_2}v'(d_1). \tag{A16}$$

Hence $v'(d_2) < v'(d_1)$ and $d_2 > d_1$.

Next, combine equation (21) with equation (23), and equation (22) with equation (24), to get

$$v'(d_1) = \frac{u'(c_1)}{\overline{n}}\frac{\pi_1 - \lambda_2\frac{u'(c_{21})}{u'(c_1)}}{\pi_1 - \lambda_2}, \tag{A17}$$

$$v'(d_2) = \frac{u'(c_2)}{\overline{n}}. \tag{A18}$$

Substituting for $v'(d_1)$ from (A17), and for $v'(d_2)$ from (A18), into (A16) and simplifying results in

$$\frac{u'(c_2)}{\overline{n}} = \frac{u'(c_1)}{\overline{n}} \frac{\pi_1\pi_2 - \pi_2\lambda_2\frac{u'(c_{21})}{u'(c_1)}}{\pi_1\pi_2 + \pi_1\lambda_2} < \frac{u'(c_1)}{\overline{n}},$$

so that $u'(c_2) < u'(c_1)$. Concavity of $u(\cdot)$ then implies that $c_2 > c_1$.

(iv) **The relationship between n_2 and n_1 under PAYGO:** Set $\lambda_1 = 0$ in (25)–(26) and "solve" for $h'(n_1)$ and $h'(n_2)$:

$$h'(n_1) = \frac{\pi_1\theta_1 u'(c_1) - \lambda_2\theta_2 u'(c_{21})}{\pi_1 - \lambda_2} - \frac{\mu\pi_1}{\pi_1 - \lambda_2}\frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2}, \quad \text{(A19)}$$

$$h'(n_2) = \theta_2 u'(c_2) - \frac{\mu\pi_2}{\pi_2 + \lambda_2}\frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2}. \quad \text{(A20)}$$

Subtract equation (A20) from equation (A19), add and subtract $\lambda_2\theta_1 u'(c_1)/(\pi_1 - \lambda_2)$ to the right-hand side. We have, after a bit of algebraic manipulation,

$$h'(n_1) - h'(n_2) = \left[\theta_1 u'(c_1) - \theta_2 u'(c_2)\right] + \frac{\lambda_2}{\pi_1 - \lambda_2}\left[\theta_1 u'(c_1) - \theta_2 u'(c_{21})\right] - \frac{\mu\lambda_2}{(\pi_2 + \lambda_2)(\pi_1 - \lambda_2)}\frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2}. \quad \text{(A21)}$$

Observe that while the first two expressions on the right-hand side of (A21) are positive, the third expression is negative. Thus, one cannot sign this expression.

Now assume that

$$\theta_1 - \frac{h'(n_1)}{u'(c_1)} \leq 0.$$

It then follows from equation (29) that

$$-\frac{\mu\lambda_2}{(\pi_2 + \lambda_2)(\pi_1 - \lambda_2)}\frac{\pi_1 d_1 + \pi_2 d_2}{\overline{n}^2} \geq -\frac{\mu\lambda_2}{(\pi_2 + \lambda_2)(\pi_1 - \lambda_2)}\frac{\lambda_2}{\pi_1}\left[\theta_1 u'(c_1) - \theta_2 u'(c_{21})\right]. \quad \text{(A22)}$$

Combining equation (A21) with inequality (A22) yields

$$h'(n_1) - h'(n_2) \geq \frac{\pi_2\lambda_2}{\pi_1(\pi_2 + \lambda_2)}\left[\theta_1 u'(c_1) - \theta_2 u'(c_{21})\right] + \left[\theta_1 u'(c_1) - \theta_2 u'(c_2)\right] > 0.$$

Concavity of $h(\cdot)$ then implies that $n_2 > n_1$.

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Notes

[1]Cremer *et al.* (2006).

[2]Sinn (2004), Abio *et al.* (2004), Fenge and Meier (2005) Van Groezen *et al.* (2003).

[3]As is standard, we assume a world of single parents. Alternatively, one can assume that the decision unit is a couple acting cooperatively.

[4]The results imply that the first-best utility level of type 2 parents is higher than that of type 1's; it is the standard result with a utilitarian objective.

[5]The only variable left to choose by households is then the number of children.

[6]Recall that consumption levels are equalized in the first best.

[7]This is a standard result with Cobb-Douglas utility function: expenditure shares are constant.

[8]To be more precise, treating n^j as exogenous, we have for $j = 1, 2$,

$$\begin{aligned}
-\pi_j \frac{d^{FP}}{\bar{n}^2} &= \frac{\partial \sum_k \pi_k d_k^{FP}/\overline{n}}{\partial n_j}, \\
\pi_j \theta_j &= \frac{\partial \sum_k \pi_k \theta_k n_k}{\partial n_j}, \\
\pi_j h'(n_j) &= \frac{\partial \sum_k \pi_k h(n_k)}{\partial n_j}, \\
\pi_j u'(c_j) &= \frac{\partial \sum_k \pi_k u(c_k)}{\partial c_j}.
\end{aligned}$$

[9]This argument assumes a linear Pigouvian subsidy scheme.

[10]Cremer *et al.* (2006) discuss this matter in some detail.

[11]This follows directly from the assumption of no private savings. This is plain when we have a PAYGO system. With the storage, the observability follows through the assumption that the second-period consumption is financed by a fully-funded "collective" scheme.

[12]It follows from (14) that

$$\pi_1 + \lambda_1 - \lambda_2 > 0,$$

and from (15) that

$$\pi_2 + \lambda_2 - \lambda_1 > 0.$$

Moreover, with $c_j = y - \theta_j n_j - T_j$ and $c_{jk} = y - \theta_j n_k - T_k$,

$$\begin{aligned}
c_{21} - c_1 &= (\theta_1 - \theta_2) n_1 > 0, \\
c_{12} - c_2 &= (\theta_2 - \theta_1) n_2 < 0.
\end{aligned}$$

[13]We show in the Appendix that a solution where both incentive constraints are binding is not possible; see Lemma 3.

[14]We owe this result to a referee.

[15]With $\lambda_1 = 0$, from (14), $\pi_1 - \lambda_2 > 0$.

[16]The same pattern of distortion arises in Cigno *et al.* (2004). In their model, this is required to induce the optimal level of investment in raising children by parents who have different abilities for it.

[17]For example, in the "normal case" of Stiglitz's two-group model where the redistribution is from high- to low-ability persons, low-ability individuals face a positive marginal income tax rate while their average tax rate is negative.

[18]Recall that s is the marginal *subsidy.* A negative value thus means a positive marginal tax.

[19]Recall that we have the "no distortion at the top" property for type 1 here.