Fertility, human capital accumulation, and the pension system*

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Abstract

This paper provides a unified treatment of externalities associated with fertility and human capital accumulation within pas-as-you-go pension systems. It considers an overlapping generations model in which every generation consists of high earners and low earners with the proportion of types being determined endogenously. The number of children is deterministically chosen but the children's future ability is in part stochastic, in part determined by the family background, and in part through education. In addition to the customary externality source associated with a change in average fertility rate, this setup highlights another externality source. This is due to the effect of a parent's choice of number and educational attainment of his children on the proportion of high-ability individuals in the steady state. Our other results include: (i) Investments in education of high- and low-ability parents must be subsidized; (ii) direct child subsidies to one or both parent types can be negative; i.e., they can be taxes; (iii) net subsidies to children (direct child subsidies plus education subsidies) to at least one type of parents must be positive; (iv) parents who have a higher number of children should invest less in their education.

JEL classification: H2, H5.

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1 Introduction

One of the most pressing problems facing the economies of the industrialized world is the fiscal solvency of their pay-as-you-go (PAYGO) social security systems.[1] An important contributing factor to this problem has been the recent drastic fertility declines in Western Europe and Japan. What truly determines fertility, and what accounts for the observed evolution in fertility behavior, are still open questions. What is clear, however, is that, faced with a PAYGO social security system, parents do not have the right incentives to choose a fertility rate that is optimal. In such systems, each person's fertility decision affects the economy's population growth rate and with it everybody's pension benefits. Specifically, an increase in the rate of population growth increases the number of future workers who will have to support a retired person. No individual, however, takes this impact into account and that leads to a decentralized equilibrium outcome with too few children.[2]

The above problem is exacerbated by another externality associated with the "quality" of children, and their human capital accumulation, through the education decisions of parents. The rate of return of a pay-as-you-go system depends not just on the fertility rate, but also on productivity growth. The more productive the children, the higher will be their ability to produce and to pay taxes. This reinforces the public good nature of a family's child-rearing activities.[3]

Most of the literature has thus far treated the quality and quantity issues separately; or else have lumped the investments in quantity and quality together as if one decision

[1] This has led to reforms in a number of countries. See Penner (2007) who surveys the recent reforms in Canada, Germany, Italy, Japan, Sweden, and the UK.

[2] In addition to this "intergenerational transfer" effect, the literature has also noted an offsetting force called "capital dilution" effect: A higher fertility rate, given the aggregate capital saved by the previous generation, implies a lower capital to labor ratio reducing per capita output; see Michel and Pestieau (1993) and Cigno (1993).

[3] To internalize the quantity and quality effects, some economists have advocated a policy of linking pension benefits (or contributions) to individuals' fertility choices. See, among others, Abio *et al.* (2004), Bental (1989), Cigno *et al.* (2003), Fenge and Meier (2004), Kolmar (1997), van Groezen *et al.* (2000, 2003).

determines both.[4] A basic shortcoming of this approach is that it cannot distinguish between child subsidies, which correct externalities emanating from fertility decisions, and education subsidies which correct for externalities due to investing in education. This lack of distinction becomes more of a serious problem when the two types of externalities interact as they often do.

To be sure, there are a number of studies in the literature that distinguish between quantity and quality decisions and study them both in one unified framework. Peters (1995) is an early example of this. In his model, both fertility and education choices are made deterministically. The main shortcomings of his approach are the deterministic nature of both quantity and quality decisions, and the lack of any heterogeneity among parents. Cigno *et al.* (2003) also allow for both fertility and quality. Fertility is fully deterministic, but children's quality, which Cigno *et al.* define in terms of "lifetime tax contributions", is in part random and in part determined through actions of parents. The limitations of their study come from the static nature of their model, in looking at the decisions of the initial parent only, and their not allowing for heterogeneity among parents.

Cigno and Luporini (2003), while building on Cigno *et al.* (2003), allow for parents' heterogeneity in terms of their ability to influence their children's probability of success in life.[5] However, their model remains static in nature as they too do not go beyond the decisions of the initial parents. In Meier and Wrede (2008) both fertility and types are partly stochastic and partly determined by investments. The limitation of their model comes from their ignoring the impact of fertility and education investments on the distribution of types in the economy. But this induced change in the distribution of types constitutes an important component of fertility and education externalities.[6]

[4] Cremer *et al.* (2003, 2008) are examples of this latter approach, while Cremer *et al.* (2006) is concerned only with quantity decisions.

[5] They also drop Cigno *et al.*'s (2003) assumption that fertility is fully deterministic.

[6] Sinn (2004) also considers a model that allows or both fertility and quality. In his setup fertility is fully random and quality fully deterministic. However, Sinn is interested more in examining the

The current paper addresses the quantity and quality questions in an overlapping generations model with high- and low-ability individuals. The unique feature of our study is its endogenous determination of the distribution of types. Specifically, we allow for this distribution to be affected by both education and fertility decisions. This framework gives rise to three sources of externality. First, there is the customary externality associated with the change in average fertility—the intergenerational transfer effect. It arises from the fertility decisions of parents. This source of externality disappears if the pension system is a pre-funded one. The second source of externality emanates from decisions that change the distribution of types even if average fertility is kept constant. It arises from both education decisions and fertility decisions. Its unique feature is that it does not depend on the institution of social security and exists for pre-funded systems as well. The third source of externality is due to interaction between average fertility and the distribution of types. It too arises from both education decisions and fertility decisions. It is different from the second externality source in that it exists because of the PAYGO institution and disappears if one moves to a pre-funded system. It is also different from the first externality source because it will not exist if the distribution of types were immutable.

One distinguishing element between quantity and quality decisions is that of timing. One decides on the number of children quite early; the quality of children, i.e. their future earning capacity, is determined much later. We incorporate this timing sequence in our two-period overlapping generations model by assuming a sequential decision making process: At the end of the first-period, the young decide on starting a family and having children first and then on the extent of their children's education.

We assume that parents choose the number of their children deterministically. It is true that the actual number of children in a family does not necessarily coincide with the number that parents initially intended to have.[7] However, this choice is intrinsically

properties of a traditional PAYGO system rather than the properties of an *optimal* pension plan.

[7] Infertility, premature death, misplanning and multiple births are some of the reasons explaining

more deterministic and less susceptible to random and other shocks than determining the quality of one's children. As to the quality, it is unrealistic to expect that one can determine the future earning abilities of one's children in a deterministic fashion simply by investing in their education and training. We assume that quality is determined by three factors. One is random; the second is due to education; and the third is pre-determined by one's "genes" and family background. Nevertheless all children of a particular parent turn out to be either of high- or of low-ability.

Finally, we study the properties of an optimal pension system assuming that intergenerational transfer of resources occur only through the PAYGO scheme. This simplifies the analysis drastically by allowing us to ignore the issues relating to the choice between PAYGO and fully- or partially-funded pension systems. The determinants of this choice are multi-dimensional and, given our focus on endogenous fertility and education, any attempt to address this choice is bound to be inadequate.[8]

2 The model

2.1 Preliminaries

Consider, within an overlapping generations framework, the sequence of decisions a child has to face after he is born. First, upon reaching adulthood, he has to decide on starting a family and having children. Subsequently, as a parent, he has to decide on the extent of his children's education. Finally, the retirement period arrives. Such a rich model allows for children, adults, parents, and the retired (grand parents) to overlap, requiring a four-period overlapping generations model. However, analyzing a full-fledged four period model quickly becomes cumbersome and too detailed for developing insights.

this gap.

[8] One important question here is whether or not one should have a PAYGO system if the rate of return to capital exceeds the population growth rate. In a stripped down model such as ours, a PAYGO pension plan is undesirable unless the economy is characterized by dynamic inefficiency. Although Weil (2008) has recently argued that this possibility may arise even in advanced countries, we do not want to cope with this issue.

We thus take a short cut and transform the four-period setup we have in mind into a simple two-period overlapping generations model. To do this we assume the decisions of having children and educating them occur sequentially just prior to the beginning of one's retirement. This saves us from having to distinguish between working as an adult and working as a parent.

Assume each generation consists of two types of people; they posses either a high or a low earning ability. Denote high- and low-ability types by subscripts h and l and let $j = h, l$. All children of a particular parent will turn out to be either of high- or of low-ability; no mix of high- and low-ability children is possible. There are three factors that determine if a child turns into a high- or a low-ability individual. One is due education; the second is a random element; and the third is pre-determined by one's "genes" and family background. The effect of education on ability is, *ceteris paribus*, most certainly positive. To introduce randomness into this process, we assume that investing in education does not necessarily transform a child into a high-ability type; instead, it only increases the *probability* of its occurrence. Thus, when a j-type parent invests e "units" in educating his child, the child will have a $\pi_j = \pi_j(e)$ *probability* of turning out to be of *high-ability*. Naturally, the probability that the child will be of low-ability is $1 - \pi_j$. We assume that $\pi_j(\cdot)$ is an increasing and strictly concave function with $\pi_j(0) > 0$.

The third factor, the child's family background, manifests itself through the functional form of $\pi_j(e)$ and that is why the function is indexed by j. Specifically, one would expect that $\pi_h(e) > \pi_l(e)$. That is, for the same level of (formal) education, children of high-ability parents have a higher chance of becoming more able. This reflects the fact that high-ability parents tend to spend more time reading to their children and engage them in activities that builds up their human capital. To say more about the structure of $\pi_j(e)$, one needs to know the precise nature of the interaction between (formal) education and family background on a child's ability. Decompose $\pi_j(e)$ into two distinct elements: an educational component $\pi(e)$ and a family background component represented by a parameter θ_j, with $\theta_l < \theta_h$. We assume that the interaction between $\pi(e)$ and θ_j is additive so that $\pi_j = \pi(e) + \theta_j$.[9] According to this formulation, the marginal productivity of spending e dollars on educating one's children is the same regardless of the parent's type.[10]

Assume generation T consists of N_T people. Denote the proportion of high-ability persons in generation T by δ_T $(0 < \delta_T < 1)$ so that the number of high-ability persons in generation T is $\delta_T N_T$. Parents choose the number of the children they want to have and do so deterministically. Denote the number of children each j-type parent will have by n_j. Thus $\delta_T N_T$ high-ability parents of generation T end up with $(\delta_T N_T) n_h \pi_h$ high-ability children and $(\delta_T N_T) n_h (1 - \pi_h)$ low-ability children. Similarly, $(1 - \delta_T) N_T$ low-ability persons of generation T end up with $(1 - \delta_T) N_T n_l \pi_l$ high-ability children and $(1 - \delta_T) N_T n_l (1 - \pi_l)$ low-ability children. Consequently, the proportion of high-ability children in the next generation will be

$$\delta_{T+1} = \frac{\delta_T N_T n_h \pi_h + (1 - \delta_T) N_T n_l \pi_l}{\delta_T N_T n_h + (1 - \delta_T) N_T n_l} = \frac{\delta_T n_h \pi_h + (1 - \delta_T) n_l \pi_l}{\delta_T n_h + (1 - \delta_T) n_l}. \tag{1}$$

2.2 Steady state

In the steady state, $\delta_{T+1} = \delta_T \equiv \delta$. It then follows from equation (1) relating δ_{T+1} to δ_T that

$$\frac{\delta n_h \pi_h + (1 - \delta) n_l \pi_l}{\delta n_h + (1 - \delta) n_l} = \delta. \tag{2}$$

Observe that δ is a weighted average of π_h and π_l and thus bracketed by them. Moreover, equation (2) indicates that δ is homogeneous of degree zero in (n_l, n_h). It follows from

[9] Observe that in this case $\theta_l < \theta_h \leqslant 1 - \pi(e)$.

[10] Alternatively, one can posit a multiplicative relationship between $\pi(e)$ and θ_j so that $\pi_j = \theta_j \pi(e)$ (with $\theta_l < \theta_h \leqslant 1/\pi(e)$).This assumption states that the marginal productivity of spending e dollars is higher for the more able parents. Its main import is to enhance the educational investment of high-ability parents relative to low-ability ones. Otherwise, it has similar implications for the nature of externalities. In an earlier version of the paper, we explored this issue as well.

Euler's Theorem that

$$n_h \frac{\partial \delta}{\partial n_h} + n_l \frac{\partial \delta}{\partial n_l} = 0. \tag{3}$$

It follows from this equation that $\partial\delta/\partial n_h$ and $\partial\delta/\partial n_l$ are of opposite signs.

Let e_j denote the j-type's investment in the education of his children. Solve equation (2) for δ and write the solution as $\delta = \delta(e_h, e_l, n_h, n_l)$. Introduce

$$Z \equiv 2\delta(n_h - n_l) + n_l(1 + \pi_l) - n_h\pi_h. \tag{4}$$

Differentiating (2) yields the following partial derivatives:

$$\frac{\partial \delta}{\partial e_h} = \frac{\delta n_h \pi_h'(e_h)}{Z}, \tag{5}$$

$$\frac{\partial \delta}{\partial e_l} = \frac{(1-\delta) n_l \pi_l'(e_l)}{Z}, \tag{6}$$

$$\frac{\partial \delta}{\partial n_h} = \frac{\delta(\pi_h - \delta)}{Z}, \tag{7}$$

$$\frac{\partial \delta}{\partial n_l} = \frac{(1-\delta)(\pi_l - \delta)}{Z}. \tag{8}$$

We prove in Appendix A that a necessary condition for the stability of steady-state solution for δ, namely $|\partial\delta_{T+1}/\partial\delta_T| < 1$, is that $Z > 0$. Thus, assuming a stable steady state implies that $Z > 0$ so that

$$\frac{\partial \delta}{\partial e_h} > 0, \text{ and } \frac{\partial \delta}{\partial e_l} > 0.$$

2.3 Laissez faire

To establish a benchmark, we start by studying the properties of laissez-faire equilibrium of the economy. Individuals have preferences over consumption when young, c, consumption when retired, d, and the number of children, n. They also care about the quality of their children. We represent this by assigning a higher weight to the subutility for children if they turn out to be of high ability. Specifically, the preferences of a j-type parent for having i-type children are represented by

$$U_j = u(c_j) + v(d_j) + \gamma^i \varphi(n_j), \tag{9}$$

where $\gamma^h > \gamma^l$ with $\gamma^h - \gamma^l$ indicating the strength of preferences for higher-ability children. Under this circumstance, given the partly stochastic nature of children ability, each j-type will have an ex-ante expected utility depending on the outcome of his investment in children. Setting $\gamma^l = 1$, and $\gamma^h = \gamma > 1$, we have

$$\begin{aligned} EU_j &= u(c) + v(d) + \pi_j \gamma \varphi(n) + (1 - \pi_j)\, \varphi(n) \\ &= u(c) + v(d) + [1 + (\gamma - 1)\,(\pi(e) + \theta_j)]\, \varphi(n). \end{aligned} \tag{10}$$

Assume each j-type person earns an income equal to $\beta_j I$ when young, where $\beta_h > \beta_l$.[11] Without any loss of generality, set $\beta_l = 1$ and $\beta_h = \beta > 1$. Denote the non-education cost of raising a child by a and the "quantity" of education provided to a child by e. Choose the units of measurement for c, d, and e such that their producer prices are one. The young individual spends a portion of his income on his immediate consumption, c, a portion on raising his children, an, and another portion on educating his children, en. He saves the rest of his income receiving a rate of return equal to r. Upon retirement, the individual receives and spends all his savings plus interest, leaving no bequests.

Denote the rate of interest by r. The budget constraint for the j-type is given by

$$\beta_j I = c_j + \frac{d_j}{1+r} + e_j n_j + a n_j. \tag{11}$$

The j-type young individual chooses c_j, d_j, n_j, and e_j to maximizes his utility (10) subject to his budget constraint (11). This problem is summarized by the Lagrangian

$$\begin{aligned} \mathcal{L} &= u(c) + v(d) + [1 + (\gamma - 1)\,(\pi(e) + \theta_j)]\, \varphi(n) \\ &\quad + \mu \left[\beta_j I - c - \frac{d}{1+r} - n\ a - e\ n \right]. \end{aligned} \tag{12}$$

[11] We assume that I does not depend on the economy's capital stock. In this sense, our overlapping generations model is of Samuelson's (1958) variety rather than Diamond's (1965).

Manipulating the first-order conditions with respect to c_j, d_j, e_j, and n_j, the laissez faire solutions for these variables are found from

$$\frac{v'(d_j)}{u'(c_j)} = \frac{1}{1+r}, \tag{13}$$

$$\frac{\pi'(e_j)}{u'(c_j)} = \frac{n_j}{(\gamma-1)\varphi(n_j)}, \tag{14}$$

$$\frac{\varphi'(n_j)}{u'(c_j)} = \frac{a+e_j}{1+(\gamma-1)\left(\pi(e_j)+\theta_j\right)}, \tag{15}$$

$$\beta_j I = c_j + \frac{d_j}{1+r} + n_j(a+e_j). \tag{16}$$

At this level of generality, the effect of a higher level of income on educational investment is not clearcut. There are different forces at work. Consequently, one cannot determine which type invests more in education or has more children.[12]

The results of this section are summarized as

Proposition 1 *Consider an overlapping generations model in the steady state with two types of people in each generation: high- and low-ability. Each type receives an income commensurate with his ability when young and has preferences over consumption during working years and retirement, as well as the number of children he will have and their ability type. Each type can have children of either ability. The probability of having a high-ability child depends positively on investment in education and is higher, ceteris paribus, for high-ability parents. Then:*

(i) Investment in education by either type of parents increases the proportion of high-ability persons in the steady state, δ.

[12] If parents care about having children but not about their ability types, the ambiguity goes away. This is a special case of our model in which $\gamma = 1$. Under this circumstance, one can easily see that the solution for education expenditures requires $e = 0$. This is not surprising given that education is costly to the parent but bestows no utility upon him. Observe also that in this case, the first-order condition (15) will be simplified to $\varphi'(n_j)/\varphi'(n_j) = a$. One can then show that, given strong separability and concavity of all subutility functions, c, d, and n are all normal goods so that $c_h > c_l, d_h > d_l$, and $n_h > n_l$.

(ii) Increasing the number of children increases δ for one type of parents and decreases it for the other.

(iii) The laissez-faire solution is found from equations (13)–(16).

3 Utilitarian First Best

Denote the population growth rate by

$$\overline{n} \equiv \delta n_h + (1-\delta) n_l. \tag{17}$$

The economy's resource constraint in the steady state is then written as

$$[1+(\beta-1)\delta)]\, I \geqq \delta \left[c_h + n_h\,(a+e_h) + \frac{d_h}{\overline{n}}\right] + (1-\delta)\left[c_l + n_l\,(a+e_l) + \frac{d_l}{\overline{n}}\right]. \tag{18}$$

Thus the consumption of the retired is financed from taxes imposed on the young as in a pay-as-you-go retirement system. In what follows, we simplify our analysis by concentrating on the steady-state equilibrium, ignoring the welfare of generations who live on the transitional path from one steady state to another. This approach is equivalent to assuming that the government's social welfare function is defined over unweighted average utilities of all current and future generations. Clearly, the extent of redistribution across generations are susceptible to this particular choice of social welfare function.

3.1 The problem and its solution

Using the economy's resource constraint (18), the government's optimization problem is summarized by the Lagrangian

$$\begin{aligned} \pounds \;\; = \;\; & \delta \left\{u(c_h) + v(d_h) + [1+(\gamma-1)\left(\pi(e_h)+\theta_h\right)]\,\varphi(n_h)\right\} + \\ & (1-\delta)\left\{u(c_l) + v(d_l) + [1+(\gamma-1)\pi(e_l)+\theta_l]\,\varphi(n_l)\right\} + \\ & \mu\Big\{\,[1+\beta-1)\delta]\,I - \delta\left[c_h + n_h\,(a+e_h) + \frac{d_h}{\overline{n}}\right] \\ & -(1-\delta)\left[c_l + n_l\,(a+e_l) + \frac{d_l}{\overline{n}}\right]\Big\}, \end{aligned} \tag{19}$$

leading to the following first-order conditions with respect c_h, c_l, d_h and d_l:

$$\frac{\partial \pounds}{\partial c_h} = \delta[u'(c_h) - \mu] = 0, \tag{20}$$

$$\frac{\partial \pounds}{\partial c_l} = (1-\delta)[u'(c_l) - \mu] = 0, \tag{21}$$

$$\frac{\partial \pounds}{\partial d_h} = \delta[v'(d_h) - \frac{\mu}{\overline{n}}] = 0, \tag{22}$$

$$\frac{\partial \pounds}{\partial d_l} = (1-\delta)[v'(d_l) - \frac{\mu}{\overline{n}}] = 0. \tag{23}$$

Manipulating these conditions yields

$$c_h = c_l = c, \text{ and } d_h = d_l = d.$$

3.2 Externalities due to education and having children

Introduce

$$\begin{aligned} D \equiv \frac{\partial \pounds}{\partial \delta} &= [1 + (\gamma - 1)(\pi(e_h) + \theta_h)]\,\varphi(n_h) - [1 + (\gamma - 1)\pi(e_l) + \theta_l]\,\varphi(n_l) \\ &+ u'(c)\left\{(\beta - 1)I - [n_h(a + e_h) - n_l(a + e_l)] + \frac{(n_h - n_l)\,d}{\overline{n}^2}\right\}. \end{aligned} \tag{24}$$

Observe that D shows the change in social welfare due to an increase in the proportion of high-ability persons in the population so that it must be positive.[13] With $c_h = c_l$ and $d_h = d_l$, the first bracketed term on the right-hand side of (24) shows the net change in utilities. The second bracketed expression shows the *net* change in resources; i.e. the increase in the available resources minus the extra resources required in consumption.[14] Using the definition of D and the previous findings that $c_h = c_l = c$, $d_h = d_l = d$, and $\mu = u'(c)$, one can write the first-order conditions for the maximization of social welfare

[13] Being a proportion, this is matched by a reduction in the proportion of low-ability persons.

[14] This term arises only in conjunction with pensions. A change in δ changes $\overline{n} = n_l + \delta(n_h - n_l)$, the number of future working people who support a retired person under a PAYGO pension plan.

with respect to n_h, n_l, e_h, and e_l as

$$\frac{\partial \pounds}{\partial e_h} = \delta\left[(\gamma-1)\varphi(n_h)\pi'(e_h) - n_h u'(c)\right] + D\frac{\partial \delta}{\partial e_h} = 0, \tag{25}$$

$$\frac{\partial \pounds}{\partial e_l} = (1-\delta)\left[(\gamma-1)\varphi(n_l)\pi'(e_l) - n_l u'(c)\right] + D\frac{\partial \delta}{\partial e_l} = 0, \tag{26}$$

$$\frac{\partial \pounds}{\partial n_h} = \delta\left\{\left[1+(\gamma-1)\left(\pi(e_h)+\theta_h\right)\right]\varphi'(n_h) - \left(a + e_h - \frac{d}{\overline{n}^2}\right)u'(c)\right\} + D\frac{\partial \delta}{\partial n_h} = 0, \tag{27}$$

$$\frac{\partial \pounds}{\partial n_l} = (1-\delta)\left\{\left[1+(\gamma-1)\left(\pi(e_l)+\theta_l\right)\right]\varphi'(n_l) - \left(a + e_l - \frac{d}{\overline{n}^2}\right)u'(c)\right\} + D\frac{\partial \delta}{\partial n_l} = 0. \tag{28}$$

Investing in education raises the probability of one's children to be of high ability. To the extent that parents prefer to have high ability children, this increases their utility as measured by $(\gamma-1)\varphi(n_j)\pi'(e_j)$. At the same time, investment in education is costly. Spending e_j to educate each of one's children imposes a utility cost of $u'(c)n_j$ on the parent. Thus the first expression in equations (25) and (26) show the net private benefit of investment in education. The second expressions in these equations reveal the existence of an externality represented by

$$\frac{D}{\delta}\frac{\partial \delta}{\partial e_h} \quad \text{for increasing } e_h, \tag{29}$$

$$\frac{D}{1-\delta}\frac{\partial \delta}{\partial e_l} \quad \text{for increasing } e_l. \tag{30}$$

This externality arises through the effect of e_j on δ. Moreover, given that $\partial\delta/\partial e_j > 0$ and $D > 0$, this is a positive externality.

The externality terms (29)–(30) coming through δ may be divided into two parts. One is due to the direct change in δ as e_j changes. When there is an increase in the proportion of high-ability persons in the population, matched of course by a reduction in the proportion of low-ability persons, social welfare changes by the difference in the utilities of high- and low-ability types *and* the change in the *net* resources (income minus consumption). This effect does not work through fertility; it is present also in the absence of PAYGO pension plans when all second-period consumptions are financed by private savings. The second part, on the other hand, works through changing average fertility. Its existence depends on having a PAYGO pension plan in place. It arises indirectly as the change in δ changes $\overline{n}$ as well. Remember that $\overline{n}$ depends on δ and δ depends on e_j (as well as n_j). This change in $\overline{n}$ is also neglected in private calculations. With $\overline{n} = n_l + \delta(n_h - n_l)$, this effect depends on the difference between n_h and n_l. The various terms in D represent these two direct and indirect externalities. The latter is captured by the $(n_h - n_l)\, d/\overline{n}^2$ term that appears in the definition of D, and the former by the remaining expressions therein.

Similarly, increasing n_j has externalities of its own. When a j-type individual increases his fertility rate, he does not take the effect of his decision on $\overline{n}$ into consideration. He thus perceives the effect of increasing n_j in his *net* welfare to consist of an increase in his utility, $[1 + (\gamma - 1)\pi_j]\, \varphi'(n_j)$, minus an increase in his expenditures on n_j, measured by $(a + e_j)\, u'(c)$. Comparing this with the expressions in equations (27) and (28) reveals the existence of externalities represented by[15]

$$\frac{d}{\overline{n}^2} u'(c) + \frac{D}{\delta} \frac{\partial \delta}{\partial n_h} \qquad \text{for increasing } n_h, \tag{31}$$

$$\frac{d}{\overline{n}^2} u'(c) + \frac{D}{1 - \delta} \frac{\partial \delta}{\partial n_l} \quad \text{for increasing } n_l. \tag{32}$$

The externalities associated with n_j, as depicted by expressions (31)–(32), consist of two distinct elements. While the first element has no counterpart in the externalities associated with e_j, the second element is identical in nature to the externality coming from e_j. The term $d/\overline{n}^2$ represents the first element and captures the effect of increasing n_h or n_l on $\overline{n}$, and through it on the aggregate resources available for distribution between the young and the old under PAYGO. Specifically, this externality tells us that increasing fertility increases the number of future working people who support a retired person. This is the familiar positive "intergenerational transfer" effect that appears in the literature on growth with endogenous fertility; see Cigno (1993) and Michel and Pestieau (1993). The second externality source, represented by the second expressions

[15] The term $d/\overline{n}^2$ is present only in conjunction with pensions.

in (31)–(32), is due to the change in δ. It is the same type of externality discussed previously in relation to the effect of e_j on δ. The crucial point is that these externalities emanate from a change in δ which can come about from a change in either n_j or e_j. This is why each of the second expressions in (31)–(32) is identical to its counterpart in (29)–(30) except that $\partial\delta/\partial n_h$ and $\partial\delta/\partial n_l$ have replaced $\partial\delta/\partial e_h$ and $\partial\delta/\partial e_l$. Finally, observe that with $D > 0$, this externality source is positive if $\partial\delta/\partial n_j > 0$ and is negative if $\partial\delta/\partial n_j > 0$. Recall also that $\partial\delta/\partial n_h$ and $\partial\delta/\partial n_l$ are of opposite signs; hence one ability type exerts a positive externality, and the other a negative externality, on the society through their fertility decisions when mediated through δ.

The results thus far in this section are summarized as

Proposition 2 *(i) Under the utilitarian first-best solution with PAYGO, the number of children that high- and low-ability parents have and the amounts of investment they make in the education of their children are characterized by equations (25)–(28).*

(ii) Investing in education of children by either type of parents bestows a positive externality on everybody else. This externality has two components, one of which exists only in the presence of PAYGO pension plans.

(iii) A parent's fertility choice imposes two kinds of externalities on everyone else. One is the familiar positive externality known as "intergenerational transfer" effect. The other emanates from a change in the proportion of high-ability children. This externality too has two components, one of which exists only in the presence of PAYGO pension plans.

3.3 Who should have more children and invest in education?

One interesting question concerns the relative size of n_h to n_l, and e_h to e_l; that is, which type should have more children and which type should invest more in education. To examine this question, substitute the expressions for $\partial\delta/\partial n_h$ and $\partial\delta/\partial n_l$ from (7)–(8)

into equations (27)–(28) and simplify. Then subtract one equation from another to get

$$\left\{[1+(\gamma-1)\pi_h]\,\varphi'(n_h)-[1+(\gamma-1)\pi_l]\,\varphi'(n_l)\right\}-(e_h-e_l)\,u'(c)+\frac{D}{Z}\,(\pi_h-\pi_l)=0. \tag{33}$$

To see the intuition for this result, consider a concomitant increase in n_h and a reduction in n_l. On the one hand, this changes the utilities of the two types of parents by $[1+(\gamma-1)\pi_h]\,\varphi'(n_h)\ -[1+(\gamma-1)\pi_l]\,\varphi'(n_l)$. On the other hand, there will be an increase in resource cost to the economy because educational expenditures increase by e_h-e_l which is worth $(e_h-e_l)\,u'(c)$ in terms of utilities. This should be subtracted from the utility benefit. Additionally, there is a gain to the economy through the externalities that emanate from a change in δ. This is measured by the last expression in (33). The above relationship tells us that at the optimum the sum of all the marginal effects must be zero. However, (33) does not allow us to determine which type should have more children. The source of this ambiguity is in the fact that fertility rates and educational investment levels move in opposite direction. We elaborate on this point below.

Divide equation (25) by (26) and substitute the expressions for $\partial\delta/\partial e_h$ and $\partial\delta/\partial e_l$ from (5)–(6) in the resulting equation. Simplifying yields

$$\mu n_h n_l\left[\pi'(e_h)-\pi'(e_l)\right]=(\gamma-1)\pi'(e_h)\pi'(e_l)\left[n_h\varphi(n_l)-n_l\varphi(n_h)\right]. \tag{34}$$

It follows from the concavity of $\pi(\cdot)$ that the left-hand side of (34) has the same sign as (e_l-e_h). Similarly, concavity of $\varphi(\cdot)$ implies that the right-hand side of (34) has the same sign as (n_h-n_l). Consequently, at the first-best, (e_h-e_l) and (n_h-n_l) are of opposite signs. That is, those parents who have a higher number of children should invest less in their education.That these two decisions go in opposite directions cause an ambiguity in determining which type of parents should have more children and which type should invest more in education. This ambiguity disappears in the special case when parents care only for the number of children they have but not their type. Under this circumstance, the decisions on fertility and education do not run in opposite directions. One can then show that under the utilitarian first-best solution with PAYGO: (i) Both types of parents invest equally in education; (ii) High-ability parents have more children; (iii) Increasing the fertility rate of high-ability parents increases the proportion of high-ability children in the economy and bestows a positive externality on everybody else; (iv) Increasing the fertility rate of low-ability parents, reduces the proportion of high-ability children and imposes a negative externality on everybody else. See Cremer *et al.* (2010).

3.4 Decentralization

As observed earlier, we assume that second-period consumption levels are financed through the PAYGO pension system. This requires the government to impose a one-hundred percent tax on savings and their returns. Recall also that the optimum requires equal consumption levels for the two ability types both during working years and retirement. Consequently, the government must provide everyone with the same pension $P = d_h = d_l = d$ where d is evaluated at its first-best value. Next, to induce the correct choice of fertility and education, two types of subsidies are required. One is a subsidy on education at the rate τ_j for the j-type, the other is a direct child subsidy to the j-type equal to t_j dollars per child. Finally, first-period lump-sum taxes, T_j, are required to ensure that consumption levels during working years are the same for both types. Below, we show how these instruments decentralize the first-best allocations.

Give these instruments, pensions are fixed and parents decide only on their first-period consumption, fertility, and children's education. Let α_j denote the Lagrangian multiplier associated with the budget constraint of a j-type parent. The optimization problem of this parent is summarized by the Lagrangian expression,

$$\begin{aligned}\mathcal{L}_j &= u(c_j) + [1 + (\gamma - 1)(\pi(e_j) + \theta_j)]\,\varphi(n_j) \\ &\quad + \alpha_j \left[\beta_j I - c_j - n_j(a - t_j) - (1 - \tau_j)e_j n_j - T_j\right].\end{aligned}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}_j}{\partial c_j} = u'(c_j) - \alpha_j = 0, \tag{35}$$

$$\frac{\partial \mathcal{L}_j}{\partial e_j} = (\gamma - 1)\varphi(n_j)\pi'(e_j) - \alpha_j(1-\tau_j)n_j = 0, \tag{36}$$

$$\frac{\partial \mathcal{L}_j}{\partial n_j} = \left[1 + (\gamma - 1)\left(\pi(e_j) + \theta_j\right)\right]\varphi'(n_j) - \alpha_j\left[a - t_j + (1-\tau_j)e_j\right] = 0. \tag{37}$$

The question one needs to examine is how to set the tax rates such that the solution to the individual's first-order conditions (35)–(37) above coincide with the first-best solution (c, e_j, n_j) from equations (20)–(28).

First, compare equation (36), using (35), with (25) and (26). This tells us that education costs must be subsidized at a rate equal to

$$\tau_h = \frac{D}{u'(c)}\frac{1}{\delta n_h}\frac{\partial \delta}{\partial e_h}, \tag{38}$$

$$\tau_l = \frac{D}{u'(c)}\frac{1}{(1-\delta)\, n_l}\frac{\partial \delta}{\partial e_l}, \tag{39}$$

where c is set at its first-best value. To understand the intuition behind equations (38)–(39), note that the algebraic expressions in these equations are precisely the externality terms that come into play through δ as e_h and e_l change. The equations then tell us that at the optimum the subsidy rates on education must equate their marginal externality benefits. Observe also that with $\partial\delta/\partial e_h > 0, \partial\delta/\partial e_l > 0$, and $D > 0$, (38)–(39) tell us that $\tau_h > 0$ and $\tau_l > 0$. These results make sense and are due to the positive effect of investment in education on the proportion of high-ability persons in the economy.

Second, compare equation (37), using (35), with (27) and (28). We will have

$$t_h + \tau_h e_h = \frac{d}{\overline{n}^2} + \frac{D}{u'(c)}\frac{1}{\delta}\frac{\partial \delta}{\partial n_h}, \tag{40}$$

$$t_l + \tau_l e_l = \frac{d}{\overline{n}^2} + \frac{D}{u'(c)}\frac{1}{1-\delta}\frac{\partial \delta}{\partial n_l}, \tag{41}$$

The left-hand sides of (40) and (41), $t_h + \tau_h e_h$ and $t_l + \tau_l e_l$, show the *net* subsidy given to an h-type and to an l-type parent for each of his children. The right-hand sides of

(40) and (41) consist of the two externality sources described previously; they both are present when n_h and n_l change. These equations thus tell us that, at the optimum, we should subsidize the cost of having a child by an amount equal to its net externality benefit.

Recall that the cost of raising and educating a child is $a + e_j$. A child subsidy of t dollars per child reduces this cost. Similarly, a subsidy to education reduces this cost but through lowering the price of one particular element of it, namely, education cost. Thus a subsidy to education is also a subsidy to children. The difference is that the education subsidy lowers the share of education cost in total cost. On the other hand, a subsidy to children is "neutral" between the two sources of costs.

With either $\partial\delta/\partial n_h$ or $\partial\delta/\partial n_l$ being positive, equations (40)–(41) tell us that at least one of the two expressions $t_h + \tau_h e_h$ or $t_l + \tau_l e_l$ must be positive. That is, at least one of the two h- or l-type parents receive a *net* subsidy for each of their children. The other parent type, on the other hand, may receive either a net subsidy or a net tax depending on the relative size of the two expressions on the right-hand side of (40) and (41).

Finally, substituting first-best values for τ_h and τ_l from equations (38)–(39) into equations (40)–(41) yield first-best values for t_h and t_l. We have

$$t_h = \frac{d}{\overline{n}^2} + \frac{D}{u'(c)}\frac{1}{\delta}\left(\frac{\partial\delta}{\partial n_h} - \frac{e_h}{n_h}\frac{\partial\delta}{\partial e_h}\right), \tag{42}$$

$$t_l = \frac{d}{\overline{n}^2} + \frac{D}{u'(c)}\frac{1}{1-\delta}\left(\frac{\partial\delta}{\partial n_l} - \frac{e_l}{n_l}\frac{\partial\delta}{\partial e_l}\right), \tag{43}$$

where c, e_j and n_j are set at their first-best values. These equations do not allow us to determine the signs of t_h and t_l. Indeed, either one or both can be positive (*i.e.* a subsidy) as well as negative (*i.e.* a tax).

Finally, to ensure that the two types will have identical consumption levels during working years, one has to set first-period lump-sum taxes such that both individual types spend the same amount of money on c. It follows from the j-type parent's budget constraint that T_j must be set equal to

$$T_j = \beta_j I - n_j(a - t_j) - (1 - \tau_j)e_j n_j - c, \tag{44}$$

where τ_j and t_j are given according to equations (40)–(43) and e_j, n_j, and c are set at their first-best values.

The following proposition summarizes our results on decentralization.

Proposition 3 *(i) In the first-best, the parent type who has more children should invest less in education.*

(ii) Investments in education of high- and low-ability parents must be subsidized at a rate equal to the externalities they bestow to everyone as given by expressions (38)–(39).

(iii) Let t_j denote the direct child subsidy to a j-type parent in dollars. Its value must be set according to (42)–(43). Both t_h and t_l can be subsidies as well as taxes.

(iv) Direct child subsidies and education subsidies both reduce the cost of raising children. Thus a subsidy to education is also a subsidy to fertility. The difference is that the education subsidy lowers the share of education cost in the total cost. On the other hand, a subsidy to children is "neutral" between the two sources of costs.

(v) Denote the subsidy rate on education investment for the j-type by τ_j. Net subsidies to children are then equal to $t_j + \tau_j e_j$. They must be set equal to the net externalities associated with increasing n_j as shown by expressions (40) and (41). At least one of the two expressions $t_h + \tau_h e_h$ or $t_l + \tau_l e_l$ must be positive. That is, at least one of the two h- or l-type parents receive a net subsidy for each of their children; the other parent type may receive either a net subsidy or a net tax.

4 Concluding remarks

In discussing PAYGO pension plans, models with endogenous fertility have emphasized the positive externality that each person's fertility decision bestows on everybody by increasing everybody's pension benefits through a higher population growth rate. This type of externality, it has been argued, may be internalized through child subsidies. Similarly, models with endogenous human capital formation have emphasized the positive externality of investing in education of one's children (because parents cannot expropriate the children's extra earnings due to parents' education expenditures). The same argument has been put forward in cases when parents build their own human capital which they subsequently pass on to their children. These types of externalities may be internalized through education subsidies.

In this paper, we have combined the different externality sources to learn what their interactions teach us about the combination of child and education subsidies one must use to internalize them both. We have also been concerned with the question of heterogeneity of parents and how this may come into play in connection with externality-correcting policies. This is particularly relevant when child and education subsidies change the distribution of parent types. To this end, the paper has modeled endogenous fertility and human capital formation in an overlapping-generations framework wherein every generation consists of high earners and low earners with the proportion of types being determined endogenously. We have found, among other results, that:

(1) Investing in education of children by either type of parents increases the proportion of high-ability children in the economy and bestows a positive externality on everybody else. This externality has two components, one of which is specific to PAYGO pension plans.

(2) Increasing the fertility rate of one type of parents increase the proportion of high-ability children in the economy and bestows a positive externality on everybody else. An increase in the fertility rate of the other type reduces the proportion of high-ability children and imposes a negative externality on everybody else.

(3) The ambiguity in determining which parents impose a positive externality, and which ones a negative externality, by having more children is due to the fact that the type who has more children invests less in education.

(4) Direct child subsidies and education subsidies both reduce the cost of raising children. Thus a subsidy to education is also a subsidy to fertility. The difference is that the education subsidy lowers the share of education cost in the fertility subsidy. On the other hand, a subsidy to children is "neutral" between the two sources of costs.

(5) Investments in education of high- and low-ability parents must always be subsidized because they entail positive externalities.

(6) Direct child subsidies to one or both parent types can be negative; i.e., they can be taxes.

(7) Net subsidies to children of a particular parent type (direct child subsidies plus education subsidies) must be set equal to the net externalities associated with increasing the fertility rate of that type. Net child subsidies to at least one type of parents must be positive; net child subsidies to the other type can be positive or negative.

As a final observation, we remind our readers that our study has been conducted in a first-best environment. Although the main thrust of our observations should carry over to second-best environments wherein educational investments and/or types are not publicly observable, other interesting issues would also surface. We have left the examination of these other issues to a subsequent paper.

Appendix A

Proof of $2\delta(n_h - n_l) + n_l(1+\pi_l) - n_h\pi_h > 0$: Rewrite equation(1) as

$$\delta_{T+1} = \frac{\delta_T n_h \pi_h + (1-\delta_T) n_l \pi_l}{\delta_T n_h + (1-\delta_T) n_l} \equiv f\left(\delta_T, n_h, \pi_h, n_l, \pi_l\right). \tag{A1}$$

The steady-state value of δ is found from

$$\begin{cases} \delta_{T+1} = & f\left(\delta_T, n_h, \pi_h, n_l, \pi_l\right), \\ \delta_{T+1} = & \delta_T = \delta. \end{cases}$$

Differentiating δ totally with respect to π_h yields

$$\frac{d\delta}{d\pi_h} = \frac{\partial f}{\partial \delta_T}\frac{d\delta}{d\pi_h} + \frac{\partial f}{\partial \pi_h}. \tag{A2}$$

Then one finds $d\delta/d\pi_h$ from equation (A2) as

$$\frac{d\delta}{d\pi_h} = \frac{\partial f/\partial \pi_h}{1 - \partial f/\partial \delta_T}. \tag{A3}$$

Next, partially differentiate equation (A1) with respect to π_h to arrive at

$$\frac{\partial \delta_{T+1}}{\partial \pi_h} = \frac{\partial f}{\partial \pi_h} = \frac{\delta_T n_h}{\overline{n}}. \tag{A4}$$

Substituting from (A3) into (A4) yields

$$\frac{d\delta}{d\pi_h} = \frac{\delta_T n_h/\overline{n}}{1 - \partial f/\partial \delta_T},$$

or, alternatively,

$$\frac{d\delta}{de_h} = \frac{d\delta}{d\pi_h}\theta\pi'(e_h) = \frac{\delta_T n_h \theta \pi'(e_h)}{\overline{n}\left[1 - \partial f/\partial \delta_T\right]}. \tag{A5}$$

Comparing the expressions for $d\delta/de_h$ as given by equation (A5) above and equation (7) derived in the text tells us that the denominator in equations (7)–(8) is equal to the denominator of (A5). That is,

$$Z \equiv 2\delta(n_h - n_l) + n_l(1+\pi_l) - n_h\pi_h = \overline{n}\left[1 - \partial f/\partial \delta_T\right].$$

Now if $\partial f/\partial \delta_T < 0$, then $1 - \partial f/\partial \delta_T > 0 \Rightarrow Z > 0$. On the other hand, if $\partial f/\partial \delta_T > 0$, the stability condition $|\partial \delta_{T+1}/\partial \delta_T| = |\partial f/\partial \delta_T| < 1$ implies that $1 - \partial f/\partial \delta_T > 0$ and we again have $Z > 0$.

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