On the marginal cost of public funds and the optimal provision of public goods

Firouz Gahvari*

Department of Economics
University of Illinois at Urbana-Champaign
Urbana, IL 61801, USA

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Abstract

This paper argues that in models with heterogeneous agents, the concept of the marginal cost of public funds ($MCPF$) will only be useful if it is compared with an analogous concept for the benefit side. The $MCPF$ does not assume a unique value and is not particularly illuminating in and out of itself. Also gone is the benchmark status of $MCPF = 1$. Turning to the provision of public goods, using a mechanism design approach, the paper constructs a two-stage proof for Kaplow's (1996) proposition concerning the "irrelevance" of labor supply and distributional concerns in public good provision. This highlights the two fundamental ingredients for his result. First, the provision of public goods per se, when it satisfies the Samuelson's rule, is only *potentially* Pareto-improving. Second, the actual Pareto improvement will materialize when, or if, one reforms the income tax structure. If the reform is not forthcoming, the decision on public goods provision must rely on redistributional concerns. Finally, the paper generalizes Boadway and Keen's (1993) result to a model with many types of agents, many private goods, and without making any assumptions regarding which self-selection constraints are or are not binding.

JEL classification: H21; H41.

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1 Introduction

The concept of the marginal cost of public funds ($MCPF$) has been studied at length in the literature, both in its own right, as well as in the role it plays in decisions regarding the provision of public goods.[1] Recent contributions have centered around two issues. The first generalizes the concept of the $MCPF$ to take account of the government's redistributive concerns [see, e.g., Dahlby (1998), Sandmo (1998) and Slemrod and Yitzhaki (2001)]. The second studies the question of the optimal provision of public goods in the presence of a *general* income tax.[2] The central contribution here is Christiansen (1981) and Boadway and Keen (1993).[3] They have proved that, distortionary taxation notwithstanding, the optimal provision of public goods continues to be characterized by the simple Samuelson (1954) rule of equality between the marginal rate of transformation and the sum of individuals' marginal rates of substitution between public and private goods (as long as the government levies an optimal general income tax, and that preferences are separable in labor supply and other goods, including the public good). Kaplow (1996, 2004) has extended this result to circumstances under which neither the tax system and nor the provision of public goods are optimal. He argues that, in a benchmark case in which a distribution neutral income tax adjustment is employed, labor supply and distributional concerns should play no role in decisions regarding the provision of a public good. Instead, the decision must rest solely on a simple benefit cost rule that compares the marginal rate of transformation and the sum of individuals' marginal rates of substitution between public and private goods.

[1]The early literature has been cast within the confine of a representative-consumer economy with linear taxes. A selective sample includes Atkinson and Stern (1974), Hansson and Stuart (1985), Browning (1987), Gahvari (1990, 1995), Mayshar (1990), Fullerton (1991) and Ballard and Fullerton (1992).

[2]A third recent application of the $MCPF$ concept is found in the literature on optimal environmental taxation in the presence of distortionary taxes. The value of the $MCPF$ has assumed a central role in the discussions on the size of the optimal environmental tax relative to the Pigouvian tax. On this see, among others, Sandmo (1975), Bovenberg and de Mooij (1994), Kaplow (1996, 2004), Fullerton (1997), and Cremer, Gahvari and Ladoux (2001).

[3]See also Tuomala (1990).

Kaplow's paper has created a lot of controversy as it seems to fly in the face of previous contributions in this area, particularly the seminal paper by Atkinson and Stern (1974). Two recent examples of this are Auerbach and Hines (2002) who write in support of Kaplow's result and Slemrod and Yitzhaki (2001) who seem to reject it.

This paper assesses the usefulness of the concept of the $MCPF$ in models with heterogeneous agents within a second-best framework à la Mirrlees (1971).[4] Second, I use a mechanism design approach to present an alternative demonstartion and explanation of Kaplow's claim. In this way, I am able to emphasize the two fundamental ingredients of his argument.[5] This is important because the key to resolving the controversy surrounding Kaplow's rule lies in the roles that the two ingredients play in attaining his claim. The first ingredient calls for provision of public goods according to Samuelson's rule, and the second for an adjustment in the income tax schedule. This second aspect is crucial. It is this feature that allows a potential Pareto improvement to be realized. The argument is that one must follow the simple benefit cost rule *provided that* a concomitant adjustment occurs in the tax system. If the adjustment does not take place, the benefit cost rule will no longer be free of distributional concerns. It appears that many authors have interpreted Kaplow as favoring the simple benefit cost rule even in

[4]Christiansen (1999) also studies the concept of the $MCPF$ in Mirrlees's framework. Dahlby (1998) develops a formula for the $MCPF$ when different taxpayers are in different income tax brackets. However, most of the contributors to this literature have remained within the Ramsey framework with linear taxes.

[5]The mechanism design approach adopted here is particularly suited for this purpose. The existing proofs in the literature, working directly with an existing tax function, do not adequately highlight the relevance of the two ingredients for the attainment of the result. Kaplow's exposition of his proof proceeds as follows. Assume that the government increases the supply of public goods while simultaneously changing all individuals' income taxes in such a way as to keep their net utility constant *on the assumption that their labor supplies do not change.* He then argues that "for each ability level (w), each choice of l [labor supply] produces the same utility when g [public goods] is raised and T [the income tax] is adjusted in an offsetting manner as it does when g is not raised. Because individuals' opportunity sets are thus unaffected, they will choose the same l" (1996, p. 531). In Auerbach and Hines's (2002, pp 1390–1391) proof, the income tax is conditioned on the public goods provision. They then show, as initially suggested by Kaplow (1996, note 4, p. 533), that the individuals' first-order condition for utility maximization remains satisfied for the initial levels of labor supplies and utilities, as the public goods supply and consequently the individuals' tax payments change.

the absence of the tax adjustment. Second, I generalize Boadway and Keen's original framework by considering a model with many types of agents and many private goods.[6] Their demonstration rested on a two-group model with one private good.[7] It was also based on a tax regime in which the downward self-selection constraint was assumed to be binding. I make no assumptions in this regard; plainly, one does not in general know which self-selection constraints are binding. Nor, beyond the two-group model, will it be efficient or indeed feasible to list all combinations of the self-selection constraints that may be binding, and to study the properties of each and every corresponding solution.

Third, I will discuss the usefulness of the recent attempts to generalize the concept of the $MCPF$ to economies with heterogeneous agents. The purpose of $MCPF$ is to serve as a number which transforms the money cost of a project into its welfare cost, in order to make it comparable with its welfare benefit. However, with heterogeneous agents, one cannot derive a unique number for the $MCPF$. Assigning different welfare weights to different agents results in different values for the $MCPF$. Moreover, each set of welfare weights also produces a different valuation for the social benefit of the same public project. One has to apply identical welfare weights to both sides; the two cannot be studied independently of one another. Also gone will be the benchmark status of $MCPF = 1$. In representative agent models, $MCPF \gtrless 1$ implies under- or over-provision of the public good at the second best—at least in terms of "rule" if not "level"; see Atkinson and Stern (1974). This is no longer the case with heterogeneous

[6]Boadway and Keen (1993), as well as Christiansen (1981), may be seen as a natural basis for what one may call Kaplow's *message.* The message, as opposed to his specific result, is that in deciding on the provision of public goods, one should follow the simple Samuelson's rule *provided that* one can reform the income tax system. But this follows from Boadway and Keen (even though their proof requires the income to be optimal). *If* one is able to reform the income tax system with no restrictions, then there always exists at least one tax reform which is Pareto improving and requires that one follows the Samuelson's rule. This is of course the optimal income tax system! On the other hand, if there are restrictions on the way that one can reform the income tax system, one may not be able to change the income tax system in the manner required by Kaplow (1996) either. Of course, Kaplow's result is more general and his approach is different. He shows that there are *many* income tax structures (and not just the optimal one) which are consistent with following the Samuelson's rule.

[7]They did nevertheless observe that their results will hold in a more general setting.

agents.

Finally, I will argue that while Kaplow's proposition is analytically correct, it is of limited application. As Kaplow (1996, 2004) recognized and discussed, the change in the income tax structure required for the achievement of a Pareto-improving outcome is rarely forthcoming.

2 Preliminaries: A one-consumer economy

Define the $MCPF$ to denote the social cost (measured in dollars) of spending one extra dollar on any given public good, G. Similarly, denote the marginal benefit of spending one extra dollar on the publicly-provided good by $MBPG$; see Slemrod and Yitzhaki (2001). It is almost a truism that one should increase (lower) expenditures on the publicly-provided good as long as the $MBPG$ exceeds (falls short of) the $MCPF$, with the optimal provision being characterized by $MBPG = MCPF$.

Let μ denote the cost to the society, measured in "utils", of raising \$1 in tax revenue; call this "the shadow price of public funds". Denote the marginal utility of private income by α; this measures the "private" cost, again measured in "utils", of having \$1 more to spend. The $MCPF$ is equal to μ/α.

To be more specific, assume the economy is inhabited by identical individuals whose preferences depend on n private goods, x_i's $(i = 1, 2, \ldots, n)$, one public good, G, and labor supply, L. All non-labor goods are produced by a linear technology with the producer prices of private goods being normalized to one. Denote the producer price of the public good by p_G, the gross-of-tax-wage by w, the net-of-tax-wage by w_n, the consumer price of x_i by q_i (with $\underline{q}$ as the vector of consumer prices), the tax rate on x_i by t_i, the tax rate on labor supply by t_w, the lump-sum tax (if any) by T, and the exogenous income (if any) by M. Further, denote the representative agent's indirect utility function by $v = v(\underline{q}, w_n, M - T, G)$, and the government's *net* tax revenues (taxes minus expenditures on G) by R so that net expenditures (the negative of net tax revenues) are equal to $-R = p_G G - \sum_i t_i x_i - t_w w L - T$. Raising one extra dollar by adjusting t (where t stands for t_i, t_w or T) results in a $MCPF$ equal to

$$MCPF = \frac{\mu}{\alpha} = \frac{1}{\alpha}\left(\frac{\partial v/\partial t}{-\partial R/\partial t}\right). \tag{1}$$

Similarly, spending one extra dollar on the provision of public goods results in

$$MBPG = \frac{1}{\alpha}\left(\frac{\partial v/\partial G}{-\partial R/\partial G}\right). \tag{2}$$

2.1 Tax normalization, $MCPF$ and $MBPG$

If the tax instrument employed is a lump-sum tax, assuming such taxes are available, then $\mu = \alpha$. This implies that $MCPF = 1$, and , on the benefit side, $MBPG = v_G/p_G$ (where $v_G = \partial v/\partial G$). In the absence of lump-sum taxation and exogenous income, the government has an extra degree of freedom in setting its tax instruments. Consequently, one can always set (without any loss of generality) one of the tax instruments $t_1, t_2, \ldots, t_n$ or t_w equal to zero. With individuals being endowed in time and not in goods, it is important to distinguish between two tax normalization possibilities: Setting the wage tax, or setting one of the consumption good taxes, say t_1, equal to zero.

(i) $t_w = 0$. In this case, assuming the extra tax revenue is raised from increasing t_j, one obtains from equations (1) and (2) that

$$\begin{aligned} MCPF_j &= \frac{1}{1 + \sum_{i=1}^n \frac{t_i}{x_j}\frac{\partial x_i}{\partial q_j}}, \\ MBPG_j &= \frac{1}{1 - \sum_{i=1}^n \frac{t_i}{p_G}\frac{\partial x_i}{\partial G}}\,\frac{v_G/\alpha}{p_G}. \end{aligned}$$

(ii) $t_1 = 0$. In this case, assuming the extra revenue is raised from the wage tax, equations (1) and (2) yield

$$\begin{aligned} MCPF_w &= \frac{1}{1 - \sum_{i=2}^n \frac{t_i}{L}\frac{\partial x_i}{\partial w_n} - \frac{t_w w}{L}\frac{\partial L}{\partial w_n}}, \\ MBPG_w &= \frac{1}{1 - \sum_{i=2}^n \frac{t_i}{p_G}\frac{\partial x_i}{\partial G} - \frac{t_w w}{p_G}\frac{\partial L}{\partial G}}\,\frac{v_G/\alpha}{p_G}. \end{aligned}$$

Observe that the values of the $MCPF$ and the $MBPG$ depend on the "tax normalization". To gain intuition, consider two identical tax regimes: one consisting of a wage tax and the other a uniform consumption tax. A consumer can buy less with one dollar under the consumption tax as compared to the wage tax. This indicates that the value of the marginal utility of income, α, depends on the tax normalization. By contrast the value of the shadow cost of public funds, μ, is independent of the tax normalization: The relevant prices to the government are always the tax-exclusive producer prices. In consequence, the $MCPF$ depends on which good goes "tax free".[8] Similarly, the $MBPG$ is also dependent on the tax normalization through α.

That the $MCPF$ depends on the tax normalization rule, however, does not affect the uses to which this concept has been put in a one-consumer economy. First, the tax normalization affects the benefit side as well; changing the $MBPG$ by the same factor as the $MCPF$ (both due to α.) The choice of a project is thus not affected. Secondly, the result that $MCPF \gtrless 1$ implies under- or over-provision of the public good at the second best (in terms of "rule") does not depend on the tax normalization.[9] To see this,

[8] Let subscripts c and w under a variable denote that variable's value under equivalent consumption and wage tax systems. Assume $t_1 = t_2 = \ldots = t_n = t_c$ and that $t_w = t_c/(1 + t_c)$ so that the consumption tax and the wage tax systems are identical. One can easily show that $\alpha_c = \alpha_w/(1 + t_c)$ and $MCPF_c = (1 + t_c)MCPF_w$; see Cremer, Gahvari and Ladoux (2001), Gahvari (1995, 2002) and Williams (2001).

[9] In this context, it is important to distinguish between $MCPF > 1$ and the positive marginal excess burden of distortionary taxes. Specifically, let MEB denote the "marginal excess burden of taxation due to the last dollar raised". Then, in general, $MCPF \neq 1 + MEB$. The reason is that, as Auerbach and Hines (2002) have pointed out, these two concepts describe two entirely different thought experiments. The $MCPF$ measures the welfare cost of raising one extra dollar to be spent on public goods provision. This cost is to be "balanced" by the benefit of public provision. In the case of MEB, the thought experiment is one of substituting a (hypothetical) lump-sum for distortionary taxes. The MEB measures the gain due to a "tax reform" where one switches, at the margin, from distortionary taxation into a lump-sum tax. *If the existing second-best tax structure is optimal,* the MEB must necessarily be positive. However, this does not mean that $MCPF$ is necessarily greater than one. Essentially, the difference is due to the substitution of lump-sum for distortionary taxes in the "excess burden" exercise; the rebated dollar will also be spent on taxed goods generating extra tax revenues.

Ballard and Fullerton (1992) present a clear description of these two thought experiments in terms of two different "traditions". The conceptual exercise associated with the $MCPF$ is, in their terminology, the "Stiglitz-Dasgupta-Atkinson-Stern" approach; the "Pigou-Harberger-Browning" tradition is observe that at the (second-best) optimum $MCPF = MBPG$ so that from (1)–(2), and regardless of the tax normalization rule,

$$\frac{v_G}{\alpha} = MCPF \left(-\frac{\partial R}{\partial G} \right).$$

Now assuming that preferences are weakly separable in G and other goods, $-\partial R/\partial G = p_G$, and the above simplifies to

$$\frac{v_G}{\alpha} = MCPF \times p_G,$$

with $MCPF \gtreqless 1 \Leftrightarrow v_G/\alpha \gtreqless p_G$.[10]

3 Second-best à la Mirrlees

Assume now that individuals differ in earning ability with an individual of type h earning w^h $(h = 1, 2, \ldots, H)$. Tax instruments are limited only to the extent that the required information for their imposition may not be publicly available. Specifically, assume, as is standard in the optimal tax literature à la Mirrlees (1971), that an individual's type (i.e. earning ability) and labor input, L, are not observable by the government. His before-tax income, $I = wL$, on the other hand, is. It is then convenient to introduce a

the thought experiment associated with the MEB. Nevertheless, they use the "$MCPF$" terminology to refer to both approaches. In their terminology, $MCPF$ and MEB are indistinguishable according to the thought experiments and, by definition, $MCPF = 1 + MEB$. One of the two $MCPF$ and MEB terms then becomes redundant; see Fullerton (1991). Thus there are two thought experiment (or "approaches"), *but* one terminology. This is somewhat confusing. It seems more appropriate to keep both $MCPF$ and MEB terminologies; each referring to one thought experiment.

[10]Similarly, and contrary to what Auerbach and Hines (2002, p. 1388) appear to be arguing, whether in imposing second-best environmental taxes one should "undercorrect" for the externality or not, does not depend on the tax normalization rule. In their Cobb-Douglas example, the optimal environmental tax (the difference between the tax on the dirty good and the tax on the clean good), can be shown to be equal to $-Hv_E/\lambda$ for both wage and consumption tax systems (where, in their notation, λ is the marginal utility of income, H the number of identical individuals, and $-v_E$ the marginal disutility of the externality). Now it is true that λ, the private marginal utility of income, takes different values under the two tax systems. However, the optimal environmental tax *relative to the price of clean goods,* is the same under both tax systems and equal to $-Hv_E/\mu$, where μ is, as in here, the shadow price of public funds.

type-specific utility function describing preferences over the observables according to

$$\boldsymbol{u^h}(\underline{x}, I, G) \equiv U\big(\underline{x}, \frac{I}{w^h}, G\big). \tag{3}$$

In characterizing the optimal tax rates, I follow the standard equivalent problem of determining the optimal allocations subject to resource and incentive compatibility constraints. Thus denote the utility level of a h-type individual by u^h when he chooses the allocation intended for him, and by u^{hk} when he chooses a k-type person's bundle. That is, for all $h \neq k = 1, 2, \ldots, H$,

$$u^h = \boldsymbol{u^h}(\underline{x}^h, I^h, G), \tag{4a}$$

$$u^{hk} = \boldsymbol{u^h}(\underline{x}^k, I^k, G). \tag{4b}$$

3.1 Optimal tax-cum-public-good-provision

Let π^h denote the proportion of people of type h in the economy, and $W(.)$ represent a concave function. The government's objective is one of maximizing a general social welfare function[11]

$$\Omega = \sum_{h=1}^{H} \pi^h W\left(u^h\right), \tag{5}$$

with respect to $\underline{x}^h, I^h$ and G, subject to the resource constraint

$$\mathcal{R} \equiv \sum_{h=1}^{H} \pi^h \big(I^h - \sum_i x_i^h\big) - p_G G \geq 0, \tag{6}$$

and the self-selection constraints

$$u^h \geq u^{hk}, \quad h \neq k;\ h, k = 1, 2, \ldots, H. \tag{7}$$

[11]This procedure assumes that the personal consumption levels are publicly observable so that the government can levy nonlinear consumption taxes. This may not be a good assumption for most consumer goods. If this is the case, one must impose the linearity of consumption taxes as an additional constraint on the problem. On this, see Cremer and Gahvari (1997). Observe, however, that the proof of Kaplow's proposition, and the extension of Boadway and Keen's result, are based on weak separability of preferences in labor and other goods. This assumption makes commodity taxes redundant whether they are linear or nonlinear.

The Lagrangian expression, $\mathcal{L}$, for the government's problem is

$$\mathcal{L} = \sum_h \pi^h W(u^h) + \sum_h \sum_{k \neq h} \lambda^{hk}(u^h - u^{hk}) + \mu \sum_h \pi^h \big[I^h - \sum_i x_i^h - p_G G \big]. \tag{8}$$

The first-order conditions are, for all $h, k = 1, 2, \ldots, H$, and $j = 1, 2, \ldots, n$,

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_j^h} &= \pi^h W'(u^h) u_j^h - \mu \pi^h + \sum_{k \neq h} \lambda^{hk} u_j^h - \sum_{k \neq h} \lambda^{kh} u_j^{kh} = 0, && \text{(9a)} \\
\frac{\partial \mathcal{L}}{\partial I^h} &= \pi^h W'(u^h) u_I^h + \mu \pi^h + \sum_{k \neq h} \lambda^{hk} u_I^h - \sum_{k \neq h} \lambda^{kh} u_I^{kh} = 0, && \text{(9b)} \\
\frac{\partial \mathcal{L}}{\partial G} &= \sum_h \pi^h W'(u^h) u_G^h - \mu p_G + \sum_h \sum_{k \neq h} \lambda^{hk} u_G^h - \sum_h \sum_{k \neq h} \lambda^{kh} u_G^{kh} = 0, && \text{(9c)}
\end{aligned}$$

where the subscripts j, I and G on u denote its partial derivatives with respect to x_j, I and G. Optimal allocations are found from equations (9a), (9b) and (9c).

Let $\beta^h \equiv W'(v^h)\alpha^h$ denote individual h's social marginal utility of income, and α^h his private marginal utility of income. To derive the rule for optimal provision of public goods, rewrite the first-order conditions (9a)–(9c) as

$$\begin{aligned}
\mu &= \beta^h \frac{u_j^h}{\alpha^h} + \frac{1}{\pi^h} \sum_{k \neq h} \left(\lambda^{hk} u_j^h - \lambda^{kh} u_j^{kh} \right), && \text{(10a)} \\
&= \beta^h \frac{u_I^h}{\alpha^h} + \frac{1}{\pi^h} \sum_{k \neq h} \left(\lambda^{hk} u_I^h - \lambda^{kh} u_I^{kh} \right), && \text{(10b)} \\
&= \frac{1}{p_G} \sum_h \pi^h \beta^h \frac{u_G^h}{\alpha^h} + \frac{1}{p_G} \sum_h \sum_{k \neq h} \left(\lambda^{hk} u_G^h - \lambda^{kh} u_G^{kh} \right). && \text{(10c)}
\end{aligned}$$

Next, multiply equation (10a) through by $\pi^h u_G^h / u_j^h$ and sum over h to get

$$\mu \sum_h \pi^h \frac{u_G^h}{u_j^h} = \sum_h \pi^h \beta^h \frac{u_G^h}{\alpha^h} + \sum_h \sum_{k \neq h} \lambda^{hk} u_G^h - \sum_h \sum_{k \neq h} \lambda^{kh} u_j^{kh} \frac{u_G^h}{u_j^h}. \tag{11}$$

Substituting for $\sum_h \pi^h \beta^h u_G^h/\alpha^h + \sum_h \sum_{k\neq h} \lambda^{hk} u_G^h$ from (11) in (10c), one gets

$$\begin{aligned}\mu &= \frac{1}{p_G}\left[\mu \sum_h \pi^h \frac{u_G^h}{u_j^h} + \sum_h \sum_{k\neq h} \lambda^{kh} u_j^{kh} \frac{u_G^h}{u_j^h} - \sum_h \sum_{k\neq h} \lambda^{kh} u_G^{kh}\right] \\ &= \frac{\mu}{p_G} \sum_h \pi^h \frac{u_G^h}{u_j^h} + \frac{1}{p_G} \sum_h \sum_{k\neq h} \lambda^{kh} u_j^{kh} \left(\frac{u_G^h}{u_j^h} - \frac{u_G^{kh}}{u_j^{kh}}\right). \qquad (12)\end{aligned}$$

3.2 Weak-separability in labor

If preferences are weakly separable in in labor supply and other goods as in $U = U\big(f\big(\underline{x}, G\big), L\big)$, one has

$$\frac{u_G^h}{u_j^h} = \frac{u_G^{kh}}{u_j^{kh}}, \qquad h, k = 1, 2, \ldots, H, \qquad (13)$$

and equation (12) simplifies to $\sum_h \pi^h (u_G^h/u_j^h) = p_G$. Now, Atkinson and Stiglitz (1976) have shown that, with these preferences, optimal taxation requires no differential commodity taxes. Adopting the tax normalization rule of $t_1 = 0$ (as in Subsection 2.1, (ii)) then implies $u_j^h = \alpha^h$ (for all $j = 1, 2, \ldots, n$). This further simplifies equation (12) to

$$\sum_h \pi^h \frac{u_G^h}{\alpha^h} = p_G, \qquad (14)$$

which is the Samuelson rule. This proof generalizes Boadway and Keen's (1993) result (obtained for a two-group model and one private good) to H types of agents and n private goods. Observe also that the result does not rest on any assumptions concerning which self-selection constraints bind. In their demonstration, Boadway and Keen had assumed that the downward self-selection constraint binds.

3.2.1 Non-optimal taxes

Kaplow's insight was to recognize that, even if taxes are not optimal, a *potential* Pareto improvement is possible whenever the sum of individuals' marginal rates of substitution exceeds the marginal cost of public good provision. He also proposed a method to realize this potential. Thus assume that initially neither the tax rates and nor the level of G are set optimally. The non-optimality notwithstanding, it must be the case that the tax structure satisfies both the resource constraint (equation (6)) and the self-selection constraints (equation (7)). That is, the allocation under the tax system must be feasible and, with the individuals being free to react to the tax system, the initial tax structure must be incentive compatible.

Now consider a concomitant change in G and in a numeraire good, x_j, for all individuals h ($h = 1, 2, \ldots, H$) such that

$$dx_j^h = -\frac{u_G^h}{u_j^h}\, dG. \tag{15}$$

It is plain that such a change leaves everyone equally well-off as before so that social welfare remains unchanged ($d\Omega = 0$). Two interesting questions arise with respect to this new allocation: First, if it is implementable by a general tax function (i.e., if it satisfies the incentive compatibility constraints), and second, if it is feasible (i.e., if it satisfies the resource constraint). To answer the first question, observe that

$$du^{hk} = u_j^{hk}\, dx_j^k + u_G^{hk}\, dG = u_j^{hk}\left(dx_j^k + \frac{u_G^{hk}}{u_j^{hk}} dG\right).$$

With weak separability of preferences in labor supply and other goods, (13) is satisfied and the above equation simplifies to

$$d\, u^{hk} = u_j^{hk}\left(d\, x_j^k + \frac{u_G^k}{u_j^k}\, dG\right) = 0,$$

where the second equality follows from equation (15). Consequently, the incentive compatibility condition continues to hold (if it did originally), and the allocation is implementable.

To answer the second question, observe that changing G as specified by equation (15) changes the government's budget constraint, from equation (6), according to

$$d\mathcal{R} = -\sum_h \pi^h\, dx_j^h - p_G\, dG = \left[\sum_h \pi^h\, \frac{u_G^h}{u_j^h} - p_G\right] dG. \tag{16}$$

It immediately follows from (16) that if $\sum_h \pi^h (u_G^h/u_j^h) > p_G$, then *increasing* G results in an increase in the government's net revenues. This tells us that, if one keeps $\mathcal{R}$ constant, it is *possible* to reconfigure the tax system in such a way as to improve social welfare. On the other hand, if $\sum_h \pi^h (u_G^h/u_j^h) < p_G$, then *decreasing* $G \Rightarrow d\mathcal{R} > 0$, and again *potentially* one can effect a Pareto improvement by reducing G.

The above demonstration proves Kaplow's result in the present mechanism design framework. However, it is important to emphasize here that increasing G when $\sum_h \pi^h (u_G^h/u_j^h) > p_G$, or reducing G when $\sum_h \pi^h (u_G^h/u_j^h) < p_G$, is not in and out of itself Pareto-improving—only potentially so. The Pareto-improvement materializes if (and generally only if) the income tax structure changes in a particular way.

3.3 MCPF, again

The first issue now is to redefine the concept of $MCPF$. Sandmo (1998) uses a utilitarian social welfare and defines $MCPF$ as μ (the shadow price of public funds) divided by the average of marginal utilities of incomes. A generalization of this definition divides μ by a *welfare weighted* average of marginal utilities. According to this definition, which I follow, the marginal cost of public funds is given by[12]

$$MCPF = \frac{\mu}{\sum_h \pi^h \beta^h}. \tag{17}$$

Multiply the first-order condition (equation (10a)) through by π^h and sum over h. Using the definition of $MCPF$ in equation (17) in the resulting equation, and setting $u_j^h = \alpha^h$, it immediately follows that

$$MCPF \gtreqless 1 \Leftrightarrow \sum_h \sum_{k \neq h} \lambda^{hk} (u_j^h - u_j^{hk}) \gtreqless 0. \tag{18}$$

[12] This, of course, is not the only possible definition. It is nevertheless the "closest" one comes in this setting to the definition of the $MCPF$ in the one-consumer economy. Here one divides μ by $\sum_h \pi^h \beta^h$, an expression that shows the change in welfare when everyone receives one dollar. In the one-consumer economy, μ is divided by α; this is also a variable that shows the change in welfare when everyone (who are all alike) receives a dollar. Observe also that, with μ showing the change in welfare per government dollar, the dimension of $\mu / \sum_h \pi^h \beta^h$ is "welfare per private dollar when distributed equally".

This implies that if all $\lambda^{hk} = 0$, i.e. if the allocation is first-best, then $MCPF = 1$. In the second-best, on the other hand, a subset of λ^{hk}'s must be positive. Depending on the relative size of u_j^h and u_j^{hk} (for this subset), $MCPF$ may then exceed as well as fall short of one.[13]

To be more specific, assume preferences are additive across all goods and labor supply, and that all private goods are normal. Further assume that at the optimum, redistribution is from higher- to lower-ability individuals so that $\lambda^{hk} > 0$ only if $h > k$. With the more-able individuals earning and consuming more of all private goods, $h > k \Rightarrow x_j^h > x_j^k \Rightarrow u_j^h < u_j^{hk}$. Consequently, from equation (18), $MCPF < 1$.[14] The

[13] Christiansen (1999) considers a two-group model, and uses the sum of the income equivalents of the utility losses of the two groups (as their respective taxes are increased) plus one (the extra tax revenue raised) as a measure of the $MCPF$. In this exercise, the distribution of the extra tax revenue is allowed to vary across the two groups as long as their sum total remains unchanged and they continue to satisfy the incentive compatibility constraints. Observe that as the distribution of the extra tax payments change, the "implicit" social welfare function for the economy (i.e., the social welfare function that makes the given tax distribution optimal) changes. That is, with the exception of one particular distribution for the extra tax payments, Christiansen's definition of the $MCPF$ will have the implicit social welfare function for the economy after the tax increase to be different from what it was before it. Christiansen (1999) finds that the individual utility losses depend on the tax distributions. The tax distributions also determine if the $MCPF$ exceeds or falls short of one. Moreover, he finds that the $MCPF$ moves positively with the tax payments of the high-ability individuals. The intuition is that as one increases the high-ability individuals' share of the extra taxes, one implicitly cares more about redistribution towards the poor. That is, the society's implicit social welfare function becomes more redistributive. A higher degree of redistribution entails more costly taxation.

[14] The $MCPF$ is also smaller that one in the two-group model studied by Boadway and Keen (1993). To prove this, assume, as in Boadway and Keen, there are two groups of individuals with $w^2 > w^1$, one consumption good, and that at the optimum the downward self-selection constraint binds (i.e. $\lambda^{21} > 0$ and $\lambda^{12} = 0$). Then

$$\sum_h \sum_{k \neq h} \lambda^{hk} (u_j^h - u_j^{hk}) = \lambda^{21} (u_x^2 - u_x^{21}).$$

But

$$u_x^2 - u_x^{21} = U_x \left(f(x^2, G), \frac{I^2}{w^2} \right) - U_x \left(f(x^1, G), \frac{I^1}{w^2} \right) \equiv \Delta U_x \left(f(x, G), L \right),$$

where Δ is the "difference" operator. Observe that with higher ability individuals earning more and consuming more, $I^2 > I^1$ so that $L^2 > L^{21} = I^1/w^2$ and $x^2 > x^1$. Now, given that $x^2 > x^1$ and $L^2 > L^{21}$, consider an infinitesimal change in which x and L increase:

$$\begin{aligned} dU_x \left(f(x, G), L \right) &= U_{xx} \left(f(x, G), L \right) dx + U_{xL} \left(f(x, G), L \right) dL \\ &= \frac{dx}{w} \left[w U_{xx} \left(f(x, G), L \right) + U_{xL} \left(f(x, G), L \right) \right], \end{aligned}$$

where I have set $dL/dx = -U_x/U_L = 1/w$. It is easy to check that if leisure is a normal good, important point here is *not* that $MCPF < 1$ (although this result itself is in contrast to what emerges in a representative agent model where, using the same tax normalization rule, normality of leisure implies $MCPF > 1$). *The upshot is that* $MCPF < 1$ *and yet the public good is neither under- and nor over-provided.* Recall that in a representative agent model, the condition $MCPF < 1$ implies that the public good is over-provided (in the "rule" sense and as long as preferences are separable in G and other goods).

The definition of the $MCPF$ in (17) applies when the income tax structure is optimal. With sub-optimal taxes, the $MCPF$ will depend on the existing tax structure as well as on how the extra revenues are raised. Assume that G is to be increased by dG and financed by reducing the production (and consumption) of the private good x_j. It then follows from the social welfare function (5) that $d\Omega = \sum_h \pi^h \beta^h dx_j^h$, and from the economy's resource constraint (6) that $d\mathcal{R} = \sum_h \pi^h dx_j^h$. Now suppose the share of individual h in $d\mathcal{R}$ is s_j^h so that $dx_j^h = s_j^h d\mathcal{R}$. It then follows that $d\Omega = \sum_h \pi^h \beta^h s_j^h d\mathcal{R}$, and

$$MCPF = \frac{1}{\sum_h \pi^h \beta^h} \frac{d\Omega}{d\mathcal{R}} = \frac{\sum_h \pi^h \beta^h s_j^h}{\sum_h \pi^h \beta^h}.$$

In a similar fashion, one can find the following expression for the $MBPG$,

$$MBPG = \frac{1}{p_G \sum_h \pi^h \beta^h} \frac{\partial \Omega / \partial G}{\partial \mathcal{R} / \partial G} = \frac{\sum_h \pi^h \beta^h (u_G^h / \alpha^h)}{p_G \sum_h \pi^h \beta^h}.$$

As long as there is going to be no change in the income tax structure, the decision regarding public good provision must be governed according to the relationship between $MCPF$ and $MBPG$. Specifically, more public goods should be provided if, and only if, $MBPG > MCPF$ or

$$\sum_h \pi^h \beta^h (u_G^h / \alpha^h) > p_G \sum_h \pi^h \beta^h s_j^h.$$

This is, of course, a different rule from Samuelson's. It holds despite the fact that the income tax is nonlinear, and that preferences may exhibit weak separability in labor and other goods. The rule embodies the distributional objectives of the society.

$wU_{xx}\left(f(x, G), L\right) + U_{xL}\left(f(x, G), L\right) < 0$. It then follows that $u_x^2 - u_x^{21} < 0$ so that $MCPF < 1$.

4 Concluding remarks

Two important lessons emerge from this study. One is that in models with heterogeneous agents, the concept of the marginal cost of public funds does not assume a unique value; assigning different welfare weights to different individuals in the economy results in different values for the $MCPF$ associated with the same project. Moreover, there is no use in calculating the $MCPF$ unless this is done in conjunction with calculating the marginal benefit of public spending ($MBPG$), using the same welfare weights.

The second lesson concerns Kaplow's prescription. The paper constructs an intuitive proof for Kaplow's proposition which distinguishes between the two ingredients of his argument. One is that increasing G yields a *potential* Pareto improvement (whenever the Samuelson rule is satisfied and preferences exhibit weak separability in labor and other goods). The second ingredient calls for reforming the income tax structure in such a way as to realize this potential Pareto improvement. If this reform is not forthcoming, one cannot rely on the simple Samuelson rule for a decision on the public good provision. This decision must be governed by the comparison between the $MCPF$ and the $MBPG$ of the project, both of which embody distributional concerns of the society.[15]

[15] This point is made forcefully by Slemrod and Yitzhaki (2001) who make similar comparisons between $MCPF$ and $MBPG$ but for linear income taxes. The existence of distributional concerns is also recognized and discussed by Kaplow (1996, 2004).

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