Optimal Taxation with Consumption Time as a Leisure or Labor Substitute

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Abstract

This paper studies the optimal commodity taxation problem when time taken in consumption is a perfect substitute for either labor or leisure. It shows that while labor substitutability affects the optimal tax structure, leisure substitutability leaves the classical optimal tax results intact. In the Ramsey tax framework with linear income taxes, whether the consumers have the same or different earning abilities, labor substitutes tend to be taxed at a higher rate than leisure substitutes with the tax differential being increasing in consumption time. This is not necessarily the case when one allows for nonlinear income taxation.

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1 Introduction

Standard optimal tax analysis—summarized in Atkinson and Stiglitz (1980) and more recently, Auerbach and Hines (2002)—ignores the fact that consumption of goods takes time, and treats all time as being devoted to either labor or leisure. These results have been called into question in studies by Gahvari and Yang (1993) and Kleven (2004) who derive optimal commodity tax structures when consumption is time-consuming in the manner suggested by Becker (1965). Optimal commodity tax rates then depend on time spent consuming each good in a rather complicated way. Traditional results such as the uniformity of commodity taxation when labor supply (but not leisure) is perfectly inelastic in supply, or when utility functions are weakly separable between leisure and goods and homothetic in goods, no longer apply.[1] In the case of separability, Gahvari (2006) has shown that it matters if one formulates utility as a weakly separable function of goods and leisure, or goods and labor supply. When consumption is time consuming, the two formulations are no longer equivalent.[2] Specifically, if separability is in terms of goods and leisure, as in Kleven (2004), it leads to a violation of the standard results. On the other hand, with separability in goods and labor supply, the standard results apply.

Clearly then, the manner in which one formulates the utility function, including the way in which separability may apply, is critical. At the heart of this problem lies the substitute/complement relations that exist among times spent in consumption, labor, and leisure, especially whether any given use of consumption time is a substitute for labor or for leisure. The Becker approach is silent on this issue, and as we shall argue implicitly treats all consumption time, regardless of the activity, as a substitute for

[1]The first result is due to Gahvari and Yang (1993), and the second to Kleven (2004).

[2]In the standard model when consumption is not time consuming, writing utility in terms of consumption and leisure, or consumption and labor, are equivalent. This follows because leisure is just time available less labor. Indeed, the underlying utility function can be thought of as having goods, labor and leisure as arguments, with either leisure or labor having been eliminated via the household's time constraint. We return to the consequences of the formulation of the utility function below.

labor.[3] Yet, one's everyday experiences suggests that the nature of activity is very important in how one is affected by consumption time. Contrast the time spent driving to a vacation spot with time spent vacationing once one is there. In the former, time is simply an unavoidable necessity for the purpose of transportation with no inherent utility of its own. On the other hand, time spent vacationing is an integral part of getting pleasure from that activity, much the same way as sitting and doing nothing (which would be pure leisure time).

The above distinction applies to many other consumption activities. Time spent at a dentist's office is qualitatively different from time spent listening to music. The same is true of time spent doing chores around the house versus time spent going out. Such a distinction follows quite naturally if one applies Lancaster's (1966) goods-characteristics formulation of consumption to Becker's (1965) consumption activity approach. What is required is to conceive of the consumption activity as having two characteristics. One is represented by the good purchased directly and the other by the time spent using it, with both entering the utility function directly. This is the approach we take here.

To emphasize the relevance of our approach, we focus on a case that is extreme in two senses. First, we assume that consumption time is either a perfect substitute for labor or a perfect substitute for leisure. The distinction between labor and leisure substitutes makes a crucial difference for the results, as we shall see later, and one that is brought out most clearly by assuming perfect substitutability. Second, time spent consuming any particular good is taken to be a fixed proportion of the quantity of the good—the proportion being potentially different for all goods. This is the case studied by Gahvari and Yang (1993), Kleven (2004) and Gahvari (2006), all of whom used it as a special case of the Becker (1985) formulation in which each good purchased can be combined with time in variable proportions to produce consumption services. It thus gives us a natural basis for comparing our results to theirs who, we shall argue, implicitly assume

[3] This distinction is not made in the applications of Becker (1985) to optimal taxation by Gahvari and Yang (1993), Kleven (2004) and Gahvari (2006) .

that all time used in consumption is a perfect substitute for labor. On the basis of this formulation, we are able to show that while labor substitutability affects the optimal tax structure, leisure substitutability leaves the classical optimal tax results intact.

We begin with the analog of the classical Ramsey problem of choosing the optimal commodity tax structure in an economy of identical households, the case considered by Gahvari and Yang (1993) and Kleven (2004).[4] This case nicely illustrates the intuition and forms the basis for the two heterogeneous-household cases we consider. In the first of these, optimal commodity taxes as well as an equal per capita lump-sum tax (or subsidy) can be applied. This yields the analog of the so-called many-person Ramsey rule (Diamond, 1975). We then turn to the second heterogeneous-household case where income can be observed so an optimal nonlinear income tax can be used alongside commodity taxes. Here the relevant separability result is the well-known theorem of Atkinson and Stiglitz (1976) that if household preferences are weakly separable in goods and leisure, one can dispense with differential commodity taxes. The validity of this result will also depend upon whether consumption time is a substitute for labor or leisure.

2 The identical-household economy

We begin with the classical identical-household optimal commodity tax problem of Ramsey (1927) extended to incorporate time used to consume goods. Although the identical household case is obviously unrealistic, it is a useful way to introduce the model, to understand some of the intuition, and to draw some comparisons with the existing literature. The representative household consumes a bundle of goods, each of which requires some time. He devotes the remainder of his time to either market labor or pure leisure.[5]

[4] Gahvari (2006) discusses the heterogeneous-household models as well.

[5] One can argue that there is no such thing as pure leisure, that is, that all non-market uses of time involve the use of at least some commodities. We allow for pure leisure so that our analysis is as general as possible. The case of no leisure, studied by Kleven (2004), then becomes a special case, as discussed further below.

Specifically, the household purchases a vector of n goods, denoted $\underline{x} \equiv (x_1, x_2, \ldots, x_n)$, at the consumer prices $q_i = 1+t_i$, $i = 1, \ldots, n$, where all producer prices are normalized at unity and t_i is an excise tax on x_i (either per unit or equivalently as a proportion of producer price). The amount of time devoted to the consumption of good i is denoted $\underline{\theta} \equiv (\theta_1, \theta_2, \ldots, \theta_n)$. We focus on the case where $\theta_i > 0$ for all i, although as we shall see, allowing for goods that require no time is straightforward.

In addition to devoting time to consumption of each good, the representative household supplies L units of labor to the market at the wage rate w and takes r units of leisure (rest). As usual, one can assume that there is no tax on labor by suitable normalizations of producer and consumer prices. The household faces two constraints, a time constraint and a budget constraint. Assuming the household is endowed with one unit of time, the time constraint can be written as

$$L + r + \sum_{i=1}^{n} \theta_i = 1. \tag{1}$$

Similarly, with labor supply being the sole source of income, the budget constraint is written as

$$\sum_{i=1}^{n} q_i x_i = wL. \tag{2}$$

The household derives utility (or disutility) from all uses of time as well as from consumption of goods. A utility function reflecting the household's preferences might then be written most generally as

$$F(L, r, \underline{\theta}, \underline{x}),$$

This formulation is distinct from that of Becker (1985), who assumed that the arguments in the utility function were consumption activities, $Z_i(x_i, \theta_i)$ in his notation, rather than goods and time separately. As mentioned, this allows us to emphasize the distinction between consumption time that gives pleasure and that which is unpleasant. We assume that $F(\cdot)$ is increasing in goods, $\underline{x}$, and leisure, r, and decreasing in labor supply, L).

Time spent consuming a good, θ_i, could either be utility-decreasing or utility-increasing depending on the particular activity.

To capture this difference succinctly, assume that time taken in consumption is either a perfect substitute for time at work, or a perfect substitute for leisure. We refer to goods that fall into the first category as L-substitutes, and those that fall into the second category as r-substitutes. We also label the goods such that the first m goods (with the consumption levels x_i, $i = 1, \ldots, m$) are L-substitutes and the remaining $n - m$ goods (with the consumption levels x_i, $i = m + 1, \ldots, n$) are r-substitutes. An example of an L-substitute could be doing household work, while an example of an r-substitute is listening to music or going to a museum. Given the perfect substitutability assumption, utility may be rewritten as

$$F(L, r, \underline{\theta}, \underline{x}) = \Omega \left(L + \sum_{i=1}^{m} \theta_i,\ r + \sum_{i=m+1}^{n} \theta_i,\ \underline{x} \right) \tag{3}$$

where $\Omega(\cdot)$ is decreasing in the first argument, and increasing in the rest. We also assume that $\Omega(\cdot)$ is strictly monotonic, quasi-concave and twice differentiable.

It is convenient to assume that the consumption of a unit of good i requires a fixed amount of time a_i, so that $\theta_i = a_i x_i$ is the time taken consuming i. This fixed-proportion assumption is a simplification of Becker's general theory of time, but it is a useful simplification for analytical purposes. It also corresponds with the cases found in Gahvari and Yang (1993), Kleven (2004) and Gahvari (2006). In a more general analysis, one would want to allow for some substitutability between the quantity of goods consumed and time, as well as joint consumption of several goods and time (as in the case of household production). These extensions, although realistic and important in other contexts (e.g., the analysis of daycare policy in Bergstrom and Blomquist, 1996 or the taxation of household production in Kleven *et al.*, 2000), obscure the transparency of the optimal tax results we are able to derive. Given fixed proportions, the utility function (3) may be written as

$$F(L, r, \underline{ax}, \underline{x}) = \Omega\left(L + \sum_{i=1}^{m} a_i x_i,\ r + \sum_{i=m+1}^{n} a_i x_i,\ \underline{x}\right) \tag{4}$$

and the time constraint as

$$L + r + \sum_{i=1}^{n} a_i x_i = 1. \tag{5}$$

Next, denote the first two arguments of $\Omega(\cdot)$ by

$$Y \equiv L + \sum_{i=1}^{m} a_i x_i, \tag{6a}$$

$$y \equiv r + \sum_{i=m+1}^{n} a_i x_i, \tag{6b}$$

where Y is total time devoted to labor-equivalent activities, and y is total time devoted to leisure-equivalent activities. Using these definitions, we rewrite the utility function (4) as $\Omega(Y, y, \underline{x})$, with $\Omega_Y < 0, \Omega_y > 0, \Omega_i > 0$, for $i = 1, 2, \ldots, n$. Moreover, since the time constraint (5) is now simply $Y + y = 1$, one can rewrite the utility function $\Omega(.)$ in terms of y and $\underline{x}$ only:[6]

$$\Omega(Y, y, \underline{x}) = \Omega(1 - y, y, \underline{x}) \equiv U(y, \underline{x}). \tag{7}$$

Note for future reference that if $\Omega(.)$ is weakly separable in (Y, y) and $\underline{x}$, then $U(.)$ is weakly separable in y and $\underline{x}$. Note also that if all goods are L-substitutes, (7) reduces to $U(r, \underline{x})$. This is equivalent to the formulation in Gahvari and Yang (1993) and Kleven (2004).[7]

[6]The representation of utility in terms of "effective leisure" y and goods $\underline{x}$ depends critically on the assumption that time spent consuming is a perfect substitute for either labor or leisure. Observe also that, under this assumption, the utility could alternatively be written as a function of "effective labor" Y and $\underline{x}$ by substituting $y = 1 - Y$ in $\Omega(Y, y, \underline{x})$. For the analysis that follows, it turns out to be more fruitful to eliminate Y.

[7]They write utility as a function of consumption activities in the manner of Becker (1985), of which pure leisure may be one. However, given that consumption time and goods are in fixed proportion, that is equivalent to including only goods in the utility function. See Gahvari (2006).

Similarly, one may rewrite the budget constraint (2), in terms of y and $\underline{x}$. Thus, add $w \sum_{i=1}^{m} a_i x_i$ to both sides of Eq. (2) and use the definition of y in Eq. (6a) to arrive at

$$\sum_{i=1}^{n} q_i x_i + w \sum_{i=1}^{m} a_i x_i = wY, \tag{8}$$

or,

$$\sum_{i=1}^{m} \widetilde{q}_i x_i + \sum_{i=m+1}^{n} q_i x_i = w(1-y), \tag{9}$$

where

$$\widetilde{q}_i \equiv q_i + w a_i, \qquad i = 1, 2, \ldots, m \tag{10}$$

One can think of $\widetilde{q}_i$ as the "full price" or the full cost of consuming good i, including the cost of the time devoted to its consumption. Thus, $q_i/\widetilde{q}_i$ can be thought of, following Kleven (2004), as the share of monetary cost in the total cost, including the time cost, of consuming good i.

2.1 The optimal tax problem

Consider first the problem of the household, which is to choose y and $\underline{x}$ to maximize utility (7), subject to the budget constraint (9).[8] It is summarized by the Lagrangian expression

$$\mathcal{L} = U(y, \underline{x}) + \alpha \left[w(1-y) - \sum_{i=1}^{m} \widetilde{q}_i x_i - \sum_{i=m+1}^{n} q_i x_i \right].$$

The first-order conditions of this problem reduce to the following equations for the L- and r-substitutes, respectively,

$$\frac{U_j}{U_y} = \frac{\widetilde{q}_j}{w}, \qquad j = 1, 2, \ldots, m \tag{11a}$$

$$\frac{U_s}{U_y} = \frac{q_s}{w}, \qquad s = m+1, \ldots, n. \tag{11b}$$

[8] Equivalently, the household can be thought of as choosing an allocation of time between y and Y, given that $y + Y = 1$, and a vector of consumption, $\underline{x}$.

These determine consumer demands for full leisure and goods $y(\underset{\sim}{\widetilde{q}}, \underset{\sim}{q}, w)$ and $x_i(\underset{\sim}{\widetilde{q}}, \underset{\sim}{q}, w)$, and indirect utility $v(\underset{\sim}{\widetilde{q}}, \underset{\sim}{q}, w)$, given the vector of consumer prices $\underset{\sim}{\widetilde{q}} = (\widetilde{q}_1, \ldots, \widetilde{q}_m)$ and $\underset{\sim}{q} = (q_{m+1}, \ldots, q_n)$. Using the envelope theorem, one can easily show that Roy's identity applies to this setting so that

$$\frac{\partial v}{\partial \widetilde{q}_j} = -\alpha x_j, \quad \frac{\partial v}{\partial q_s} = -\alpha x_s, \tag{12}$$

where α is the representative household's marginal utility of income.

Next, consider the government's problem. It chooses commodity tax rates $t_i = q_i - 1$ to maximize the indirect utility $v(\underset{\sim}{\widetilde{q}}, \underset{\sim}{q}, w)$ subject to its revenue constraint, $\sum_{i=1}^n t_i x_i = \bar{R}$, where $\bar{R}$ is a given revenue requirement. The structure of this problem is analogous to the standard optimal commodity tax problem, found for example in Sandmo (1974) and Atkinson and Stiglitz (1980), and is summarized by the Lagrangian

$$\mathcal{L} = v(\underset{\sim}{\widetilde{q}}, \underset{\sim}{q}, w) + \mu \left[\sum_{i=1}^n t_i x_i - \bar{R} \right].$$

The first-order conditions for optimal taxes on L- and r-substitutes are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial t_j} &= \frac{\partial v}{\partial \widetilde{q}_j} + \mu \left[\sum_{i=1}^n t_i \frac{\partial x_i}{\partial \widetilde{q}_j} + x_j \right] = 0, \qquad j = 1, 2, \ldots, m, \\
\frac{\partial \mathcal{L}}{\partial t_s} &= \frac{\partial v}{\partial q_s} + \mu \left[\sum_{i=1}^n t_i \frac{\partial x_i}{\partial q_s} + x_s \right] = 0, \qquad s = m+1, \ldots, n.
\end{aligned}$$

Simplifying these equations, using Roy's identity, results in

$$\begin{aligned}
\sum_{i=1}^n t_i \frac{\partial x_i}{\partial \widetilde{q}_j} &= -\frac{\mu - \alpha}{\mu} x_j, \quad j = 1, 2, \ldots, m \\
\sum_{i=1}^n t_i \frac{\partial x_i}{\partial q_s} &= -\frac{\mu - \alpha}{\mu} x_s, \quad s = m+1, \ldots, n.
\end{aligned}$$

These are the standard Ramsey tax equations except that $\widetilde{q}_j$ replaces q_j for all L-substitutes ($j = 1, 2, \ldots, m$).

The above equations can further be rewritten, using the Slutsky equation and the symmetry of the substitution effects, as the analogs of the standard form of optimal commodity tax expressions in terms of *compensated* demand functions. We have

$$\sum_{i=1}^{n} \frac{t_i}{x_j} \frac{\partial x_j^c}{\partial \widetilde{q}_i} = \sum_{i=1}^{n} \frac{t_i}{\widetilde{q}_j} \varepsilon_{ji} = -\frac{\mu - \gamma}{\mu}, \quad j = 1, 2, \ldots, m, \tag{13a}$$

$$\sum_{i=1}^{n} \frac{t_i}{x_s} \frac{\partial x_s^c}{\partial q_i} = \sum_{i=1}^{n} \frac{t_i}{q_s} \varepsilon_{si} = -\frac{\mu - \gamma}{\mu}, \quad s = m+1, \ldots, n, \tag{13b}$$

where x_i^c denotes the compensated demand for good i, ε_{ki} is the compensated elasticity of demand for good k with respect to the full consumer price of good i, and $\gamma \equiv \alpha/\mu + \sum_i t_i(\partial x_i/\partial M)$ is the well-known net social marginal utility of income, with M denoting household non-labor income. The left-hand sides of these expressions are the so-called indices of discouragement for L- and r-substitutes. For the latter, the optimal tax rules are the standard ones. For the former, tax rules mimic the standard ones except that $\widetilde{q}_j$ replaces q_j $(j = 1, 2, \ldots, m)$. Moreover, since $\widetilde{q}_j > q_j$, nominal tax rates t_j/q_j (as opposed to effective tax rates $t_j/\widetilde{q}_j$) tend to be higher than those given by the standard rules. Of course, if there are some goods whose consumption does not involve any time, so that $a_i = 0$, they would also be taxed according to the same rules as r-substitutes.

The analog of the Corlett and Hague (1953) result, in the present setting, is immediately obtained from optimal tax rules (13a) and (13b). Suppose there are two goods, $i = 1, 2$, either of which could be and r- or L-substitute. Then, Eqs. (13a) and (13b) can be written:

$$\frac{t_1}{\widetilde{q}_1}(\varepsilon_{11} - \varepsilon_{21}) = \frac{t_2}{\widetilde{q}_2}(\varepsilon_{22} - \varepsilon_{12}),$$

where $\widetilde{q}_i = q_i$ for the case of an r-substitute. Using the homogeneity property of compensated demand functions, where 0 stands for y, $\varepsilon_{i0} + \varepsilon_{i1} + \varepsilon_{i2} = 0$, we obtain:

$$\frac{t_1/q_1}{t_2/q_2} = \frac{\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{10}}{\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{20}} \frac{q_2/\widetilde{q}_2}{q_1/\widetilde{q}_1}.$$

In the case where both goods are r-substitutes, this is the standard Corlett and Hague result whereby a higher tax is put on the good which is relatively more complementary with leisure. On the other hand, if both goods are L-substitutes, this result must be modified to take account of the fact that the good that is more time-intensive bears a higher tax rate on that account (Kleven, 2004, Proposition 4). Similarly, if only one is an L-substitute, that will tend to cause it to have a higher tax rate and will also modify Corlett and Hague.

2.2 The inverse elasticity rule

Consider the quasi-linear case in which $U(y, \underline{x})$ is additive and linear in y. Then, the demand for x_i depends only on its own (full) price, and the optimal tax rules (13a) and (13b) can be written as

$$\frac{t_j}{\widetilde{q}_j} = -\frac{\mu-\gamma}{\mu}\frac{1}{\varepsilon_{jj}}, \qquad j = 1, 2, \ldots, m, \tag{14a}$$

$$\frac{t_s}{q_s} = -\frac{\mu-\gamma}{\mu}\frac{1}{\varepsilon_{ss}}, \qquad s = m+1, \ldots, n, \tag{14b}$$

where ε_{jj} and ε_{ss} denote both the compensated and uncompensated own price elasticities of demand (given that there are no income effects). Thus, tax rates in terms of full consumer prices are all proportional to the inverse of the elasticity of demand where the factor of proportionality is the same for all goods.

Observe that, for L-substitutes, one can rewrite the optimal tax rates in terms of market prices q_j, using the definition of $\widetilde{q}_j$, as[9]

$$\frac{t_j}{q_j} = -\frac{\mu-\gamma}{\mu}\frac{1}{\varepsilon_{jj}}\left(1+\frac{wa_j}{q_j}\right), \qquad j = 1, 2, \ldots, m. \tag{15}$$

[9] Equivalently, this equation can be written, following Kleven (2004), Proposition 3, as

$$\frac{t_j}{q_j} = -\frac{\mu-\gamma}{\mu}\frac{1}{\varepsilon_{jj}}\frac{1}{\alpha_j}$$

where $\alpha_j = q_j/\widetilde{q}_j$ is the share of the monetary costs in total cost of consuming x_j (where t_j/q_j here corresponds to t^j in Kleven.)

Consequently, optimal tax rates in terms of consumer prices are inversely proportional to the elasticity of demand, but the factor of proportionality for L-substitutes is systematically higher than that for r-substitutes and is increasing in the time intensity of consumption a_j. Intuitively, the inverse elasticity rule in this context requires that optimal tax rates *in terms of full prices* be proportional to the inverse of the elasticity of demand. Thus, if an L-substitute and an r-substitute have the same elasticity of demand, their uniform tax rate in term of full prices translates into the L-substitute having a *higher* tax rate as a proportion of market prices.

2.3 Uniform taxation

A further well-known special case arises when utility is weakly separable in goods and leisure, and is homothetic in goods. Sandmo (1974) showed that in this case, optimal commodity taxes in the standard Ramsey problem will be uniform. The analog here is that if preferences are weakly separable in $\underline{x}$ and full leisure y, and homothetic in $\underline{x}$, optimal tax rates will be characterized by[10]

$$\frac{t_j}{\widetilde{q}_j} = \frac{t_s}{q_s} = \tau, \qquad j = 1, 2, \ldots, m \text{ and } \quad s = m+1, \ldots, n, \tag{16}$$

where τ is the uniform tax rate based on full consumer prices $\widetilde{q}$ and q.[11] The proof of Eq. (16) is given in the Appendix. It implies, given the definition of $\widetilde{q}_j$ in Eq. (10), the following specific commodity taxes (or equivalently, taxes as a proportion of producer

[10]Separability in this context means that the marginal rate of substitution between any two goods purchased from the market is independent of leisure time broadly defined to include pure leisure and time spent on r-substitutes. It is different from separability in the traditional leisure/labor model. In particular, unlike the traditional model in which leisure and labor separability are equivalent, r-substitute separability and L-substitute separability are two different assumptions with different implications. On this, see also Gahvari (2006).

[11]In the case of L-substitutes, one can write Eq. (16) as

$$\frac{t_j/q_j}{t_k/q_k} = \frac{\alpha_k}{\alpha_j},$$

where α_i is the cost share defined in the previous footnote. This is identical to the result found in Kleven (2004), Proposition 2, whose formulation, we have argued above, is equivalent to assuming all goods are L-substitutes.

prices since the latter are all unity),

$$t_j = (1 + wa_j)t, \quad j = 1, 2, \ldots, m \tag{17a}$$

$$t_s = t, \quad s = m + 1, \ldots, n \tag{17b}$$

where t is a uniform tax rate applied to producer prices.

These results suggest that L-substitute goods should be taxed at a higher rate than r-substitutes, and that the more time-consuming is consumption of an L-substitute good, the higher should be its tax rate. The intuition is that, while the consumption of both types of goods involves the use of scarce time, in the case of r-substitutes, there is no opportunity cost involved. Any time spent in consumption reduces the amount of time spent on pure leisure, but the total amount of leisure-equivalent time remains unchanged so utility is not affected on that account. In the case of L-substitutes, the time spent consuming comes at the expense of time working. While the total labor-equivalent time does not change and utility is not affected on that account, less income is earned and the household is worse off. The same reasoning tells us that the more time-consuming is consumption of an L-substitute good, the higher should be its tax rate.

The empirical content of the differential tax treatment of L- and r-substitutes is far reaching. It suggests that, contrary to widely-held views, one should tax goods like books, movies, CD's and sport events at the same rate as goods whose consumption may not take much time, and at a *lower* rate than goods like transportation. Observe also that this result does not violate the message of Corlett and Hague. They refer to different forces at work in optimal taxation.

2.4 Absence of pure leisure

Suppose there is no pure leisure so that $r = 0$, and one spends all of his time endowment in conjunction with some consumption activity. In this case, Eq. (6b), which shows time devoted to leisure-equivalent activities, simplifies to $y = \sum_{i=m+1}^{n} a_i x_i$. Two special cases of this formulation are illuminating. In the first, all n goods are L-substitutes, which is equivalent to the case of Gahvari and Yang (1993) and Kleven (2004). Then, $y = 0$, and household utility (7) may be written, with some abuse of notation, simply as $U(\underline{x})$. Similarly, the budget constraint (9) simplifies to $\sum_{i=1}^{n} \widetilde{q}_i x_i = w$. The problem of the household is to maximize $U(\underline{x})$ subject to $\sum_{i=1}^{n} \widetilde{q}_i x_i = w$. This is equivalent to a standard consumer problem with fixed income and the first-order conditions are $U_i/U_j = \widetilde{q}_i/\widetilde{q}_j, \forall i, j$. It is plain that, given this setup, tax revenues can be raised in a non-distortionary way: Simply apply a tax structure in which tax rates, as a proportion of full consumer prices, are uniform. That is, set $t_i/\widetilde{q}_i = t_j/\widetilde{q}_j = \tau, \forall i, j$. Two features of this result are worth noting. First, in contrast to the previous subsection where we had to assume separable and homothetic-in-goods preferences, one gets a uniform taxation result here without imposing any restrictions on households' preferences. Second, and again in contrast with the previous subsection where uniform taxation entailed an excess burden, we now have a first-best outcome.[12]

In the second special case, all n goods are r-substitutes. Then, the budget constraint (9) becomes $\sum_{i=1}^{n} q_i x_i = w(1 - y)$, where $y = \sum_{i=1}^{n} a_i x_i$. The household maximizes $U(y, \underline{x})$ subject to $\sum_{i=1}^{n} q_i x_i = w(1 - y)$. This problem is analytically the same as a utility-maximization one with variable labor supply. A government seeking to raise a given amount of revenue using commodity taxes faces the same type of problem as in the standard Ramsey case, and the traditional optimal commodity tax rules apply. Unlike in the previous case with all L-substitutes, a first-best outcome cannot be achieved.

It is clear from the above discussion that as long as there exist some r-substitutes, all possible commodity tax structures will be second best. The first-best outcome result holds *only* in the special case where all goods are L-substitutes. Once again we see that

[12] One can write the tax as a proportion of the good's market prices as $t_i/q_i = \tau\widetilde{q}_i/q_i, \forall i$. That is, tax rates should be inversely proportional to the ratio of a good's market price to its full price, $q_i/\widetilde{q}_i$ (Proposition 1, Kleven, 2004).

whether time in consumption is substitutable for labor or for leisure is critical for the design of an optimal commodity tax system.

The two special cases studied above illustrate a common thread that runs through all the special cases we have considered. If time spent in consumption is always a perfect substitute for leisure, all goods would be r-substitutes and the standard optimal tax results apply. Under this circumstance, there would be no need to take account of time spent in consumption. On the other hand, if all goods were L-substitutes, the results of Gahvari and Yang (1993) and Kleven (2004) would apply. With a mix of L-substitutes and r-substitutes, the results would have some features of the traditional model, along with some adjustments as suggested by these authors. Proposition 1 summarizes the results of this section.

Proposition 1 *Consider a representative-household economy and assume that consumption goods are of two types: L-substitutes, where time spent consuming is a perfect substitute for work, and r-substitutes, where time is a perfect substitute for leisure. Assume that consumption of one unit of good i takes a_i units of time:*

(i) Optimal taxes are characterized by Eqs. (13a)–(13b).

(ii) If preferences are additive in goods and linear in leisure-equivalent activities, then,

- *The inverse elasticity rule applies when tax rates are expressed in terms of full consumer prices, as characterized by Eqs. (14a)–(14b).*
- *Given the same price elasticity of demand, the statutory tax on an L-substitute must be higher than that on an r-substitute. The tax increases as the consumption time for the L-substitutes increases as characterized by Eq. (15).*

(iii) If preferences are weakly separable in goods and leisure-equivalent activities, and homothetic in goods, then,

- *The r-substitutes are always taxed uniformly, as in the standard theory.*
- *The tax rates on L-substitutes are increasing in the time-intensity of the good consumed as characterized by Eq. (17a).*
- *The L-substitutes are taxed at higher rates than the r-substitutes.*

(iv) Assume there is no pure leisure. Then,

- *If all goods are L-substitutes, a tax structure in which tax rates, as a proportion of full consumer prices, are uniform is first-best and Eq. (17a) applies.*
- *As long as there exist some r-substitutes, all commodity tax structures will be second best.*

3 Heterogeneous households with linear taxes

The above analysis where all households are identical focuses solely on efficiency. Suppose now that households are heterogeneous in earning ability, so that equity becomes a consideration. Following the pedagogical approach in the optimal tax literature, in this section we consider the extended Ramsey case where the government uses only linear taxes, as in Diamond and Mirrlees (1971) and Diamond (1975), and summarized in Atkinson and Stiglitz (1980). In the next section, we allow for nonlinear income taxation. As is usual, all households have identical preferences, and we suppose those preferences are characterized as previously by Eq. (7).

Let there be H types of households, indexed by $h = 1, 2, \ldots, H$. Households of type h have an earning ability given by their wage rate w^h, which is fixed by assumption. The total population size is again normalized at one, and the proportion of persons of type h is π^h, where $\sum_{h=1}^{H} \pi^h = 1$.

3.1 The optimal tax problem

The government now levies a set of excise taxes $t_i,\ i = 1, \ldots, n$, on the n goods as before, but in addition provides everyone with a lump-sum transfer of M to make the tax system progressive. The optimization problem of an h-type household then becomes a simple extension of the identical-household case. The household chooses y^h and $\underline{x}^h$ to maximize utility $U(y^h, \underline{x}^h)$ subject to the budget constraint:

$$\sum_{i=1}^{m} \tilde{q}_i^h x_i^h + \sum_{i=m+1}^{n} q_i x_i^h = w^h(1 - y^h) + M,$$

where now

$$\tilde{q}_j^h \equiv q_j + w^h a_j, \qquad j = 1, 2, \ldots, m. \tag{18}$$

Thus, the full cost of consuming L-substitutes varies with the earning ability of households, reflecting the fact that their opportunity cost of labor time differs. The first-order conditions are analogous to Eqs. (11a) and (11b), and the indirect utility function is given by $v^h(\underline{\tilde{q}}^h, \underline{q}, M) = v(\underline{\tilde{q}}^h, \underline{q}, w^h, M)$ where $\underline{\tilde{q}}^h = (\tilde{q}_1^h, \tilde{q}_2^h, \ldots, \tilde{q}_m^h)$ and $\underline{q} = (q_{m+1}, \ldots, q_n)$. The standard envelope properties apply to this indirect utility function.

The objective function for the government is a standard additive social welfare function of the following form:

$$\sum_{h=1}^{H} \pi^h W\left(v^h(\underline{\tilde{q}}^h, \underline{q}, M)\right), \tag{19}$$

where $W(v^h(\cdot))$—the social utility of household h—is increasing, concave and twice differentiable in v^h. The optimal tax problem is then to determine the values of the lump-sum transfer M and the commodity tax vector $(t_1, \ldots, t_n)$ that maximize social welfare subject to the government's exogenous revenue requirement, $\bar{R}$,

$$\sum_{i=1}^{n} t_i \left(\sum_{h=1}^{H} \pi^h x_i^h\right) - M \geq \bar{R}. \tag{20}$$

This problem takes the same form as the standard many-type Ramsey optimal tax problem, except that full consumer prices $\underline{\tilde{q}}^h$ apply for L-substitutes. We can make use of existing results to obtain some insight into the effect of time-consuming consumption on the structure of optimal commodity taxes. Note first that if all goods were r-substitutes, the standard optimal tax analysis would apply and time differences in consumption would be irrelevant. This follows from the analysis in the previous section.

We are now in a position to reexamine the special cases we considered in the previous section for the representative Ramsey case. The following definitions help in presentation. Define the marginal social utility of income of an h-type household by

$$\beta^h \equiv W'(v^h)\frac{\partial v^h}{\partial M}.$$

Denote the shadow price of government revenue (the Lagrange multiplier on the revenue constraint) by μ. Then

$$\gamma^h \equiv \beta^h + \mu \sum_i t_i \frac{\partial x_i^h}{\partial M}$$

is household h's net social marginal utility of income, as conventionally defined in the literature. One can interpret γ^h as the social value in terms of the numeraire good of transferring a unit of income to a household of type h. The mean value of the γ^h's is $\overline{\gamma} = \sum_h \pi^h \gamma^h$.

3.2 Inverse elasticity rule

Assume, as in the representative-household case, that $U(y, \underline{x})$ is additive and linear in y. Let $X_j = \sum_h \pi^h x_j^h$ and $X_s = \sum_h \pi^h x_s^h$ denote aggregate (over all households) consumption of goods $j = 1, 2, \ldots, m$ and $s = m+1, \ldots, n$. Similarly, let ε_{jj}^h and ε_{ss}^h denote household h's price elasticity of demand with respect to $\widetilde{q}_j^h$ and q_s (both compensated and uncompensated given that there are no income effects). We show in the Appendix that, corresponding to Eqs. (15) and (14b), we now have

$$\frac{t_j}{\widetilde{q}_j} = -\frac{1}{\mu}\frac{\mu - \sum_h \gamma^h \pi^h x_j^h / X_j}{\sum_h (q_j/\widetilde{q}_j^h)(\pi^h x_j^h / X_j)\varepsilon_{jj}^h}, \qquad j = 1, 2, \ldots, m, \tag{21a}$$

$$\frac{t_s}{q_s} = -\frac{1}{\mu}\frac{\mu - \sum_h \gamma^h \pi^h x_s^h / X_s}{\sum_h (\pi^h x_s^h / X_s)\varepsilon_{ss}^h}, \qquad s = m+1, \ldots, n, \tag{21b}$$

with $\mu = \bar{\gamma}$.

It follows from Eq. (21b) that the characterization of the optimal tax on r-substitutes remains as in the traditional model where we have a generalized version of the inverse elasticity rule adjusted by equity considerations and every household's share in consumption of these goods. In the case of L-substitutes, the optimal tax characterization changes in that h-household's elasticity of demand is now weighted by the ratio of the good's market price to the full price for household h. With $\widetilde{q}_j^h > q_j$ (for all h and j), it again follows that, if an L-substitute and an r-substitute have the same elasticity of demand and the same consumption share (for all households), the L-substitute will have a *higher* tax rate than the r-substitute as a proportion of market prices.

3.3 Weakly-separable preferences

Suppose we again assume that $U(y, \underline{x})$ is weakly separable in y and $\underline{x}$ and that the subutility in $\underline{x}$ is homothetic, so that the problem has the same structure as Deaton (1977). We prove in the Appendix that in this case, the optimal commodity taxes are characterized by

$$t_j = \frac{\sum_h \pi^h(\overline{\gamma} - \gamma^h)/\varepsilon^h}{\overline{\gamma} - \sum_h \pi^h(\overline{\gamma} - \gamma^h)/\varepsilon^h}(1 + w^h a_j), \quad j = 1, 2, \ldots, m \tag{22a}$$

$$t_s = \frac{\sum_h \pi^h(\overline{\gamma} - \gamma^h)/\varepsilon^h}{\overline{\gamma} - \sum_h \pi^h(\overline{\gamma} - \gamma^h)/\varepsilon^h}, \quad s = m + 1, \ldots, n \tag{22b}$$

where ε^h is the cross-price elasticity of the h-type's demand for leisure with respect to any one of the produced goods. Note that the separability and homotheticity assumptions imply that ε^h is the same for all goods.

To interpret Eqs. (22a) and (22b), define

$$t \equiv \frac{\sum_h \pi^h(\overline{\gamma} - \gamma^h)/\varepsilon^h}{\overline{\gamma} - \sum_h \pi^h(\overline{\gamma} - \gamma^h)/\varepsilon^h}, \tag{23a}$$

$$\Theta \equiv \frac{\sum_h \pi^h(\overline{\gamma} - \gamma^h)w^h/\varepsilon^h}{\overline{\gamma} - \sum_h \pi^k(\overline{\gamma} - \gamma^k)/\varepsilon^k}. \tag{23b}$$

Then, optimal tax rates t_j and t_s can be written as

$$t_j = t + \Theta a_j, \quad j = 1, \ldots, m \tag{24a}$$

$$t_s = t, \quad s = m+1, \ldots, n \tag{24b}$$

These are the analogs of Eqs. (17a)–(17b) in the representative-household case. The r-substitutes are always taxed uniformly. As in the traditional model, the tax rate t is affected by distributional considerations via the γ^h terms, and by the efficiency terms via the ϵ terms. For L-substitutes, we have a more complex version of the representative-household case. As in that setting, all L-substitutes whose consumption take the same time should be taxed at the same rate. Moreover, tax rates must increase with time taken in consumption. However, unlike the representative-household case, relative tax rates for the L-substitutes are not governed simply by relative time intensity factors. One now has $t_j/t_i = (t+\Theta a_j)/(t+\Theta a_i)$ so that distributional and efficiency considerations also enter through Θ. Every different configuration of γ^h's or ϵ^h's imply a different value for Θ and, with it, a different ratio of tax rates for the *same* a_s and a_i (as long as $a_s \neq a_i$).[13] Finally, observe that Eqs. (24a)–(24b) imply that tax rates for L-substitutes are all higher than the tax rate on the r-substitutes.

3.4 Absence of pure leisure

It is plain that the first-best result of the representative-household case, when leisure does not enter preferences and all goods are L-substitutes, will no longer hold when households are heterogeneous. This is easy to see. With the full consumer price of each good, $\widetilde{q}_j^h$, being different for different consumers, there will be no tax scheme which can raise all full consumer prices proportionally for all consumers. If tax rates are chosen such that the full consumer prices increase by the same proportion for one household,

[13] Time differences in consumption become irrelevant if the social planner has no equity objectives. Setting $\gamma^h = \overline{\gamma}$ (for all $h = 1, 2, \ldots, H$) in Eqs. (23a)–(23b) implies that $t = \Theta = 0$ and the tax on all goods (r- as well as L-substitutes) are set at zero. All revenues are then raised from a head tax.

then they will change at varying proportions for different households. Proposition 2 summarizes the results of this Section.

Proposition 2 *Assume the economy is inhabited by households who differ in earning ability but have identical preferences, and that tax instruments are constrained to be linear.*

(i) Assume preferences are additive in goods and linear in leisure-equivalent activities. There will be a generalized version of the inverse elasticity rule for optimal taxes on L- and r-substitutes as characterized by Eqs. (21a)–(21b). They show that if an L-substitute and an r-substitute have the same elasticity of demand and the same consumption share (for all households), the L-substitute will have a higher tax rate than the r-substitute as a proportion of market prices.

(ii) Assume preferences are weakly separable in leisure-equivalent activities and goods. Then,

- *The r-substitutes are always taxed uniformly and characterized by Eq. (23a), while the optimal taxes on L-substitutes are characterized by Eqs. (23a)–(24a).*
- *All L-substitutes whose consumption take the same time should be taxed at the same rate, and the tax rates increase with time taken in consumption.*
- *All L-substitutes are taxed at higher rates than the uniform tax rate on r-substitutes.*
- *Time differences in consumption become irrelevant if the social planner has no equity objectives; all revenues are then raised from a head tax.*

(iii) Assume that pure leisure does not enter preferences and that all goods are L-substitutes. No first-best outcome can be attained.

4 Heterogeneous households with a nonlinear income tax

The presence of a general income tax changes the landscape for optimal taxation. In the traditional model, weak separability between goods and leisure (or labor supply) is sufficient for the redundancy of commodity taxes: homotheticity of the goods subutility function is no longer required (Atkinson and Stiglitz, 1976). The standard method for the derivation of optimal taxes in models with a discrete number of types is to first derive the optimal allocations and then consider the properties of the tax functions that implement this allocation (e.g., Stiglitz, 1987). With more than one consumption good, however, this procedure will yield allocations whose implementation generally requires *nonlinear commodity taxes.* Whether or not goods may be taxed nonlinearly depends crucially on the type of information that is available to the tax administration. Specifically, whether the available information is on personal consumption levels or not. While such information may exist for certain commodities (e.g., electricity consumption by households), it is more likely that the tax administration has information on anonymous transactions only. The possibility of reselling commodities exacerbates the problem. Under this circumstance, nonlinear commodity taxes are not feasible. This is the informational structure that we shall assume in this paper.

As previously, households have identical preferences $U(y, \underline{x})$ defined by Eqs. (6a)–(7) with $y = 1 - Y = 1 - L - \sum_{i=1}^{m} a_i x_i$. Define $I \equiv wL$, and let w^k denote the wage of a household of "type" k, with $w^k > w^h$ whenever $k > h$. Introduce a type-specific utility function describing preferences over x_i's and I,

$$\boldsymbol{u^h}(I, \underline{x}) \equiv U\left(1 - \frac{I}{w^h} - \sum_{i=1}^{m} a_i x_i, \underline{x}\right). \tag{25}$$

Denote the utility level of a h-type household by u^h when he chooses the allocation intended for him, and by u^{hk} when he chooses a k-type person's bundle, namely,

$$u^h = \boldsymbol{u^h}(I^h, \underline{x}^h), \tag{26a}$$

$$u^{hk} = \boldsymbol{u^h}(I^k, \underline{x}^k). \tag{26b}$$

We follow Cremer and Gahvari's (1997) method for the characterization of Pareto-efficient allocations that are constrained not only by the standard self-selection constraints and the resource balance, but also by the linearity of commodity taxes. Thus, we derive an optimal revelation mechanism that consists of a set of type-specific before-tax incomes, I^h's, aggregate expenditures on private goods, c^h's, and a vector of commodity tax rates (same for everyone), $\underline{t} = (t_1, t_2, \ldots, t_n)$. This procedure determines the commodity tax rates right from the outset. A complete solution to the optimal tax problem then requires only the design of a general income tax function. Note that instead of commodity taxes, the mechanism may equivalently specify the consumer prices: $\underline{q} = (q_1, q_2, \ldots, q_n)$, where $q_i = 1 + t_i$ $(i = 1, 2, \ldots, n)$. The mechanism assigns $(\underline{q}, c^h, I^h)$ to a household who reports type h; the consumer then allocates c^h between the produced goods, $\underline{x}$.[14]As usual, homogeneity of degree zero of demands in consumer prices, and supplies in producer prices, allows us to normalize both sets of prices. This enables us to normalize commodity tax rates by setting t_n to zero so that $q_n = 1$.

Formally, given any vector $(\underline{q}, c, I)$, a household of type h solves

$$\max_{\underline{x}} \qquad \boldsymbol{u}^h(\underline{x}, I) \tag{27a}$$

$$\text{subject to} \quad \sum_{i=1}^{n} q_i x_i = c. \tag{27b}$$

Denote, with some abuse of notation, the resulting "conditional" demand functions by

[14]Strictly speaking, this procedure does not characterize allocations as such; the optimization is over a mix of quantities *and* prices. However, given the commodity prices, utility maximizing households would choose the quantities themselves. We can thus think of the procedure as indirectly determining the final allocations.

$\boldsymbol{x}_i^h(\underline{q}, c, I)$,[15] and the indirect utility function by

$$\boldsymbol{v}^h(\underline{q}, c, I) \equiv \boldsymbol{u}^h\left(\underline{x}^h(\underline{q}, c, I), I\right).$$

Next, define

$$\begin{aligned} x_i^h &= \boldsymbol{x}_i^h(\underline{q}, c^h, I^h), \quad x_i^{hk} = \boldsymbol{x}_i^h(\underline{q}, c^k, I^k), \\ v^h &= \boldsymbol{v}^h(\underline{q}, c^h, I^h), \quad v^{hk} = \boldsymbol{v}^h(\underline{q}, c^k, I^k). \end{aligned}$$

Pareto-efficient allocations (constrained by incentive compatibility and linearity of commodity taxes) can then be described as follows. Let δ^h $(h = 1, 2, \ldots, H)$ denote a positive constant with the normalization $\sum_{h=1}^H \delta^h = 1$. Maximize

$$\sum_h \delta^h v^h, \tag{28}$$

with respect to c^h, I^h, and $q_1, q_2, \ldots, q_{n-1}$, subject to the resource constraint

$$\sum_h \pi^h \big[(I^h - c^h) + \sum_{i=1}^{n-1}(q_i - 1)x_i^h\big] \geq \bar{R}, \tag{29}$$

and the self-selection constraints

$$v^h \geq v^{hk}, \qquad h, k = 1, 2, \ldots, H. \tag{30}$$

Denote the Lagrange multipliers associated with the resource balance (29) by μ and the self-selection constraints (30) by λ^{hk} $(h, k = 1, 2, \ldots, H)$. Let $\hat{x}_i^h$ denote the *compensated* version of household h's conditional demand for x_i as determined by problem (27a)–(27b). Finally, define

$$\boldsymbol{A} = \begin{pmatrix} \sum_h \pi^h \frac{\partial \hat{x}_1^h}{\partial q_1} & \sum_h \pi^h \frac{\partial \hat{x}_1^h}{\partial q_2} & \cdots & \sum_h \pi^h \frac{\partial \hat{x}_1^h}{\partial q_{n-1}} \\ \sum_h \pi^h \frac{\partial \hat{x}_2^h}{\partial q_1} & \sum_h \pi^h \frac{\partial \hat{x}_2^h}{\partial q_2} & \cdots & \sum_h \pi^h \frac{\partial \hat{x}_2^h}{\partial q_{n-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_h \pi^h \frac{\partial \hat{x}_{n-1}^h}{\partial q_1} & \sum_h \pi^h \frac{\partial \hat{x}_{n-1}^h}{\partial q_2} & \cdots & \sum_h \pi^h \frac{\partial \hat{x}_{n-1}^h}{\partial q_{n-1}} \end{pmatrix}. \tag{31}$$

[15] These functions are *conditional* on c^h and I^h; they differ from the customary Marshallian demand functions. Specifically, $\boldsymbol{x}_i^h(.)$ as defined here has a different functional form from its counterpart in Section 3.

In the Appendix, we prove that an interior solution satisfies the following conditions,

$$\begin{pmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n-1} \end{pmatrix} = \boldsymbol{A^{-1}} \begin{pmatrix} -\sum_h \sum_{k\neq h} \lambda^{kh}(x_1^{kh} - x_1^h)\frac{v_c^{kh}}{\mu} \\ -\sum_h \sum_{k\neq h} \lambda^{kh}(x_2^{kh} - x_2^h)\frac{v_c^{kh}}{\mu} \\ \vdots \\ -\sum_h \sum_{k\neq h} \lambda^{kh}(x_{n-1}^{kh} - x_{n-1}^h)\frac{v_c^{kh}}{\mu} \end{pmatrix}, \tag{32}$$

where we will also give a characterization for x_i^h and x_i^{kh} solutions. It is clear from Eq. (32) that the structure of commodity taxes depend in the usual way on incentive and redistributive concerns. They also depend, in a complicated way, on the time intensity of the consumption goods. To be able to say anything more about these taxes, we again have to consider some special cases. The inverse elasticity rule has no counterpart in the optimal nonlinear tax problem. We shall thus consider only the two special cases of weakly separable preferences and absence of pure leisure.

4.1 Weakly-separable preferences in y and $\underline{x}$

Assume U is weakly separable in y and $\underline{x}$, so that

$$U(y, \underline{x}) = U\left(y, f(\underline{x})\right). \tag{33}$$

In this case, the first-order conditions to problem (27a)–(27b) for household h are, $j = 1, 2, \ldots, m$ and $s = m+1, \cdots, n$,[16]

$$\begin{aligned} -a_j \frac{U_y\left(1 - I^h/w^h - \sum_{i=1}^m a_i x_i^h, f(\underline{x}^h)\right)}{U_n\left(1 - I^h/w^h - \sum_{i=1}^m a_i x_i^h, f(\underline{x}^h)\right)} + \frac{f_j(\underline{x}^h)}{f_n(\underline{x}^h)} &= q_j, & \text{(34a)} \\ \frac{f_s(\underline{x}^h)}{f_n(\underline{x}^h)} &= q_s, & \text{(34b)} \\ \sum_{i=1}^n q_i x_i^h &= c^h. & \text{(34c)} \end{aligned}$$

These equations then determine $\underline{x}^h$. Observe also that the first-order conditions for the problem of household k mimicking h will be exactly the same except that the arguments

[16] These can be derived from Eqs. (A38)–(A40) in the Appendix.

of U_y and U_n will be $(1 - I^h/w^k - \sum_{i=1}^m a_i x_i^{kh}, \underline{x}^{kh})$, the arguments of f_j, f_n and f_s will be $\underline{x}^{kh}$, and x_i^{kh} replaces x_i^{kh} in Eq. (34c).

Equations (34a)–(34c) indicate that if there are no L-substitute goods, that is, if $m = 0$, then $\underline{x}^h = \underline{x}^{kh}$ for all $k, h = 1, 2, \ldots, H$. In turn, this implies, via Eq. (32), that no differential commodity taxation is needed. Thus, as in the two previous sections with linear income taxes, the standard optimal tax results apply if all goods are r-substitutes and there is no need to take account of time spent consuming. Once some goods are L-substitutes, there are no clearcut results. In particular, unlike the two linear income tax cases, weak separability here does not imply that r-substitutes should necessarily be taxed uniformly (based on full consumer prices). Nor does it imply that L-substitutes must as a rule be taxed at higher rates than r-substitutes. Similarly, it will no longer be the case that the tax rates on L-substitutes necessarily move positively with their time consumption coefficients. If, $a_j = a_k$, for example, it does *not* follow that $t_j = t_k$ unless $f_j(\underline{x}^h) = f_k(\underline{x}^h)$, for all $h = 1, 2, \ldots, H$. Despite lack of general results in this case, one can nevertheless get some intuition into the structure of optimal taxes through a simple example with two persons and two goods.

4.2 A two-person, two-good example

Assume there are only two household types, $h = 1, 2$, with $w^2 > w^1$, enjoying leisure and consuming two goods with time coefficients a_1, a_2. Assume also that the government wants to redistribute from type 2's to type 1's so that the only binding incentive constraint will be that on the type 2's, implying $\lambda^{21} > 0, \lambda^{12} = 0$. Since one commodity tax rate is redundant, one can set $t_2 = 0$. Then, by Eq. (32), $t_1 > 0$ if $x_1^{21} > x_1^1$.[17] That is, good 1 will have a positive tax rate if mimicking type 2's consume more of the taxed good than do the type 1's whom they are mimicking. This parallels the result in

[17] In the standard optimal tax literature, when consumption is not time consuming, the Atkinson-Stiglitz (1976) Theorem applies and weak separability is sufficient to ensure that $x_1^{21} = x_1^1$, so that, at the optimum, $t_1 = 0$.

Edwards *et al.* (1994), and the intuition is the same. Imposing a higher tax rate on a good that is consumed more by the mimicker will weaken the incentive constraint and make redistribution more efficient.

To investigate what determines the size of x_1^{21} relative to x_1^1, let $\sigma(x_1, x_2; L)$ denote the absolute value of the slope of an indifference curve in $(x_2, x_1)-$space:

$$\sigma(x_1, x_2; L) \equiv -\left.\frac{\partial x_2}{\partial x_1}\right|_{dU=0}.$$

Recall that the mimickers and type 1's receive the same after-tax income, but that the mimickers supply less labor (because both types earn the same before-tax incomes). Consequently, whether the mimickers or type 1's consume more of one good or the other, depends on how σ varies with L. Specifically, if $\partial\sigma/\partial L > 0$, mimickers will have flatter indifference curves than type 1's, consuming more x_2 and less x_1. On the other hand, if $\partial\sigma/\partial L < 0$, mimickers will have steeper indifference curves than type 1's, and consume less x_2 and more x_1. To sum, we have,

$$\frac{\partial\sigma}{\partial L} \gtreqless 0 \Rightarrow x_1^{21} \lesseqgtr x_1^1 \Rightarrow t_1 \lesseqgtr 0.$$

Now consider the case where the goods are L-substitutes. Assuming weak separability, utility may then be written, using Eq. (7), as $U(1 - Y, \underline{x}) = U(1 - L - a_1x_1 - a_2x_2, f(x_1, x_2))$. Differentiating this, we obtain the expression for σ corresponding to it,

$$\sigma(x_1, x_2; L) = \frac{U_f f_1 - a_1U_y}{U_f f_2 - a_2U_y}, \tag{35}$$

where $f_i = \partial f/\partial x_i$. Observe that, with a_1 and/or $a_2 \neq 0$, σ varies with L. Thus, despite the weak separability of preferences, type 1's and type 2's preferences over x_1 and x_2 depend on their labor supplies. Differentiating (35) with respect to L, one obtains

$$\frac{\partial\sigma}{\partial L} = \frac{U_{fy}U_y - U_{yy}U_f}{(U_f f_2 - a_2U_y)^2}(a_2f_1 - a_1f_2). \tag{36}$$

Assuming consumption goods are normal, the first expression on the right-hand side of Eq. (36) will be positive. The sign of $\partial\sigma/\partial L$ will then be the same as the sign of the second expression. The following result emerges,

$$\frac{a_1}{a_2} \gtreqless \frac{f_1}{f_2} \Rightarrow \frac{\partial \sigma}{\partial L} \lesseqgtr 0 \Rightarrow t_1 \gtreqless 0. \tag{37}$$

This relationship tells us that the tax on good 1 will be positive (negative) if the relative intensity with which x_1 uses time exceeds (falls short of) the rate at which x_1 can be substituted for x_2 in the sub-utility function. In this sense, a higher commodity tax should be imposed on the good that is most time-intensive as in the earlier sections, albeit for different reasons.

The same type of calculations establishes the relationship between tax rates on L- and r-substitutes (when there are only two goods and two types). Thus assume good 2 is an r-substitute. Then, although $a_2 \neq 0$, it does not appear in $U(y, \underline{x}) = U(1 - L - a_1 x_1, f(x_1, x_2))$. One can then find the expressions for σ and $\partial\sigma/\partial L$ by replacing a_2 with zero in Eqs (35)–(36). Moreover, it is plain that in this case, $a_1/a_2 \geq f_1/f_2$ so that from Eq. (37), $t_1 > 0$. This tell us that, as in the previous sections, the L-substitute good must be taxed at a higher rate than the r-substitute good.

Finally, observe that if both goods are r-substitutes, from Eq. (35), $\sigma = f_1/f_2$ and independent of y, and hence of labor supplies. We will then have $\partial\sigma/\partial L = 0$ so that $t_1 = t_2 = 0$, as argued for the general case.

4.3 Absence of pure leisure

Assume again that pure leisure does not enter preferences. It is again clear that as long as there exist any r-type goods, one cannot say much about the structure of commodity taxes. The interesting case is again when one also assumes that there are no r-substitutes. In this case, y disappears as an argument from the households' preferences. It also follows from the individual's time constraint (5) that $L^h = 1 - \sum_{i=1}^n a_i x_i^h$ for $h = 1, 2, \ldots, H$.

Now consider our earlier revelation mechanism (28)–(30). With $I^h = w^h L^h = w^h[1 - \sum_{i=1}^n a_i x_i^h]$ being determined through household h's choice of x_i^h $(i = 1, 2, \ldots, n)$, we no longer have to assign I^h to a household, only c^h and $\underline{q}$. Consequently, the optimum is characterized by a pooling solution where everyone is assigned the same amount of $c^h = c$. Given the same c and the same consumer prices, everyone will end up with the same consumption levels and the same labor supply. Moreover, everyone will pay a tax equal to $I^h - c$. These taxes are lump-sum (but differential). Consequently, the optimal tax policy for the government will be to levy no commodity taxes and attain a first-best outcome. Consequently, while we have a first-best outcome (unlike the heterogeneous Ramsey case), we have no commodity taxes (unlike the representative household case).

Proposition 3 summarizes the results of this Section.

Proposition 3 *Assume the economy is inhabited by households who differ in earning ability but have identical preferences. Income taxes are nonlinear, but commodity taxes are linear.*

(i) Assume further that preferences are weakly separable in leisure-equivalent activities and goods. Then,

- *Optimal commodity taxes on L- and r-substitutes are characterized by Eq. (32).*
- *If there are no L-substitute goods, differential commodity taxes are redundant.*

(ii) If pure leisure does not enter preferences and all goods are L-substitutes, the optimal policy will be first-best requiring no commodity taxes.

5 Conclusion

As Becker (1965) first emphasized, the consumption of goods is an activity that takes time, and consumer choice involves **allocating** both a goods' budget and a time budget. The benefits obtained from time-consuming activities are analogous to services obtained from the joint consumption of many goods as in Lancaster's (1966) characteristics approach to consumer demand. In principle, this should affect the design of optimal commodity taxes. Informational constraints preclude the direct taxation of the services of time-consuming household consumption activities, so for practical purposes, taxation must be based on the purchase of goods alone. Standard consumer theory has analyzed the optimal taxation of goods in a setting in which time taken in consumption is ignored. Instead, utility is based on labor time or leisure time along with goods alone, and time spent in consumption plays no role in the determination of optimal taxes.

Recent analyses by Gahvari and Yang (1993) and Kleven (2004) have indeed shown that the traditional optimal commodity tax rules can be violated once consumption time is taken into account. For example, uniformity of taxation when utility is separable in goods and leisure and homothetic in goods (the Atkinson and Stiglitz (1976) result) no longer applies. They argue that, in this case, goods that are more time-intensive should generally be taxed at higher rates. Gahvari (2006), however, has shown that this depends on the assumption that utility is separable in goods and leisure rather than goods and labor. In the latter case, standard results apply.

In this paper, we have argued that the substitutability relationships among consumption time, leisure and labor are critical in determining the optimal taxation of goods. This is seen most clearly by assuming that consumption time can either be a perfect substitute for leisure or for labor, and that time and goods are used in fixed proportions. In these circumstances, the standard optimal commodity tax rules would apply if all goods were r-substitutes, and this would be the case whether income taxes were linear or nonlinear. However, as soon as some goods are L-substitutes, optimal tax rules must take account of time spent consuming (and the fact that it is not taxed directly), and the manner the tax rules are affected depends on whether linear or nonlinear income taxes are available.

The implications of the analysis for the design of an optimal tax structure are in striking contrast to the standard results. Goods for which the time spent consuming them is unpleasant should be taxed at a higher rate than those for which it is pleasant.

Thus, goods that require household work, such as food and shelter, should be taxed at a higher rate on that account than those that involve leisurely time, such as consumer electronics and books. Moreover, the more time do L-substitutes take, the greater the need for the tax rate to be higher. It may well be that this consideration conflicts with standard optimal tax prescriptions that, for example, suggest taxing necessities at a lower rate than luxuries. It is conceivable that many necessities are L-substitutes and many luxury goods are r-substitutes.

The simple assumptions that we have built into our analysis are suggestive only and are not meant to reflect reality. In a more general analysis, consumption time could be an imperfect substitute for labor or leisure, and time and goods could be substitutable in consumption as in the original formulation of Becker (1965). Consumption activities could themselves involve more than one good. One such activity could involve household production activities in which time is presumably substitutable for work. The implications for optimal taxation would be more complicated than those we have been able to obtain here but would take us beyond the scope of the present paper.

Appendix

Proof of Eq. (16): The optimal tax problem is summarized by the Lagrangian

$$\mathcal{L} = v(\underline{\widetilde{q}}, \underline{q}, w) + \mu \left[\sum_{i=1}^{n} t_i x_i - \bar{R} \right],$$

where $\underline{\widetilde{q}} = (\widetilde{q}_1, \widetilde{q}_2, \ldots, \widetilde{q}_m)$ and $\underline{q} = (q_{m+1}, q_{m+2}, \ldots, q_n)$. The first-order conditions for this problem are,

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial t_j} &= \frac{\partial v}{\partial \widetilde{q}_j} + \mu \left[\sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial \widetilde{q}_j} + x_j \right] = 0, \quad j = 1, 2, \ldots, m, \\
\frac{\partial \mathcal{L}}{\partial t_s} &= \frac{\partial v}{\partial q_s} + \mu \left[\sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_s} + x_s \right] = 0, \quad s = m+1, \ldots, n.
\end{aligned}$$

Simplifying these equations, using Roy's identity, results in

$$\begin{aligned}
\sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial \widetilde{q}_j} &= -\frac{\mu - \alpha}{\mu} x_j, \quad j = 1, 2, \ldots, m, \\
\sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_s} &= -\frac{\mu - \alpha}{\mu} x_s, \quad s = m+1, \ldots, n,
\end{aligned}$$

where α is the representative household's marginal utility of income. These equations can be rewritten, using the Slutsky equation, as

$$\sum_{i=1}^{n} t_i \frac{\partial x_i^c}{\partial \widetilde{q}_j} = -\frac{\mu - \gamma}{\mu} x_j, \quad j = 1, 2, \ldots, m, \tag{A1}$$

$$\sum_{i=1}^{n} t_i \frac{\partial x_i^c}{\partial q_s} = -\frac{\mu - \gamma}{\mu} x_s, \quad s = m+1, \ldots, n, \tag{A2}$$

where x_i^c denotes the compensated demand for good i, and $\gamma \equiv \alpha/\mu + \sum_i t_i(\partial x_i/\partial M)$ is the net social marginal utility of income, with M denoting household's exogenous income.

Next, from the homogeneity of compensated demands (of degree zero in prices) and the symmetry of substitution terms, one has

$$\sum_{i=1}^{n} \tilde{q}_i \frac{\partial x_i^c}{\partial \widetilde{q}_j} + w \frac{\partial y^c}{\partial \widetilde{q}_j} = 0, \quad j = 1, 2, \ldots, m, \tag{A3}$$

$$\sum_{i=1}^{n} q_i \frac{\partial x_i^c}{\partial q_s} + w \frac{\partial y^c}{\partial q_s} = 0, \quad s = m+1, \ldots, n, \tag{A4}$$

where $y^c(.)$ is the compensated demand for y. When preferences are separable in y and $\underline{x}$, with the subutility in $\underline{x}$ being homothetic, we have, following Sandmo (1974),

$$\frac{\partial y^c}{\partial \widetilde{q}_j} = \eta x_j, \quad j = 1, 2, \ldots, m, \tag{A5}$$

$$\frac{\partial y^c}{\partial q_s} = \eta x_s, \quad s = m+1, \ldots, n, \tag{A6}$$

where η is independent of j and s. Substituting in Eqs. (A3)–(A4)

$$\sum_{i=1}^{n} \widetilde{q}_i \frac{\partial x_i^c}{\partial \widetilde{q}_j} = -\eta w x_j, \quad j = 1, 2, \ldots, m, \tag{A7}$$

$$\sum_{i=1}^{n} q_i \frac{\partial x_i^c}{\partial q_s} = -\eta w x_s, \quad s = m+1, \ldots, n. \tag{A8}$$

Eliminating x_j between Eqs. (A1) and (A7), and x_s between Eqs.(A2) and (A8), yields,

$$\sum_{i=1}^{n} \left[\frac{\mu t_i}{\mu - \gamma} - \frac{\widetilde{q}_i}{\eta w} \right] \frac{\partial x_i^c}{\partial \widetilde{q}_j} = 0, \qquad j = 1, 2, \ldots, m, \tag{A9}$$

$$\sum_{i=1}^{n} \left[\frac{\mu t_i}{\mu - \gamma} - \frac{q_i}{\eta w} \right] \frac{\partial x_i^c}{\partial q_s} = 0, \qquad s = m+1, \ldots, n. \tag{A10}$$

Assuming that the matrix $[\partial x_i^c / \partial \widetilde{q}_j, \partial x_i^c / \partial q_s]$ is non-singular, the solution to the system of equations (A9)–(A10) is characterized by

$$\frac{t_j}{\widetilde{q}_j} = \frac{1}{\eta w} \frac{\mu - \gamma}{\mu}, \qquad j = 1, 2, \ldots, m, \tag{A11}$$

$$\frac{t_s}{q_s} = \frac{1}{\eta w} \frac{\mu - \gamma}{\mu}, \qquad s = m+1, m+2, \ldots, n. \tag{A12}$$

where from Eqs. (A5)–(A6) and symmetry of the Slutsky matrix,

$$\begin{aligned} \eta w &= \frac{w}{x_j}\frac{\partial y^c}{\partial \tilde{q}_j} = \frac{w}{x_j}\frac{\partial x_j^c}{\partial w} = \epsilon, \\ \eta w &= \frac{w}{x_s}\frac{\partial y^c}{\partial q_s} = \frac{w}{x_s}\frac{\partial x_s^c}{\partial w} = \epsilon, \end{aligned}$$

where ϵ is the cross-price elasticity of compensated demand for y with respect to any one of the consumption goods (same for all goods). Using Eq. (A11)–(A12), one arrives at Eq. (16) in the text.

Proof of Eqs. (21a)–(21b): The optimal tax problem is summarized by the Lagrangian

$$\mathcal{L} = \sum_{h=1}^{H} \pi^h W\left(v^h(\cdot)\right) + \mu\left[\sum_{i=1}^{n} t_i \left(\sum_{h=1}^{H} \pi^h x_i^h\right) - M - \bar{R}\right],$$

where $v^h(\cdot) = v(\underline{\tilde{q}}^h, \underline{q}, w^h, M)$. The first-order conditions are, for $j = 1, 2, \ldots, m$ and $s = m+1, \ldots, n$,

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial t_j} &= \sum_{h=1}^{H} \pi^h W'\left(v^h(\cdot)\right)\frac{\partial v^h}{\partial \tilde{q}_j^h} + \mu\left[\sum_{h=1}^{H} \pi^h x_j^h + \sum_{i=1}^{n} t_i \left(\sum_{h=1}^{H} \pi^h \frac{\partial x_i^h}{\partial \tilde{q}_j^h}\right)\right] = 0, \\ \frac{\partial \mathcal{L}}{\partial t_s} &= \sum_{h=1}^{H} \pi^h W'\left(v^h(\cdot)\right)\frac{\partial v^h}{\partial q_s} + \mu\left[\sum_{h=1}^{H} \pi^h x_s^h + \sum_{i=1}^{n} t_i \left(\sum_{h=1}^{H} \pi^h \frac{\partial x_i^h}{\partial q_s}\right)\right] = 0, \\ \frac{\partial \mathcal{L}}{\partial M} &= \sum_{h=1}^{H} \pi^h W'\left(v^h(\cdot)\right)\frac{\partial v^h}{\partial M} + \mu\left[\sum_{i=1}^{n} t_i \left(\sum_{h=1}^{H} \pi^h \frac{\partial x_i^h}{\partial M}\right) - 1\right] = 0. \end{aligned}$$

Manipulation of these equations, using Roy's identity, yields

$$\sum_h (\mu - \beta^h)\pi^h x_j^h + \mu \sum_i t_i \left(\sum_h \pi^h \frac{\partial x_i^h}{\partial \tilde{q}_j^h}\right) = 0, \tag{A13}$$

$$\sum_h (\mu - \beta^h)\pi^h x_s^h + \mu \sum_i t_i \left(\sum_h \pi^h \frac{\partial x_i^h}{\partial q_s}\right) = 0, \tag{A14}$$

$$\sum_h \pi^h \left[\beta^h + \mu \sum_i t_i \frac{\partial x_i^h}{\partial M}\right] = \mu. \tag{A15}$$

Slutsky decomposition of $\partial x_i^h/\partial \widetilde{q}_j^h$ and $\partial x_i^h/\partial q_s$ terms in Eqs. (A13)–(A14), denoting the elements of the associated Slutsky matrix by S_{ij}^h and S_{is}^h, allows one to simplify Eqs. (A13)–(A14) further and rewrite Eqs. (A13)–(A15) as

$$\begin{aligned}\sum_h(\mu-\gamma^h)\pi^h x_j^h+\mu\sum_h\sum_i t_i\pi^h S_{ij}^h &= 0, && \text{(A16)}\\ \sum_h(\mu-\gamma^h)\pi^h x_s^h+\mu\sum_h\sum_i t_i\pi^h S_{is}^h &= 0, && \text{(A17)}\\ \bar{\gamma}\equiv\sum_h\pi^h\gamma^h &= \mu. && \text{(A18)}\end{aligned}$$

The system of equations (A16) hold with no restrictions imposed on $U(y,\underline{x})$. Now given the quasi-linearity and additivity assumptions, $S_{ij}^h = \partial x_i^h/\partial \widetilde{q}_j^h = 0$ whenever $i\neq j$ and $S_{is}^h=\partial x_i^h/\partial q_s=0$ whenever $i\neq s$. Substituting these values in Eqs. (A16)–(A16), rewriting the resulting equations in terms of elasticities and simplifying we have Eqs. (21a)–(21b) in the text.

Proof of Eqs. (22a)–(22b): From the properties of the Slutsky matrix,

$$\begin{aligned}\sum_i(q_i+a_iw^h)S_{ij}^h+w^hS_{yj}^h &= 0,\ \ j=1,2,\ldots,m, && \text{(A19)}\\ \sum_i q_iS_{is}^h+w^hS_{ys}^h &= 0,\ \ s=m+1,\ldots,n. && \text{(A20)}\end{aligned}$$

As with Eqs. (A5)–(A6), following Sandmo (1974), one can write

$$\begin{aligned}S_{yj}^h &= \eta^h x_j^h,\ \ j=1,2,\ldots,m, && \text{(A21)}\\ S_{ys}^h &= \eta^h x_s^h,\ \ s=m+1,m+2,\ldots,n, && \text{(A22)}\end{aligned}$$

where η^h is independent of j and s. Substituting from Eqs. (A21)–(A22) into Eqs. (A19)–(A20) then results in

$$\begin{aligned}\sum_i(q_i+a_iw^h)S_{ij}^h &= -\eta^hw^hx_j^h,\ \ j=1,2,\ldots,m, && \text{(A23)}\\ \sum_i q_iS_{is}^h &= -\eta^hw^hx_s^h,\ \ s=m+1,\ldots,n. && \text{(A24)}\end{aligned}$$

Recall that conditions (A16)–(A17) were derived without making any simplifying assumptions on the preferences $U(y, \underline{x})$. Thus substitute from Eqs. (A23)–(A24) into Eqs. (A16)–(A17) and simplify to arrive at the following system of n equations that will hold only under the weak separability assumption,

$$\sum_h \sum_i \left[t_i - \frac{\mu - \gamma^h}{\mu\, \eta^h w^h}(q_i + a_i w^h)\right] \pi^h S^h_{ij} \quad = 0 \quad , \quad j = 1, 2, \ldots, m,$$

$$\sum_h \sum_i \left[t_i - \frac{\mu - \gamma^h}{\mu\, \eta^h w^h} q_i\right] \pi^h S^h_{is} \quad = 0 \quad , \quad s = m+1, m+2, \ldots, n.$$

Now, weak separability and homotheticity in goods assumption imply that $S^h_{ij} = (w^h L^h) S_{ij}$ and $S^h_{is} = (w^h L^h) S_{is}$. Assuming that $[S_{ij}, S_{is}]$ matrix is non-singular, the solution to above system of equations is given by

$$t_j \quad = \quad \sum_h \pi^h \frac{\mu - \gamma^h}{\mu\, \eta^h w^h}(q_j + a_j w^h), \;\; j = 1, 2, \ldots, m, \tag{A25}$$

$$t_s \quad = \quad \sum_h \pi^h \frac{\mu - \gamma^h}{\mu\, \eta^h w^h} q_s, \;\; s = m+1, \ldots, n. \tag{A26}$$

Moreover, from Eqs. (A21)–(A22), we have for $j = 1, 2, \ldots, m$ and $s = m+1, \ldots, n$,

$$\eta^h w^h = \frac{S^h_{yj}}{x^h_j}\, w^h = \frac{S^h_{ys}}{x^h_s}\, w^h = \epsilon^h,$$

where ϵ^h is type h's cross-price elasticity of compensated demand for y with respect to any one of the consumption gods (same for all goods). Substituting for η^h from this relationship, and for μ from Eq. (A18), into Eqs. (A25)–(A26) and rearranging, one arrives at Eqs. (22a)–(22b) in the text.

Derivation of Eq. (32): Summarize the problem by the Lagrangian

$$\mathcal{L} = \sum_h \delta^h v^h(\underline{q}, c^h, I^h) + \mu \Big\{ \sum_h \pi^h \big[(I^h - c^h) + \sum_{i=1}^{n-1} (q_i - 1) x^h_i\big] - \bar{R} \Big\} + \sum_h \sum_{k \neq h} \lambda^{hk} (v^h - v^{hk}). \tag{A27}$$

Rearranging the terms, one may usefully rewrite the Lagrangian expression as

$$\begin{aligned}\mathcal{L} &= \sum_h \big(\delta^h + \sum_{k\neq h} \lambda^{hk}\big) v^h - +\mu\big\{ \sum_h \pi^h \big[(I^h - c^h) \\ &+ \sum_{i=1}^{n-1} (q_i - 1) x_i^h\big] - \bar{R}\big\} - \sum_h \sum_{k\neq h} \lambda^{hk} v^{hk}. \end{aligned} \tag{A28}$$

The first-order conditions are, for all $t = 1, 2, \ldots, n-1$ and $h, k = 1, 2, \ldots, H$,

$$\frac{\partial \mathcal{L}}{\partial I^h} = \big(\delta^h + \sum_{k\neq h} \lambda^{hk}\big) v_I^h + \mu\pi^h\big[1 + \sum_{i=1}^{n-1} (q_i - 1)\frac{\partial x_i^h}{\partial I^h}\big] - \sum_{k\neq h} \lambda^{kh} v_I^{kh} = 0, \tag{A29}$$

$$\frac{\partial \mathcal{L}}{\partial c^h} = \big(\delta^h + \sum_{k\neq h} \lambda^{hk}\big) v_c^h + \mu\pi^h\big[-1 + \sum_{i=1}^{n-1} (q_i - 1)\frac{\partial x_i^h}{\partial c^h}\big] - \sum_{k\neq h} \lambda^{kh} v_c^{kh} = 0, \tag{A30}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial q_t} &= \sum_h \big(\delta^h + \sum_{k\neq h} \lambda^{hk}\big) v_t^h + \mu \sum_h \pi^h\big[\sum_{1=1}^{n-1} (q_i - 1)\frac{\partial x_i^h}{\partial q_t} + x_t^h\big] \\ &- \sum_h \sum_{k\neq h} \lambda^{hk} v_t^{hk} = 0. \end{aligned} \tag{A31}$$

Multiply Eq. (A30) by x_t^h, sum over h, add the resulting equation to Eq. (A31), and simplify. We will have the following system of equations for $i = 1, 2, \ldots, n-1$,

$$\begin{aligned}\sum_h \big(\delta^h + \sum_{k\neq h} \lambda^{hk}\big)\big(v_t^h + x_t^h v_c^h\big) &+ \mu \sum_h \pi^h\big[\sum_{i=1}^{n-1} (q_i - 1)\big(\frac{\partial x_i^h}{\partial q_t} + x_t^h \frac{\partial x_i^h}{\partial c^h}\big)\big] \\ &- \sum_h \sum_{k\neq h} \lambda^{kh}\big(v_t^{kh} + x_t^h v_c^{kh}\big) = 0. \end{aligned} \tag{A32}$$

Next, make use of Roy's identity to set, for all $t = 1, 2, \ldots, n$, and $h, k = 1, 2, \ldots, H$,

$$v_t^h + x_t^h v_c^h = 0, \tag{A33}$$

$$v_t^{kh} + x_t^{kh} v_c^{kh} = 0. \tag{A34}$$

Then use the Slutsky equation to write, for all $i, t = 1, 2, \ldots, n$, and $h = 1, 2, \ldots, H$,

$$\frac{\partial x_i^h}{\partial q_t} = \frac{\partial \hat{x}_i^h}{\partial q_t} - x_t^h \frac{\partial x_i^h}{\partial c^h}. \tag{A35}$$

Substituting from (A33)–(A34) and (A35) in (A32), making use of the symmetry of the Slutsky matrix, setting $q_i - 1 = t_i$, upon further simplification and rearrangement, one arrives at

$$\sum_{i=1}^{n-1} \Big(\sum_h \pi^h \frac{\partial \hat{x}_t^h}{\partial q_i} \Big) t_i = -\sum_h \sum_{k \neq h} \lambda^{kh} \big(x_t^{kh} - x_t^h \big) \frac{v_c^{kh}}{\mu}, \; t = 1, 2, \ldots, n-1. \tag{A36}$$

Equation (A36) is one way of characterizing the optimal commodity tax rates: t_i's.

To arrive at Eq. (32), use the definition of $\boldsymbol{A}$ in Eq. (31) to write out the system of equations (A36) in matrix notation:

$$\boldsymbol{A} \begin{pmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n-1} \end{pmatrix} = \begin{pmatrix} -\sum_h \sum_{k \neq h} \lambda^{kh} \big(x_1^{kh} - x_1^h \big) \frac{v_c^{kh}}{\mu} \\ -\sum_h \sum_{k \neq h} \lambda^{kh} \big(x_2^{kh} - x_2^h \big) \frac{v_c^{kh}}{\mu} \\ \vdots \\ -\sum_h \sum_{k \neq h} \lambda^{kh} \big(x_{n-1}^{kh} - x_{n-1}^h \big) \frac{v_c^{kh}}{\mu} \end{pmatrix}. \tag{A37}$$

Premultiplying Eq. (A37) by $\boldsymbol{A^{-1}}$ then yields the system of equations (32) in the text.

Characterization of $\underline{x}^h$ and $\underline{x}^{kh}$: With the first m goods being L-substitutes and the remaining $n-m$ goods r-substitutes, the first-order conditions of the problem (27a)–(27b) are simplified as, for $j = 1, 2, \ldots, m$, and $s = m+1, \ldots, n-1$,

$$-a_j \frac{U_y}{U_n} + \frac{U_j}{U_n} = \frac{q_j}{q_n} = q_j, \tag{A38}$$

$$\frac{U_s}{U_n} = \frac{q_s}{q_n} = q_s, \tag{A39}$$

$$\sum_{i=1}^{n} q_i x_i = c^h, \tag{A40}$$

where $q_n \equiv 1$. The solution to the above equations will be $\underline{x}^h$ if the arguments of U_y, U_n, U_j and U_s are $(1 - I^h/w^h - \sum_{i=1}^m a_i x_i^h, \underline{x}^h)$ and $x_i = x_i^h$. Similarly, we will have $\underline{x}^{kh}$ as the solution if the arguments of U_y, U_n, U_j and U_s are $(1 - I^h/w^k - \sum_{i=1}^m a_i x_i^{kh}, \underline{x}^{kh})$ and $x_i = x_i^{kh}$.

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