The Friedman rule in an overlapping-generations model with nonlinear taxation and income misreporting*

Firouz Gahvari† and Luca Micheletto‡

June 2012
Revised, June 2014

*We thank three anonymous referees for their many helpful comments and suggestions. We also gratefully acknowledge the useful comments of Spencer Bastani, Tuomas Kosonen, John D. Wilson, and the seminar participants at University of Oslo, University of Milan-Bicocca, the PET 2012 conference in Taipei, the SEA 2012 conference in New Orleans, and the IIPF 2013 conference in Taormina.

†Department of Economics, University of Illinois at Urbana-Champaign, USA. E-mail: fgahvari@illinois.edu

‡Department of Law, University of Milan, Italy, and Uppsala Center for Fiscal Studies, Uppsala University, Sweden. E-mail: luca.micheletto@unimi.it

Abstract

This paper models a two-period overlapping-generations economy with money populated with individuals of different skills. They face a nonlinear income tax schedule and can engage in tax evasion. Money serves two purposes: the traditional one, modeled through a money-in-the-utility-function; it also facilitates tax evasion. The main message of the paper is that income tax evasion in this framework leads to the violation of the Friedman rule. The paper also shows that even in the absence of tax evasion, when optimality requires differential commodity taxation, complementarity of real cash balances and labor supply does not guarantee the optimality of the Friedman rule as a boundary solution. An additional assumption is required.

JEL classification: H21; E52.

Keywords: The Friedman rule, tax evasion, overlapping-generations, second best, monetary policy, fiscal policy, redistribution.

1 Introduction

This paper brings two strands of public finance literature to bear on the question of the Friedman rule (1969) for the optimal money supply.[1] One is the optimal Mirrleesian taxation that started with Mirrlees (1971) and was popularized by Stiglitz (1982) in its simplified two-group version; the second is the tax evasion literature that followed the pioneering work of Allingham and Sandmo (1972). Our paper differs from the previous contributions on this topic with the "same" three ingredients in that it adopts a Mirrleesian rather than a Ramsey approach to optimal taxation.[2]

It is now well-known that the Friedman rule is a first-best prescription and may or may not hold in second-best settings. This depends on the nature of the second-best (existence of distortionary taxes or intrinsic reasons for market failure), the set of tax instruments available to the government, and the structure of individuals' preferences.[3] Chari *et al.* (1991, 1996), in the context of a model with identical and infinitely-lived individuals, related the optimality of Friedman rule in the presence of distortionary taxes to the uniform commodity tax result of Atkinson and Stiglitz (1972) and Sandmo (1974). This latter result states that if preferences are separable in labor supply and non-leisure goods, with the subutility for goods being homothetic, optimal commodity taxes are proportionately uniform. They showed that deviations from Friedman rule violates this tax principle.[4]

These studies, being carried out in an environment with identical individuals, are

[1] The Friedman rule calls for a deflationary change in money supply that ensures the nominal interest rate—the opportunity cost of money holding—is zero (equal to the marginal cost of printing money). The classic reference for the Friedman rule is Friedman (1969). The earlier literature referred to it as the Chicago rule; see Niehans (1978).

[2] See Arbex and Turdaliev (2011) and the references therein for examples of the literature that examines the relevance of tax evasion for the Friedman rule from a Ramsey tax perspective.

[3] Non-optimality of Friedman rule in the presence of distortionary taxes was first discussed by Phelps (1973). A selective reference to other sources of distortion include: van der Ploeg and Alogoskoufis (1994) for an externality underlying endogenous growth; Ireland (1996) for monopolistic competition; Erceg *et al.* (2000) and Khan *et al.* (2003) for nominal wage and price settings; Schmitt-Grohe and Uribe (2004) for imperfections in the goods market; and Shaw *et al.* (2006) for imperfect competition as well as externality.

[4] This uniformity result is derived within the context of the traditional one-consumer Ramsey problem. As such, the result embodies only efficiency considerations. Redistributive goals do not come into play.

by construct silent on the validity of the Friedman rule when monetary policy has redistributive implications.[5] A second related drawback of these studies is their reliance on the Ramsey tax framework, which assumes that all tax instruments, including the income tax, are set linearly.

In a recent contribution, da Costa and Werning (2008) break with this tradition and consider the optimality of the Friedman rule in a model with heterogeneous agents and allow the government to levy nonlinear income taxes. Interestingly, they show that the Friedman rule is optimal in their setting (for any social welfare function that redistributes from the rich to the poor). As with Chari *et al.*'s (1991, 1996) earlier result, da Costa and Werning's (2008) finding is also related to the uniform taxation result in public finance, albeit a different one. Whereas Chari *et al.* (1991, 1996) draw on Sandmo's tax uniformity (1974) result derived within a Ramsey setting, da Costa and Werning's (2008) has its roots in Atkinson and Stiglitz (1976). This classic paper on the design of tax structures was particularly concerned with the usefulness of commodity taxes in the presence of a general income tax in economies with heterogeneous agents.[6]

Atkinson and Stiglitz (1976) proved that with a general income tax, if preferences are weakly separable in labor supply and goods, then commodity taxes are not needed as instruments of optimal tax policy. With non-separability, one wants to tax the goods that are "substitutes" with labor supply and subsidize those that are "complements" with labor supply. In da Costa and Werning (2008) the uniformity result, which implies a zero nominal interest rate, holds with preference separability. With non-separable preferences, da Costa and Werning assume that real cash balances and labor supply are complements so that cash balances should be subsidized. This implies that the optimal nominal interest rate is negative. But given the non-negativity of nominal interest rate,

[5] With the exception of intergenerational redistributive issues that arise in overlapping generations models; see, e.g., Weiss (1980), Abel (1987), and Gahvari (1988, 2007, 2012).

[6] The ineffectiveness of commodity taxes and their proportionately uniform structure boil down to the same thing. In the absence of exogenous incomes, the government has an extra degree of freedom in setting its income and commodity tax instruments. This is because all demand and supply functions are homogeneous of degree zero in consumer prices. In consequence, the government can, without any loss of generality, set one of the commodity taxes at zero (i.e. set one of the commodity prices at one). Under this normalization, uniform rates imply absence of commodity taxes.

the zero interest rate emerges as the "optimal" policy.

da Costa and Werning's (2008) results as well as the earlier Chari *et al.*'s (1991, 1996) results are all derived in settings that disregard tax evasion. Yet many empirical studies over the past few decades confirm that tax evasion is a widespread phenomenon all over the world; see Shaw *et al.* (2011) for a recent survey. Now it is also the case that introducing tax evasion into the optimal tax problem often invalidates policy lessons drawn ignoring this phenomenon. For example, in the context of the uniform taxation results, Cremer and Gahvari (1993) prove that the Ramsey results are no longer valid. Similarly, Boadway *et al.* (1994) show how the presence of tax evasion destroys the celebrated Atkinson and Stiglitz (1976) theorem on the redundancy of commodity taxes in the presence of Mirrleesian optimal income tax if preferences are weakly separable in labor supply and goods. One would then expect the same fate for the Friedman rule. This is indeed the case and there are a number of papers that make this. However, they are all written in the context of Ramsey taxes. There are no such studies to date using the Mirrleesian tax framework. Demonstrating this point constitutes the major contribution of this paper; it is not, however, the only contribution of the paper.

The paper first examines the robustness of da Costa and Werning's (2008) results. It re-examines these results in the context of a two-period overlapping-generations economy populated by two types of individuals: high-skilled and low-skilled. This is in contrast to da Costa and Werning (2008) who posit an economy populated by a continuum of infinitely-lived individuals. It shows that for their results to go through an additional assumption is required. In particular, contrary to their result, complementarity of real cash balances and labor supply alone does not guarantee the optimality of the Friedman rule as a boundary solution. We derive an additional condition to ensure this result. We argue that da Costa and Werning's (2008) result to the contrary arises because there is no differential commodity taxes in their model.

Having examined the robustness question, the paper turns to the discussion of its main message; namely, that the absence of tax evasion is crucial for da Costa and Werning's (2008) results. When agents have access to a misreporting technology, which allows them to shelter part of their earned income from the tax authority, monetary policy becomes another useful instrument for redistribution. In particular, income tax evasion invalidates the uniform commodity tax result of Atkinson and Stiglitz (1976) thus rendering the monetary growth rate a redistributive power that otherwise it does not possess. As a result, the presence of tax evasion invalidates da Costa and Werning's (2008) first result on the optimality of the Friedman rule as an interior solution if the conditions for Atkinson and Stiglitz (1976) theorem hold. da Costa and Werning's (2008) second result, on the optimality of the Friedman rule as a boundary solution if real cash balances and labor supply are complements, is no longer guaranteed either. This is because, in the presence of tax evasion, one does not know which type of agents supplies more labor (at the same level of reported income). Hence the complementarity assumption does not identify the type who demands more real cash balances.

2 The model

Consider a two-period overlapping generations (hereafter OLG) model wherein individuals work in the first period and consume in both. There is no bequest motive. Preferences are represented by the strictly quasi-concave utility function $U = u\left(c_t, d_{t+1}, x_t, L_t\right)$ where c denotes consumption when young, d consumption when old, x real money balances (held for non-evading activities)[7], and L labor supply; subscript t denotes calendar time. While the utility function is assumed to be strictly increasing in c_t and d_{t+1}, and strictly decreasing in L_t, the possibility of satiation in real balances is not ruled out (i.e. $\lim_{x \to x^{sat}} \partial u / \partial x = 0$ at the "satiation level" x^{sat}). Each generation consists of two types of individuals who differ in skill levels (labor productivity). High-skilled workers are paid w_t^h and low-skilled workers w_t^ℓ; with $w_t^h > w_t^\ell$. The proportion of agents of type j, π^j, $j = h, \ell$, remains constant over time. Denote the number of young agents of type j born in period t by n_t^j and the total number of young agents by N_t. We have $n_t^j / N_t = \pi^j$. While π^j remains constant, population grows over time at a constant rate,

[7] Additionally, we will allow for money to have a separate "evasion-facilitating" usage; see subsection 2.1.1 below.

g.

Production takes place through a linear technology with different types of labor as inputs. Transfer of resources to the future occurs only through a storage technology with a fixed (real) rate of return, r.[8] We thus work with an OLG model à la Samuelson (1958) and assume away the issues related to capital accumulation.

2.1 Money and monetary policy

At the beginning of period t, before consumption takes place, the young purchase all the existing stock of money, M_t, from the old. Denote a young j-type agent's purchases by m_t^j. We have

$$M_t = n_t^h m_t^h + n_t^\ell m_t^\ell. \tag{1}$$

The rate of return on money holdings (the nominal interest rate), i_{t+1}, is related to the inflation rate, φ_{t+1}, according to Fisher equation

$$1 + i_{t+1} \equiv (1+r)\left(1+\varphi_{t+1}\right). \tag{2}$$

Denote the price level at time t by p_t; the inflation rate is defined as

$$1 + \varphi_{t+1} \equiv \frac{p_{t+1}}{p_t}. \tag{3}$$

The monetary authority injects money into (or retires money from) the economy at the constant rate of θ.[9] Money is given to (or taken from) the old—who hold all the stock of money—via lump-sum monetary transfers (or taxes). Thus a young j-type agent who purchases m_t^j at the beginning of time t receives e_{t+1}^j at the beginning of period $t+1$. Clearly, e_{t+1}^h and e_{t+1}^ℓ must satisfy the "money injection relationship",

$$n_t^h e_{t+1}^h + n_t^\ell e_{t+1}^\ell = \theta M_t. \tag{4}$$

[8] An alternative assumption is that agents borrow and lend on international capital markets at an exogenously fixed interest rate.

[9] Given a positive real interest rate, in the absence of population growth, θ will have to be negative for the nominal interest rate to be zero as required by the Friedman rule. With population growth, the Friedman rule is compatible with a positive θ (as well as a negative θ). Either way, the fact that the nominal interest rate cannot be negative sets a lower bound on θ. See also footnote 22 below.

Beyond this, we do not specify how much of the extra money injection goes to which type. Indeed, one can show that in the presence of a general income tax this division is immaterial; see Gahvari and Micheletto (2014).

With money stock changing at the rate of θ in every period, we have $M_{t+1} = (1+\theta) M_t$. Substitute for M_t and M_{t+1}, from equation (1), into this relationship:

$$n^h_{t+1} m^h_{t+1} + n^\ell_{t+1} m^\ell_{t+1} = (1+\theta) \left(n^h_t m^h_t + n^\ell_t m^\ell_t \right).$$

Given that the population of each type grows at the constant rate of g, one can rewrite this as[10]

$$n^h_t \left(m^h_{t+1} - \frac{1+\theta}{1+g} m^h_t \right) + n^\ell_t \left(m^\ell_{t+1} - \frac{1+\theta}{1+g} m^\ell_t \right) = 0.$$

Assume that the money-holdings of each type changes in the same direction.[11] It follows from the above relationship that

$$m^j_{t+1} = \frac{1+\theta}{1+g} m^j_t. \tag{5}$$

2.1.1 Money and tax evasion

As in da Costa and Werning (2008), we rationalize money by allowing real cash balances to enter the utility function.[12] However, in the presence of tax evasion, we recognize

[10] Observe that $(1+g)\, m^j_{t+1}$ is not necessarily equal to $m^j_t + e^j_{t+1}$. This will be the case only if the money disbursement to type j is set according to $e^j_{t+1} = \theta m^j_t$.

[11] This assumption applies only to the sign and not the magnitude of such possible changes. Observe also that this is an assumption on the *equilibrium money holdings* as opposed to *money purchases* that may very well go in different directions depending on who gets the new money injections (or loses them). It is a natural assumption because there are no stochastic shocks in this model so that in going from one year to the next the opportunity sets and the prices faced by different agent types change in the same manner. Nor does the government follow a capricious redistributive policy changing the social welfare weights of different groups from one year to the next. If goods including real cash balances are normal, both types end up changing all their consumption levels in the same direction.

[12] Prior studies that use money-in-the-utility-function in conjunction with an overlapping-generations model include Weiss (1980), Abel (1987) and Gahvari (1988). In these papers, however, all individuals within a generation are identical. In the late 70s and early 80s, the overlapping-generations model was considered as a substitute for Hicks-Patinkin money-in-the-utility-function and Clower cash-in-advance constructs in rationalizing money. However, it was not too long before it was realized that, in the words of McCallum (1983): "As a 'model of money', the basic OG [overlapping generations] structure–which excludes cash-in-advance or money-in-the-utility-function (MIUF) appendages–seems inadequate and potentially misleading, the reason being that it neglects the medium-of-exchange property of money" (p 36), and "...there is no particular reason why cash-in-advance, MIUF, or other appendages designed to reflect the medium-of-exchange property should not be used in conjunction with the OG framework" (p 37).

another usage for real cash balances besides what is modeled through a money-in-the-utility-function construct. Specifically, we assume that the j-type's real money holdings, m_t^j/p_t, consists of two components. One component, x_t^j, enters the utility function and accounts for the traditional (non-evading) usage. The other component is solely for the purpose of facilitating tax evasion. Using cash allows transactions to go undocumented and thus easier to conceal from tax authorities. Empirically, too, it is a well-documented fact that people who engage in tax evasion often do so by conducting their transactions in cash; see, e.g., Morse *et al.* (2009).

It is worth pointing out, however, that allowing for this additional justification for money holdings is of no consequence for our results; they remain unaffected without it. We adopt this formulation only to provide an explanation for the empirical observation that tax evasion is often associated with larger cash holdings. Without such explicit modeling, and with money holdings and income concealed both being endogenous variables, there is no guarantee that tax evasion and cash holdings are positively correlated (even though our results remain the same).[13]

To simplify matters, we model this second component by assuming that it is proportional to the amount of income concealed from the tax authority.[14] Let $a_t^j > 0$ denote (real) income concealed by the j-type individual at time t.[15] To make this possible, we assume that the evader holds $\beta p_t a_t^j$ in cash over and above $p_t x_t^j$ that he holds for other reasons where β is a positive constant less than one. Consequently, total real money

[13] Of course one can impose strict restrictions on preferences to yield this result. But this procedure would be quite ad hoc and less satisfactory than our explicit modeling.

[14] Obviously, a more "realistic" model would make this component to be a function of income concealed rather than being proportional to it. However, our purpose in this paper is not to model tax evasion. Our aim is simply to account for the empirical observation of a positive relationship between real cash holdings and evasion.

[15] While we often speak of under-reporting and tax evasion ($a_t > 0$), in principle, over-reporting ($a_t < 0$) is also possible in our model. While a positive marginal income tax rate creates an incentive for under-reporting, a negative marginal income tax rate makes over-reporting desirable. In turn, the possibility of a negative marginal *income* tax rate arises because of the existence of commodity taxes, and in our model also inflation, in the system (see, e.g., Edwards *et al.* , 1994). None of our results depends on the sign of a_t. See the numerical example of Section 5, wherein skilled workers face a negative marginal income tax rate and over-report.

balances in our model is given by[16]

$$\frac{m_t^j}{p_t} = x_t^j + \beta a_t^j. \tag{6}$$

2.2 Fiscal policy

The tax authority levies income and commodity taxes to maximize a social welfare function defined over the utility of all agents in the economy. The government knows the distribution of types in the population but it does not know the identity of the types. Consequently, type-specific lump-sum taxes are not implementable. Earned incomes are not publicly observable either. Income reported by agents for tax purposes may thus deviate from true earned income due to the possibility of income-misreporting. To model income-misreporting in the simplest possible way, we follow the riskless approach introduced by Usher (1986) and subsequently used in a number of contributions to the tax evasion literature including Boadway *et al.* (1994).[17] Under this approach, once agents have incurred some pecuniary cost that depends on the amount they misreport, they face no risk of detection. What the fiscal authority can rely on is thus taxing income reported by agents, which will be denoted by I_t, via a general nonlinear income tax $T(I_t)$.

With the true income being equal to $w_t L_t$, the amount of income concealed is equal to $a_t = w_t L_t - I_t$. The cost of misreporting is expressed by means of the function $f(a_t)$. Assume that $f(\cdot)$ is non-negative, increasing in the absolute value of a_t and strictly convex with $f(0) = f'(0) = 0$. Finally, assume that the information the tax authority has on transactions, including money holdings, is of anonymous nature; it does not know the identity of the purchasers. This assumption, which is made for realism, implies that goods can be taxed only linearly (possibly at different rates).

[16] No extra cash holding is required with over-reporting (or not evading). Consequently, if $a_t \leq 0$ then $\beta = 0$ so that βa_t^j vanishes and equation (6) simplifies to $x_t^j = m_t^j / p_t$.

[17] Interestingly, our results remain unaffected if we were to model income misreporting as a risky activity, which can be discovered by the tax authority through costly audits and punished according to a penalty function. Gahvari and Micheletto (2013) show this using Cremer and Gahvari's (1995) formulation of tax evasion in a Mirrleesian framework.

2.3 Constrained Pareto-efficient allocations

To characterize the (constrained) Pareto-efficient allocations, one has to account for the economy's resource balance, the standard incentive compatibility constraints due to our informational structure, and the implementability constraints caused by linearity of commodity taxes—itself due to informational constraint, as well as the monetary expansion mechanism. To this end, we follow Cremer and Gahvari's (1997) approach and derive an optimal revelation mechanism. For our purpose, a mechanism consists of a set of type-specific before-tax reported labor incomes, I_t^j's, "assigned" after-tax incomes, z_t^j's, commodity tax rates on consumption when young and old, τ^c and τ^d, a money supply growth rate, θ, and a monetary distributive rule, e_{t+1}^j.[18] This procedure determines τ^c, τ^d, θ, and e_{t+1}^j from the outset. A complete solution to the optimal tax problem per-se, i.e. determination of I_t^j by the individuals via utility maximization, then requires only the design of a general income tax function $T\left(I_t\right)$ such that $z_t^j = I_t^j - T\left(I_t^j\right)$.[19]

To proceed further, it is necessary to consider the optimization problem of an individual for a given mechanism $\left(\tau^c, \tau^d, \theta, e_{t+1}, z_t, I_t\right)$. This is necessitated by the fact that the mechanism determines personal consumption levels only indirectly, namely through prices. The mechanism assigns the sextuple $\left(\tau^c, \tau^d, \theta, e_{t+1}^j, z_t^j, I_t^j\right)$ to a young individual who reports his type as j. The individual will then allocate z_t^j, and any other disposable income that he may have, between first- and second-period consumption, and real money balances in order to maximize his utility:

$$u_t^j = u\left(c_t, d_{t+1}, x_t, \frac{I_t^j + a_t}{w_t^j}\right), \quad j = h, \ell.$$

This procedure yields the conditional demands for the j-type's first- and second-

[18] If the money disbursements to skilled and unskilled workers were set according to $e_{t+1}^j = \theta m_t^j$, then once θ is determined, so will e_{t+1}^j. The revelation mechanism will then be reduced to a quintuple $\left(\tau^c, \tau^d, \theta, z^j, I^j\right)$.

[19] This formulation assumes that consumption expenditures are not publicly observable at a personal level. Strictly speaking, this procedure does not characterize allocations as such; the optimization is over a mix of quantities *and* prices. However, given the commodity prices, utility maximizing households would choose the quantities themselves. We can thus think of the procedure as indirectly determining the final allocations.

period consumption, real money balances, and the concealed labor income; see the Appendix. One can then write the conditional demand functions, and the concealed labor income, when facing $\left(\tau^c, \tau^d, \theta, e_{t+1}, z_t, I_t\right)$, as

$$\begin{aligned}
c_t^j &= c\left(q_t^c, q_{t+1}^d, q_t^x, z_t + b_{t+1}, I_t, w_t^j\right), && (7)\\
d_{t+1}^j &= d\left(q_t^c, q_{t+1}^d, q_t^x, z_t + b_{t+1}, I_t, w_t^j\right), && (8)\\
x_t^j &= x\left(q_t^c, q_{t+1}^d, q_t^x, z_t + b_{t+1}, I_t, w_t^j\right), && (9)\\
a_t^j &= a\left(q_t^c, q_{t+1}^d, q_t^x, z_t + b_{t+1}, I_t, w_t^j\right), && (10)
\end{aligned}$$

where

$$\begin{aligned}
q_t^c &\equiv 1 + \tau^c, && (11)\\
q_{t+1}^d &\equiv \frac{1 + \tau^d}{1 + r}, && (12)\\
q_t^x &\equiv \frac{i_{t+1}}{1 + i_{t+1}}, && (13)\\
b_{t+1}^j &\equiv \frac{e_{t+1}^j}{p_{t+1}\left(1 + r\right)}. && (14)
\end{aligned}$$

Observe that equation (10) also determines j-type's labor supply, $\left(I_t + a_t^j\right)/w_t^j$. When incomes are observable, there will not be such an equation so that $a_t^j = 0$. Assigning I_t to an individual then determines his labor supply, I_t/w_t^j.

As a final observation, it is crucially important to realize that, in this model, an individual's total expenditures on goods, his (actual) disposable income, is not just $z_t + b_{t+1}$ as it would be the case in the absence of misreporting. With $i_{t+1}/\left(1 + i_{t+1}\right)$ being the opportunity cost of holding one dollar in cash, $f\left(a_t\right) + i_{t+1}\beta a_t/\left(1 + i_{t+1}\right)$ is the total cost of concealing a_t. Consequently, the individual's disposable income is equal to

$$z_t + b_{t+1} + a_t - f\left(a_t\right) - \beta\frac{i_{t+1}}{1 + i_{t+1}}a_t, \qquad (15)$$

which includes income evaded, a_t, net of concealment costs where concealment costs also include the opportunity cost of holding money for concealment.[20] The disposable

[20] See also equation (A4) in the Appendix which shows the unified intertemporal budget constraint for the young.

income of a particular type thus depends on whether or not he evades and to what extent.

We summarize our discussion thus far regarding the determination of the temporal equilibrium of this economy as,

Proposition 1 *Consider an overlapping-generations model à la Samuelson (1958) with money wherein money holdings are rationalized by a money-in-the-utility-function approach. There are two types of agents, skilled and unskilled workers, denoted by h and ℓ. Both types grow at a constant rate so that the proportion of each type in the total population remains constant over time. Let a young j-type individual face, at time t, the sextuple $\left(\tau^c, \tau^d, \theta, e^j_{t+1}, z^j_t, I^j_t\right)$, where τ^c is the tax rate on first-period consumption, τ^d is the tax rate on second-period consumption, θ is the money growth (or contraction) rate, e^j_{t+1} is the j-type's allotment of money injection (or money withdrawal) to be given in second period, z^j_t is the j-type's net-of-tax reported income, and I^j_t is the j-type's before-tax reported income; $j = h, \ell$. Reported income differs from actual earnings by the amount misreported, a^j_t. Under the perfect foresight assumption, the period by period equilibrium of this economy is characterized by equations (1)–(3), and (7)–(10), where the last four equations hold for both $j = h, \ell$.*

2.4 Mechanism designer

It remains for us to specify how the mechanism designer chooses $\left(\tau^c, \tau^d, \theta, e^j_{t+1}, z^j_t, I^j_t\right)$. This will complete the characterization of the set of (constrained) Pareto-efficient allocations in every period under the perfect-foresight assumption.

Substituting the values of c^j_t, d^j_{t+1}, x^j_t and a^j_t, from (7)–(10), in the young j-type's utility function (A1) facing $\left(\tau^c, \tau^d, \theta, e_{t+1}, z_t, I_t\right)$ yields his conditional indirect utility function,

$$v\left(q^c_t, q^d_{t+1}, q^x_t, z_t + b_{t+1}, I_t, w^j_t\right) \equiv$$
$$u\left(\begin{array}{c} c\left(q^c_t, q^d_{t+1}, q^x_t, z_t + b_{t+1}, I_t, w^j_t\right), d\left(q^c_t, q^d_{t+1}, q^x_t, z_t + b_{t+1}, I_t, w^j_t\right), \\ x\left(q^c_t, q^d_{t+1}, q^x_t, z_t + b_{t+1}, I_t, w^j_t\right), \frac{I_t + a\left(q^c_t, q^d_{t+1}, q^x_t, z_t + b_{t+1}, I_t, w^j_t\right)}{w^j_t} \end{array}\right). \tag{16}$$

Let δ^j's denote positive constants with the normalization $\sum_{j=\ell,h} \delta^j = 1$. The mechanism designer maximizes

$$\sum_{j=\ell,h} \delta^j v\left(q_t^c, q_{t+1}^d, q_t^x, z_t^j + b_{t+1}^j, I_t^j, w_t^j\right),$$

with respect to $\tau^c, \tau^d, \theta, e_{t+1}^j, z_t^j$ and I_t^j; subject to the government's generational budget constraint,

$$n_t^h\left(I_t^h - z_t^h\right) + n_t^\ell\left(I_t^\ell - z_t^\ell\right) + \tau^c\left(n_t^h c_t^h + n_t^\ell c_t^\ell\right) + \frac{\tau^d}{1+r}\left(n_t^h d_{t+1}^h + n_t^\ell d_{t+1}^\ell\right) \geq N_t \bar{R}, \quad (17)$$

where $\bar{R}$ is an exogenous per-young revenue requirement, the money injection relationship (4), and the self-selection constraints

$$v\left(q_t^c, q_{t+1}^d, q_t^x, z_t^h + b_{t+1}^h, I_t^h, w_t^h\right) \geq v\left(q_t^c, q_{t+1}^d, q_t^x, z_t^\ell + b_{t+1}^\ell, I_t^\ell, w_t^h\right), \quad (18)$$

$$v\left(q_t^c, q_{t+1}^d, q_t^x, z_t^\ell + b_{t+1}^\ell, I_t^\ell, w_t^\ell\right) \geq v\left(q_t^c, q_{t+1}^d, q_t^x, z_t^h + b_{t+1}^h, I_t^h, w_t^\ell\right). \quad (19)$$

The constraints (18)–(19) require that each type of agents must (weakly) prefer the bundle intended for them to that intended for the other type. An agent who misrepresent his true type by choosing the bundle intended for another type is called "mimicker". In particular, in what follows, we shall refer to an agent of type j who mimics an agent of type k as a jk-agent or a jk-mimicker. We shall discuss the solution to the mechanism designer's problem after it reaches its steady-state equilibrium in Section 3 below.

3 Steady state, optimal taxes, and the Friedman rule

In the temporal equilibrium of our model, laid out in Section 2, the equality of aggregate money demand and aggregate money supply at time t requires, from equation (1), that $N_t\left(\pi^h m_t^h + \pi^\ell m_t^\ell\right) = M_t$. Divide this relationship by $N_t p_t$ and rewrite it as,

$$\pi^h \frac{m_t^h}{p_t} + \pi^\ell \frac{m_t^\ell}{p_t} = \frac{(1+\theta) M_{t-1}}{N_t p_t}. \quad (20)$$

Now, from the consumers' optimization problem at time t under the perfect-foresight assumption, one determines x_t^h, x_t^ℓ, a_t^h, and a_t^ℓ as functions of p_{t+1}. As a result, $m_t^h/p_t = x_t^h + \beta a_t^h$ and $m_t^\ell/p_t = x_t^\ell + \beta a_t^\ell$ are also determined as functions of p_{t+1}. Given that

θ, M_{t-1},and N_t are all predetermined variables, equation (20) defines a relationship between p_t and p_{t+1}. The dynamics of the model are then described by this equation and equation (5), $m_{t+1}^j = [(1+\theta)/(1+g)]\, m_t^j$. The economy reaches a steady-state equilibrium when $p_{t+1} = [(1+\theta)/(1+g)]\, p_t$ so that m_t^j/p_t remains constant over time. We shall assume that the steady-state equilibrium exists and is unique. Section 5 presents an example of such an steady-state equilibrium.[21]

With the steady-state being characterized by

$$1+\varphi = \frac{1+\theta}{1+g},$$

equation (2) establishes the steady-state value of the nominal interest rate, i, according to

$$1+i = \frac{1+r}{1+g}(1+\theta). \tag{21}$$

Similarly, from (13), the steady-state value of q_t^x is written as,

$$q^x \equiv \frac{i}{1+i}. \tag{22}$$

Let, for all $j = h, \ell$: $c_{t+1}^j = c_t^j \equiv c^j$, $d_{t+1}^j = d_t^j \equiv d^j$, $x_{t+1}^j = x_t^j \equiv x^j$, $a_{t+1}^j = a_t^j \equiv a^j$, $I_{t+1}^j = I_t^j \equiv I^j$, $z_{t+1}^j = z_t^j \equiv z^j$, and $b_{t+1}^j = b_t^j \equiv b^j$. Introduce

$$y^j \equiv z^j + b^j, \tag{23}$$

to denote the j-type's aggregate "observable" disposable income. One can then write the steady-state versions of equations (7)–(10) as,

$$c^j \equiv c\left(q^c, q^d, q^x, y^j, I^j, w^j\right), \tag{24}$$

$$d^j \equiv d\left(q^c, q^d, q^x, y^j, I^j, w^j\right), \tag{25}$$

[21] If the steady-state equilibrium is unstable, the equilibrium path will be unique. Otherwise, one cannot rule out the possibility of multiple equilibrium paths. One will also have to worry about hyperinflationary and implosive bubbles. The problem of hyperinflationary equilibria concerns the possibility of money, in the limit, being driven out of the system despite the regularity assumptions on the preferences which ensure a positive holding of real balances all along a solution path. Scheinkman (1980) and Obstfeld and Rogoff (1983) have shown that a sufficient condition— though not a necessary one—to rule this possibility out is to impose the restriction $\lim_{x\to 0} x\partial u/\partial x > 0$ on preferences. This restriction makes money, in some sense, essential to the economy. Similarly, Obstfeld and Rogoff (1986) argue that mild and reasonable conditions are sufficient to rule implosive bubbles out (bubbles where the price level follows an implosive path going to zero).

$$x^j \equiv x\left(q^c, q^d, q^x, y^j, I^j, w^j\right), \tag{26}$$

$$a^j \equiv a\left(q^c, q^d, q^x, y^j, I^j, w^j\right). \tag{27}$$

Other equations of interest are the steady-state versions of the young j-type's intertemporal budget constraint [equation (A4) in the Appendix], and his conditional indirect utility function (16). These are given by

$$q^c c^j + q^d d^j + q^x x^j = y^j + a^j - f\left(a^j\right) - q^x \beta a^j, \tag{28}$$

$$v^j = v\left(q^c, q^d, q^x, y^j, I^j, w^j\right), \tag{29}$$

where $y^j + a^j - f\left(a^j\right) - q^x \beta a^j$ is the j-type's disposable income. To derive the steady-state version of the government's budget constraint, divide equation (17) by N_t to write

$$\sum_{j=\ell,h} \pi^j \left(I^j - z^j\right) + \tau^c \sum_{j=\ell,h} \pi^j c^j + \frac{\tau^d}{1+r} \sum_{j=\ell,h} \pi^j d^j \geq \bar{R}. \tag{30}$$

Additionally, there is a relationship between money disbursements in real terms, b^j, and real cash balances, $x^j + \beta a^j$. This is equal to (see the Appendix),

$$\sum_{j=\ell,h} \pi^j b^j = \frac{1+g}{1+r} \frac{\theta}{1+\theta} \sum_{j=\ell,h} \pi^j \left(x^j + \beta a^j\right). \tag{31}$$

Finally, one can write the "jk-mimicker's" demand functions for c and d, his concealed labor income, and his conditional indirect utility function as,

$$c^{jk} = c\left(q^c, q^d, q^x, y^k, I^k, w^j\right), \tag{32}$$

$$d^{jk} = d\left(q^c, q^d, q^x, y^k, I^k, w^j\right), \tag{33}$$

$$x^{jk} = x\left(q^c, q^d, q^x, y^k, I^k, w^j\right), \tag{34}$$

$$a^{jk} = a\left(q^c, q^d, q^x, y^k, I^k, w^j\right), \tag{35}$$

$$v^{jk} = v\left(q^c, q^d, q^x, y^k, I^k, w^j\right). \tag{36}$$

We have,

Proposition 2 *Consider the overlapping generations model of Proposition 1. Its steady-state equilibrium, assuming it exists and is unique, is characterized by equations (21)–(27). Secondly, let* v^j *and* v^{jk}*, defined by equations (29) and (36), denote the conditional indirect utility function of the young* j*-type and* jk*-type agents;* $j = h,\ \ell$ *and* $k \neq j$*. Let* δ^j*'s be positive constants with the normalization* $\sum_{j=\ell,h} \delta^j = 1$*. The constrained Pareto-efficient allocations are described by the maximization of* $\sum_{j=\ell,h} \delta^j v^j$ *with respect to* $\tau^c, \tau^d, \theta, b^j, z^j$ *and* I^j*; subject to the government's budget constraint (30), the money injection constraint (31), and the self-selection constraints* $v^h \geq v^{h\ell}$ *and* $v^\ell \geq v^{\ell h}$*.*

We are now ready to investigate whether, in our model with nonlinear income tax and income misreporting, the Friedman rule (hereafter FR) is part of an optimal policy or not. According to the FR, optimality requires a zero opportunity cost of holding real cash balances. This is often stated in terms of choosing a rate of growth for money supply such that the nominal interest rate is equal to zero. This is the case because $q^x = i/(1+i)$ so that $i = 0$ implies $q^x = 0$.

3.1 Optimal tax/monetary policy

To characterize the optimal tax/monetary policy, using the mechanism design approach, we follow the common practice in the literature and ignore the "upward" incentive constraint, $v^\ell \geq v^{\ell h}$; assuming that it is automatically satisfied. Thus, the only possible binding constraint will be that of the high-skilled agents mimicking low-skilled agents. Intuitively, this implies that we are concerned only with the realistic case of redistribution from the high-skilled to the low-skilled agents. Focusing on the steady-state equilibrium, the mechanism designer's problem can then be represented as:

$$\max_{I^j, z^j, b^j, \tau^c, \tau^d, \theta} \sum_{j=\ell,h} \delta^j v\left(q^c, q^d, q^x, z^j + b^j, I^j, w^j\right),$$

subject to the government's budget constraint,

$$\sum_{j=\ell,h} \pi^j \left(I^j - z^j + \tau^c c^j + \frac{\tau^d}{1+r} d^j\right) \geq \bar{R}, \qquad (\mu)$$

the money injection relationship (31),

$$\sum_{j=\ell,h} \pi^j b^j = \frac{1+g}{1+r}\frac{\theta}{1+\theta} \sum_{j=\ell,h} \pi^j \left(x^j + \beta a^j\right), \qquad (\eta)$$

the self-selection constraint

$$v\left(q^c, q^d, q^x, z^h + b^h, I^h, w^h\right) \geq v\left(q^c, q^d, q^x, z^\ell + b^\ell, I^\ell, w^h\right), \qquad (\lambda)$$

and a final constraint for the non-negativity of the nominal interest rate i,

$$i \geq 0, \qquad (\gamma),$$

where the Greek letters on the right-hand side of each constraint denotes its corresponding Lagrange multiplier.[22]

Given the redundancy of one of the redistributive instruments b^h and b^ℓ, it is sufficient to carry out our optimization with respect to only b^h or b^ℓ. Without any loss of generality, we will choose b^h. The mechanism designer then determines $I^h, I^\ell, z^h, z^\ell, b^h, \tau^c, \tau^d$ and θ. In turn, consumers determine their demands for consumption goods c, d, real balances, x, and the amount of income they conceal, a (thus determining their labor supply as well).

With income misreporting one cannot rely on the standard argument in optimal tax models that justifies normalizing, without loss of generality, one of the commodity tax rates to zero.[23] Consequently, the mechanism designer must optimize with respect to

[22] The non-negativity constraint on i places a lower bound on the feasible value of θ. In particular, it follows from (21) that $i \geq 0 \implies \theta \geq (g-r)/(1+r)$.

[23] The normalization argument is based on the observation that the demands for various goods are homogeneous of degree zero in consumer prices and disposable income. Thus, as long as relative prices of the various goods are kept fixed, any effect of a proportionately uniform increase or decrease in the vector of commodity tax rates can be offset via a proper adjustment in the income tax schedule. That with income misreporting this property no longer holds can be seen by inspecting the j-type's optimization problem of subsection 2.3. The conditional demand functions (7)–(10), derived from maximization of (A1) subject to (A4) that yield first-order conditions (A5)–(A7), are not homogeneous of degree zero in prices q_t^c, q_{t+1}^d, q_t^x, and income $y_t = z_t + b_{t+1}$ (for a given I_t and w_t^j). For further discussion of this issue, see also footnote 32 below.

τ^c, τ^d and θ. Denote compensated (Hicksian) variables by a "tilde", so that, for instance, $\widetilde{x}^j$ denotes the j-type's compensated demand for x. The following Proposition, proved in the Appendix, characterizes the optimal policy with respect to the choice of τ^c, τ^d and θ.

Proposition 3 *Let $\alpha^{h\ell}$ denote the marginal utility of income for the $h\ell$-mimicker. In the steady-state equilibrium of Proposition 2, the optimal solution for τ^c, τ^d and θ satisfy:*

$$\sum_j \pi^j \left[\tau^c \frac{\partial \widetilde{c}^j}{\partial q^c} + \frac{\tau^d}{1+r}\frac{\partial \widetilde{d}^j}{\partial q^c} + \frac{1+g}{1+r}\frac{\theta}{1+\theta}\left(\frac{\partial \widetilde{x}^j}{\partial q^c} + \beta \frac{\partial \widetilde{a}^j}{\partial q^c}\right)\right] = \frac{\lambda \alpha^{h\ell}}{\mu}\left(c^\ell - c^{h\ell}\right), \quad (37)$$

$$\sum_j \pi^j \left[\tau^c \frac{\partial \widetilde{c}^j}{\partial q^d} + \frac{\tau^d}{1+r}\frac{\partial \widetilde{d}^j}{\partial q^d} + \frac{1+g}{1+r}\frac{\theta}{1+\theta}\left(\frac{\partial \widetilde{x}^j}{\partial q^d} + \beta \frac{\partial \widetilde{a}^j}{\partial q^d}\right)\right] = \frac{\lambda \alpha^{h\ell}}{\mu}\left(d^\ell - d^{h\ell}\right), \quad (38)$$

$$\sum_j \pi^j \left[\tau^c \frac{\partial \widetilde{c}^j}{\partial q^x} + \frac{\tau^d}{1+r}\frac{\partial \widetilde{d}^j}{\partial q^x} + \frac{1+g}{1+r}\frac{\theta}{1+\theta}\left(\frac{\partial \widetilde{x}^j}{\partial q^x} + \beta \frac{\partial \widetilde{a}^j}{\partial q^x}\right)\right] =$$
$$\frac{\lambda \alpha^{h\ell}}{\mu}\left[\left(x^\ell + \beta a^\ell\right) - \left(x^{h\ell} + \beta a^{h\ell}\right)\right] - \frac{\gamma}{\mu}, \quad (39)$$

with either (i) $i \geq 0$ and $\gamma = 0$ if the constraint $i \geq 0$ is non-binding, or (ii) $i = 0$ and $\gamma > 0$ if the constraint $i \geq 0$ is binding.

Each of the equations (37)–(39) reflects an optimal trade-off that arises from a compensated marginal increase in a particular policy instrument: between public-budget effects (represented by the left-hand side), and mimicking-deterring effects (represented by the right-hand side[24]). Note that, in reality, the last term within the square bracket on the left-hand sides of the three equations above captures the effect on the money-injection constraint. However, due to the fact that an optimizing planner always chooses the policy instruments in such a way as to achieve $\mu = -\eta$ (see the proof of Proposition 3 in the Appendix for details), one can re-interpret the effect on the money-injection constraint as a public-budget effect.[25]

[24] Apart from $-\gamma/\mu$ that appears on the right-hand side of (39) whenever $\gamma \neq 0$ (so that $i = 0$ emerges as a boundary solution.)

[25] The fact that $\mu = -\eta$ tells us that at a social optimum the planner is indifferent between raising

4 Is the Friedman rule optimal?

The literature on the golden rule has taught us that whenever the real interest rate r differs from the population growth rate g, it is possible to exploit this difference to increase the steady-state welfare through intergenerational wealth transfers.[26] In the absence of generation-specific lump-sum taxes, one way to do this is by levying distortionary commodity taxes that entail intergenerational wealth transfers. An inflation tax, i.e. deviating from the FR as in Weiss (1980), is one such mechanism.[27] Yet this reason for the suboptimality of the FR applies also when one rules tax evasion out and even if the individuals within a generation are identical. Consequently, to isolate the implications of tax evasion and agents' heterogeneity, we proceed by assuming that $r = g$ so that the economy is operating at its golden rule level. We do this to abstract away from, and not be distracted by, the golden rule considerations.

Now setting $r = g$ implies, from (21), that $\theta = i$. Under this circumstance the optimality of the FR, $q^x = i/(1+i) = 0$, is the same thing as the optimality of $\theta = 0$. It then follows from Proposition 3 that to have the FR satisfied, we must have

$$\tau^c \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^c} + \frac{\tau^d}{1+r} \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^c} = \frac{\lambda \alpha^{h\ell}}{\mu} \left(c^\ell - c^{h\ell} \right), \tag{40}$$

$$\tau^c \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^d} + \frac{\tau^d}{1+r} \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^d} = \frac{\lambda \alpha^{h\ell}}{\mu} \left(d^\ell - d^{h\ell} \right), \tag{41}$$

$$\tau^c \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^x} + \frac{\tau^d}{1+r} \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^x} = \frac{\lambda \alpha^{h\ell}}{\mu} \left[\left(x^\ell + \beta a^\ell \right) - \left(x^{h\ell} + \beta a^{h\ell} \right) \right] - \frac{\gamma}{\mu}, \tag{42}$$

with $\gamma = 0$ if the constraint $i \geq 0$ is non-binding and $\gamma > 0$ if the constraint is binding.

"Solve" equations (40)–(41) for τ^c and τ^d, then substitute in (42), to get (see the

the utility of type j-agents via a marginal increase in z^j or via a marginal increase in b^j. This is due to the fact that, at the individual level, the marginal rate of substitution between z^j and b^j is one; see equation (23).

[26] The terminology and the original formulation of the golden rule, in the context of the neoclassical growth model, is due to Phelps (1961). For discussions in the context of overlapping-generations model, see, among others, Diamond (1965), Hamada (1972), and Pestieau (1974).

[27] As shown by Gahvari (1988), existence of generation-specific lump-sum taxes makes the use of such distortionary taxes unnecessary and restores the optimality of the FR.

Appendix):

$$\gamma = \frac{\lambda \alpha^{h\ell} A}{\sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^c} \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^d} - \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^d} \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^c}}, \tag{43}$$

where

$$\begin{aligned} A \equiv & \left[\sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^d} \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^x} - \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^d} \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^x} \right] \left(c^\ell - c^{h\ell} \right) + \\ & \left[\sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^c} \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^x} - \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^c} \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^x} \right] \left(d^\ell - d^{h\ell} \right) + \\ & \left[\sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^d} \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^c} - \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^c} \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^d} \right] \left[\left(x^\ell + \beta a^\ell \right) - \left(x^{h\ell} + \beta a^{h\ell} \right) \right]. \end{aligned} \tag{44}$$

From the properties of the Slutsky matrix, the denominator in (43) is positive. Consequently, the FR holds as an interior solution if $A = 0$ and as a boundary solution if $A > 0$.

4.1 Absence of tax evasion

To set the stage for the discussion of the import of tax evasion, we first consider the applicability of the FR in our model in the absence of tax evasion. Under this circumstance $a^j = a^{jk} = 0$.

4.1.1 Separable preferences

If preferences are separable in labor supply and goods, $U = U\left(u\left(c, d, x\right) - \varphi\left(L\right)\right)$, Atkinson and Stiglitz (1976) theorem holds and $c^\ell = c^{h\ell}, d^\ell = d^{h\ell}, x^\ell = x^{h\ell}$. It then follows from (44) that $A = 0$. Consequently, as with da Costa and Werning's (2008) result, the FR holds as an interior solution. Indeed, in this case, $\tau^c = \tau^d = \theta = 0$, coupled with $\gamma = 0$, constitutes a solution to (37)–(39) as required by the Atkinson and Stiglitz (1976) theorem.

4.1.2 Non-separable preferences

In the absence of tax evasion tax normalization becomes possible. Setting $\tau^d = 0$, the tax optimization will be with respect to τ^c and θ only and we have, corresponding to equations (40) and (42),

$$\tau^c \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^c} = \frac{\lambda \alpha^{h\ell}}{\mu} \left(c^\ell - c^{h\ell} \right), \tag{45}$$

$$\tau^c \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^x} = \frac{\lambda \alpha^{h\ell}}{\mu} \left(x^\ell - x^{h\ell} \right) - \frac{\gamma}{\mu}. \tag{46}$$

Eliminating τ^c between these two equations yields,

$$\gamma = \lambda \alpha^{h\ell} \left[\left(x^\ell - x^{h\ell} \right) + \frac{\sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^x}}{-\sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^c}} \left(c^\ell - c^{h\ell} \right) \right]. \tag{47}$$

Now recall that in da Costa and Werning (2008) the FR holds as a boundary solution if preferences are non-separable *provided that* real cash balances and labor supply are complements. This second result of da Costa and Werning (2008) does not hold in our setting. Their assumption of $x^\ell > x^{h\ell}$ no longer guarantees that the right-hand side of (47) is positive so that $\gamma > 0$. An additional assumption such as $\left(c^\ell - c^{h\ell} \right) \sum_j \pi^j \left(\partial \widetilde{c}^j / \partial q^x \right) \geq 0$ is also required.

The discrepancy in the results is due to the fact that there is no differential commodity taxation in da Costa and Werning (2008). Intuitively, the reason that $i = 0$ emerges as a boundary solution in their model is the desire to encourage the consumption of real balances by subsidizing it. Due to τ^c, however, the consumption of real cash balances might already be high enough as to making it raise further by pushing i down to zero undesirable. The additional $\left(c^\ell - c^{h\ell} \right) \sum_j \pi^j \left(\partial \widetilde{c}^j / \partial q^x \right) \geq 0$ constraint is meant to preclude this possibility. To see the argument, suppose the additional constraint is violated so that $\left(c^\ell - c^{h\ell} \right) \sum_j \pi^j \left(\partial \widetilde{c}^j / \partial q^x \right) < 0$. Given that from (45) τ^c and $c^\ell - c^{h\ell}$ are of opposite signs, this is equivalent to $\tau^c \sum_j \pi^j \left(\partial \widetilde{c}^j / \partial q^x \right) = \tau^c \sum_j \pi^j \left(\partial \widetilde{x}^j / \partial q^c \right) > 0$. In turn, this implies that τ^c and $\sum_j \pi^j \left(\partial \widetilde{x}^j / \partial q^c \right)$ are of the same sign. Whether τ^c is positive or negative, its presence implies a higher consumption level for real cash balances.

Finally, notice that with no differential commodity taxes in our model, $\tau^c = \tau^d = 0$ and γ reduces to

$$\gamma = \lambda \alpha^{h\ell} \left(x^{\ell} - x^{h\ell} \right).$$

The $x^{\ell} > x^{h\ell}$ assumption is then sufficient for $\gamma > 0$.

4.2 Tax evasion

Lemma 1, proved in the Appendix, provides the key to understanding the import of tax evasion for the results concerning optimality of the FR (and redundancy of commodity taxation in general).[28]

Lemma 1 *Faced with the mechanism* $\left(\tau^c, \tau^d, \theta, b^h, z^h, z^{\ell}, I^h, I^{\ell}\right)$*, the* $h\ell$*-mimicker conceals a larger amount of income (*$a^{h\ell} > a^{\ell}$*) and has a larger disposable income than the* ℓ*-type.*

Lemma 1 implies, among other things, that tax evasion leads to the breakdown of two common results in optimal Mirrleesian income tax models. One is the celebrated Atkinson and Stiglitz (1976) theorem. Suppose preferences are separable in labor supply and goods so that individuals' marginal rates of substitution between goods are independent of their labor supplies. This implies that in the absence of tax evasion, the $h\ell$-mimicker and the ℓ-type would have identical demand for goods (which makes differential commodity ineffectual for redistributive purposes). On the other hand, with tax evasion, the $h\ell$-mimicker and the ℓ-type would have different levels of disposable income despite having the same $\left(z^{\ell}, I^{\ell}\right)$ bundle. As a result, they will have different demands for goods. This restores the usefulness of differential commodity taxes, and along with it, a role for deviating from the FR if preferences are separable in labor supply and goods.

The other is the result that the $h\ell$-mimicker has a lower labor supply than the true low-skilled agent. In the absence of tax evasion, this result follows because the mimicker,

[28] There is one exception to the result stated in Lemma 1: If the deviation from the FR is substantially large, both the ℓ-type and the $h\ell$-mimicker may find it optimal to report their earned income truthfully ($a^{\ell} = a^{h\ell} = 0$). However, given that this exception is predicated on the premise that the FR is violated, one can safely ignore it. See also footnote 44 below.

being more productive, works fewer hours to earn the same amount of income as the low-skilled. A result that allows one to compare the mimickers' and the low-skilled agents' demands for a particular good based on the complementarity/substitutability of that good with labor supply. That with income tax evasion $a^{h\ell} > a^{\ell}$ implies one can no longer conclude that the labor supply of a low-skilled agent, $L^{\ell} = \left(I^{\ell} + a^{\ell}\right)/w^{\ell}$, is unambiguously larger than that of an $h\ell$-mimicker, $L^{h\ell} = \left(I^{\ell} + a^{h\ell}\right)/w^{h}$.[29] This ambiguity deprives the complementarity/substitutability assumption of its predictive power. We will see the relevance of this finding for the optimality of the FR in the discussion of the non-separable preferences.[30]

4.2.1 Separable preferences

That Lemma 1 implies $c^{\ell} \neq c^{h\ell}, d^{\ell} \neq d^{h\ell}, x^{\ell} \neq x^{h\ell}$ even if preferences are separable in labor supply and goods alerts us to the fact that in the presence of tax evasion expression A given by (44) can be equal to zero only by chance for some special type of preferences. That is, as a general rule, the FR does not hold as an interior solution. Indeed, one can easily prove that with separability if all pairs of goods are Hicksian substitutes, the FR can *never* hold as an interior solution. Interestingly, it cannot hold

[29] More precisely, the fact that a mimicker has a higher wage rate exerts both an income and a substitution effect on labor supply. The income effect is negative and tends to make the mimicker's labor supply lower than that of a low-skilled. The substitution effect, on the other hand, may be either positive or negative (for a detailed analytical proof of this claim, see section 2.2 of Blomquist *et al.*, 2011). If the substitution effect is positive and large enough, a mimicker's labor supply will exceed the labor supply of a true low-skilled.

[30] Lemma 1 also sheds light on the reason for our earlier observation regarding the relevance of tax normalization. With income misreporting, even a uniform commodity tax rate can have a bite and normalizing one of the commodity tax rates to zero is no longer a harmless assumption. To see this, note that according to this Lemma, income misreporting implies that a mimicker has a larger disposable income than a true low-skilled agent. Now start from an initial equilibrium where commodity taxes are not used and consider introducing a small uniform commodity tax at rate τ on all goods, while at the same time raising the after-tax reported income z^{j}, $j = \ell, h$, to leave the utility of the non-mimicking agents unchanged. [In our setting, this requires adjusting z^{j} by $dz^{j} = \left[c^{j} + d^{j}/(1+r)\right]\tau$.] With the $h\ell$-mimickers' disposable income exceeding that of true low-skilled agents, and for simplicity assuming $i = 0$, the total expenditure of the mimicker on goods c and d exceeds that of a true low-skilled agent: $c^{h\ell} + d^{h\ell}/(1+r) > c^{\ell} + d^{\ell}/(1+r)$. Consequently, the increase in z^{ℓ} is not enough to fully compensate the mimicker for the introduction of the uniform commodity tax. As a result, this reform makes the $h\ell$-mimicker worse-off and slackens the previously binding self-selection constraint. This means that, in the presence of tax evasion, the absolute price levels of c and d matter and one cannot simply normalize one of the prices.

as a boundary solution either. To prove this, recall that from Lemma 1, $a^{h\ell} > a^{\ell}$ so that an $h\ell$-mimicker has a larger disposable income than a true low-skilled agent: $y^{\ell} + a^{h\ell} - f\left(a^{h\ell}\right) - \beta q^{x} a^{h\ell} > y^{\ell} + a^{\ell} - f\left(a^{\ell}\right) - \beta q^{x} a^{\ell}$. Moreover, with separability, labor supply does not directly affect the way income is spent across goods. Thus, assuming all goods are normal, $c^{h\ell} > c^{\ell}$, $d^{h\ell} > d^{\ell}$, and $x^{h\ell} + \beta a^{h\ell} \geq x^{\ell} + \beta a^{\ell}$.[31] Now, assuming all pairs of goods are Hicksian substitutes, the bracketed expressions to the left of $\left(c^{\ell} - c^{h\ell}\right)$ and $\left(d^{\ell} - d^{h\ell}\right)$ in (44) are positive. At the same time, from the properties of the Slutsky matrix, the bracketed expression appearing to the left of $\left[\left(x^{\ell} + \beta a^{\ell}\right) - \left(x^{h\ell} + \beta a^{h\ell}\right)\right]$ is also positive. Consequently, $A < 0$ and the FR holds neither as an interior solution nor as a boundary solution.

On the other hand, if at least one pair of goods are Hicksian complements, then the FR might hold as a boundary solution. For instance, if x is a Hicksian complement to both c and d, while c and d are Hicksian substitutes, the first two terms in A are positive[32] and the FR may hold as a boundary solution. To get an intuition for why this might happen, suppose one chooses τ^{c} and τ^{d} optimally conditional on $i = \theta = 0$. Both τ^{c} and τ^{d} would then be positive,[33] thus indirectly discouraging the demand for real cash balances (since x is assumed to be a Hicksian complement to both c and d). If the induced downward distortion on the demand for cash balances proves to be suboptimally large, the social planner may want to counter it by pushing i all the way down to zero.

Another instance of income misreporting negating the optimality of the FR as an interior solution with separable preferences arises when the general income tax is the only instrument used ($\tau^{c} = \tau^{d} = 0$). Under this circumstance, equations (37)–(38) in Proposition 3 no longer apply and the optimality condition for monetary growth rate is

[31] The possibility for equality arises if agents are not under-reporting so that $\beta = 0$ and if both x^{ℓ} and $x^{h\ell}$ happen to be at the satiation level.

[32] Remember that, under separability and with income misreporting, $c^{\ell} - c^{h\ell} < 0$ and $d^{\ell} - d^{h\ell} < 0$.

[33] This can be easily seen by setting $\theta = 0$ in (37)–(38) and remembering that $c^{\ell} - c^{h\ell} < 0$ and $d^{\ell} - d^{h\ell} < 0$.

given by equation (39) only. Setting $\gamma = 0$ and $\tau^c = \tau^d = 0$ in equation (39) yields,

$$\frac{\theta}{1+\theta}\sum_j \pi^j \left(\frac{\partial \widetilde{x}^j}{\partial q^x} + \beta \frac{\partial \widetilde{a}^j}{\partial q^x}\right) = \frac{\lambda \alpha^{h\ell}}{\mu}\left[\left(x^\ell + \beta a^\ell\right) - \left(x^{h\ell} + \beta a^{h\ell}\right)\right].$$

The FR is thus violated in the presence of tax evasion because, as we have seen, the right-hand side of the above equation is negative.[34] Indeed, given that $\partial \widetilde{x}^j / \partial q^x < 0$ and $\partial \widetilde{a}^j / \partial q^x < 0$, the optimal solution for θ is positive. On the other hand, without tax evasion, the right-hand side collapses to zero so that $\theta = 0$ and the FR holds. The example presented in Section 5 illustrates that this is the case.

4.2.2 Non-separable preferences

With non-separable preferences, tax evasion implies that the FR may or may not hold as a boundary solution. Inspecting the various terms in A, as given by (44), reveals that A may be positive or negative depending on the various substitutability/complementarity relationships between goods and between goods and labor supply. However, unlike the case without tax evasion, one cannot establish simple sufficient conditions for $A > 0$ (and the optimality of the FR as a boundary solution). The problem is that even full knowledge of the substitutability/complementarity relationship of labor supply to c, d, and x leaves the signs of $\left(c^\ell - c^{h\ell}\right), \left(d^\ell - d^{h\ell}\right)$ and $\left(x^\ell + \beta a^\ell\right) - \left(x^{h\ell} + \beta a^{h\ell}\right)$ in A indeterminate. In the presence of tax evasion, as we mentioned in our discussion of Lemma 1, either the low-skilled or the mimicker can have a larger labor supply.[35]

The results of this section are summarized as:

Proposition 4 *Consider the steady-state equilibrium of Proposition 2 at its golden rule level:*

(i) The Friedman rule does not generally hold as an interior solution.

(ii) The Friedman rule may or may not hold as a boundary solution.

[34] Observe that in the absence of commodity taxes, low-ability agents (as well as $h\ell$-mimickers) face a positive marginal income tax rate so that they will never be over-reporting. Consequently, $\beta > 0$ and even if x^ℓ and $x^{h\ell}$ happen to be at the satiation level, the right-hand side is always strictly negative.

[35] The problem is simpler with separable preferences because we need not know which agents have a larger or smaller labor supply. It simply does not matter.

(iii) A necessary condition for the FR to hold as a boundary solution under separability of preferences between labor supply and goods is that at least one pair of goods are Hicksian complements.

(iv) Whether or not complementarity between real cash balances and labor supply favors the optimality of the FR as a boundary solution depends on whether the labor supply of a low-skilled agent exceeds or falls short of the labor supply of an $h\ell$-mimicker.

(v) Even in the absence of tax evasion, complementarity between labor supply and consumption of real cash balances does not guarantee the optimality of the Friedman rule as a boundary solution.

5 An example

Assume that skilled and unskilled agents have identical preferences represented by:

$$u_t = \ln c_t + \ln d_{t+1} - x_t \ln x_t - 0.25 L_t^4, \quad 0 < x_t \leq \frac{1}{e} \simeq 0.367879. \tag{48}$$

There are as many skilled as unskilled workers agents in the population so that $\pi^h = \pi^\ell = 0.5$. Their real wage rates, reflecting their productivities, are constant over time and set equal to $w^h = 26$ and $w^\ell = 20$. Assume further that the government's objective function is purely utilitarian with $\delta^h = \delta^\ell = 0.5$. Moreover, given that in a two-period setting the length of each period corresponds to the length of the working-life of each generation, one can reasonably set $r = g = 0.5$. As far as the government's external revenue is concerned, we set $\bar{R} = 0$ so that optimal taxes are purely redistributive.

It will be instructive to solve this with as well as without income tax evasion. Assume, when tax evasion is feasible, the avoidance cost function is given by $f(a_t) = a_t^2/200$. That is, it costs a j-type worker who conceals a_t^j of his income, $\left(a_t^j\right)^2/200$ in real resources to make his evasion go undetected. Under this circumstance, set $\beta = 0.05$. This means that the evader will also have to hold an extra 0.05 in real balances in order to be able to conceal one unit of income.[36]

[36] This captures the second usage of money in the presence of tax evasion, as explained in the paper. If there is no such requirement and money is rationalized only through entering into the utility function,

5.1 Steady-state solution in the absence of evasion

We begin by solving for the steady-state equilibrium in the absence of evasion. If evasion is not possible, $a_t^j = 0$ and the true income of a j-type worker is equal to his reported income, I_t^j. In the absence of income tax evasion, the intertemporal budget constraint of this worker is written as[37]

$$(1+\tau^c)\, c_t^j + \frac{\left(1+\tau^d\right) d_{t+1}^j}{1+r} + \frac{i_{t+1}}{1+i_{t+1}} x_t^j = z_t^j + \frac{e_{t+1}^j}{p_{t+1}\,(1+r)}.$$

To arrive at the steady-state solution for the optimal tax/monetary instruments, we follow the two-step optimization procedure outlined in the paper (using the KNITRO package when solving for the government's optimization problem). This yields:

$$\begin{aligned} \tau^c &= \tau^d = 0, \\ \theta &= i = 0, \\ T'\left(I^h\right) &= 0, \\ T'\left(I^\ell\right) &= 6.3\%. \end{aligned}$$

The results are as one expects. Given the separable specification of preferences in the example, Atkinson and Stiglitz (1976) theorem applies: Commodity taxes are redundant and the FR holds. Moreover, $T'\left(I^h\right) = 0$ reflects the no-distortion-at-the-top result.[38]

Observe also that from the relationship $1+\varphi = (1+\theta)/(1+g)$, one calculates $\varphi = -0.33$. That is, with no increase in money supply and a population growth rate of 50%, the price level is falling at a rate of 33%. These instruments result in the following values for the goods and services that appear in the preferences:[39]

$$\begin{aligned} c^h &= 14.8523, \quad d^h = 22.2785, \quad x^h = 0.367876, \quad L^h = 1.2052, \\ c^\ell &= 12.3179, \quad d^\ell = 18.4768, \quad x^\ell = 0.367876, \quad L^\ell = 1.1503. \end{aligned}$$

$\beta = 0$. Examples of this type, i.e. with $\beta = 0$, are simpler to construct.

[37] This is derived by setting $a_t^j = 0$ in equation (A4) in the Appendix.

[38] These tax rates imply that $I^h = 31.3352, z^h = 29.7047, I^\ell = 23.0052$, and $z^\ell = 24.6358$. Moreover, with no increase in money supply, $b^h = b^\ell = 0$.

[39] As one would expect, the calculated values for $c^{h\ell}, d^{h\ell}$, and $x^{h\ell}$ are precisely identical to the corresponding for c^ℓ, d^ℓ, and x^ℓ; while $L^{h\ell} = 0.884815 < L^\ell$. It is also the case that $u^h = u^{h\ell} = 5.64221$ and $u^\ell = 5.3578$.

5.2 Steady-state solution with tax evasion

The intertemporal budget constraint of the j-type agent in the presence of tax evasion is the more complicated formulation given by equation (A4) in the Appendix. Thus, in the first-stage of our optimization problem, this agent also chooses a_t^j (in addition to c_t^j, d_{t+1}^j, and x_t^j). Again, using the KNITRO package and following the steps outlined in the paper, we find

$$\begin{aligned} \tau^c &= \tau^d = 5.27\%, \\ \theta &= i = 66.65\%, \\ T'\left(I^h\right) &= -0.36\%, \\ T'\left(I^\ell\right) &= 2.39\%. \end{aligned}$$

Note that the presence of income tax evasion necessitates the levying of commodity taxes as well as the negation of the Friedman rule.[40] Observe also that tax evasion and its concealment imply that the marginal income tax rate faced by each agent must be equal to the agent's marginal cost of concealment plus his implicit cost of holding extra money for the purpose of evasion: $T'\left(I^j\right) = f'\left(a^j\right) + \beta i/\left(1+i\right)$; see equation (A7) in the Appendix for a formal proof.[41]

This time around, from the relationship $1+\varphi = \left(1+\theta\right)/\left(1+g\right)$, and with $\theta > g$, one calculates $\varphi = 0.111$. Given the optimal positive rate of money injection, the price level is now increasing. These instruments result in the following values for the goods and services that appear in the preferences[42]:

[40] That $\tau^c = \tau^d$ in this example is due to the logarithmic specification of the utility function. Interestingly too, in this example, the average labor-income tax rates $\left(I-z\right)/I$ on both agents turn out to be negative (even though the marginal labor-income tax rates are positive).

[41] In calculating the marginal income tax rate for the skilled worker, $T'\left(I^h\right)$, we have set $\beta = 0$ for him because $a^h < 0$. Given the above reported tax rates, we now have, $I^h = 31.0085, I^\ell = 22.8557$, so that both agents earn less income as compared to the no-evasion solution, with $z^h = 32.1619, z^\ell = 24.5419$. Observe also that, with $\theta > 0$, b^h and b^ℓ are now positive and calculated to be $b^h = 0.001662$ and $b^\ell = 0.143791$.

[42] Observe that with tax evasion,

$$c^{h\ell} = 14.7884, d^{h\ell} = 22.1825, x^{h\ell} = 0.358549,$$

so that the values for $c^{h\ell}, d^{h\ell}$, and $x^{h\ell}$ are now different from the corresponding values for c^ℓ, d^ℓ, x^ℓ.

$$
\begin{aligned}
c^h &= 15.1329, \quad d^h = 22.6994, \quad x^h = 0.358759, \quad L^h = 1.1788, \quad a^h = -0.3600, \\
c^\ell &= 11.8129, \quad d^\ell = 17.7193, \quad x^\ell = 0.356236, \quad L^\ell = 1.1622, \quad a^\ell = 0.3889.
\end{aligned}
$$

6 Summary and conclusion

This paper has developed a version of Samuelson's (1958) overlapping-generations model that (i) includes money, (ii) is populated with agents who are heterogeneous in terms of earning ability, and (iii) allows for tax evasion. Money is used for two reasons. One is the traditional (non-evading) usage captured by money-in-the-utility-function; the other is to facilitate tax evasion. Money supply increases, or contracts, at a fixed rate per year through lump-sum money transfers to individuals. The policy-maker has information on the distribution of abilities in the population, on individuals' preferences, and on the technology used by agents to shelter income from the tax authority. Reported incomes are subject to a nonlinear tax schedule.

The paper has studied the nature of the economy's perfect-foresight temporal equilibrium as well as its steady state. It has characterized the informationally constrained Pareto-efficient allocations of this economy and the properties of the optimal commodity taxes that implement them. In the absence of tax evasion, it has shown that as long as preferences are separable in labor supply and goods, Atkinson and Stiglitz (1976) theorem applies and the Friedman rule becomes optimal for precisely the same reason that commodity taxes become redundant in such a setting.[43] With non-separable preferences, if consumption of real cash balances is positively related to labor supply, one would want to encourage the demand for cash balances. Absent (differentiated) com-

We calculate $a^{h\ell} = 7.03378 > a^\ell$, as must be the case according to Lemma 1. We also have, $L^{h\ell} = 1.14959 < L^\ell$. Finally, we calculate $u^h = u^{h\ell} = 5.72427$ and $u^\ell = 5.25539$. The possibility of income tax evasion increases the utility of skilled agents at the expense of the unskilled agents. The reason for this is that tax evasion increases the utility of the mimicker thus reducing what one can take away from skilled workers.

[43] As stated in the paper, to abstract away from golden-rule considerations, this and other stated results in the Summary and conclusion are derived assuming the economy is operating at its golden rule level.

modity taxation, this implies that the optimal nominal interest rate is negative, and therefore, given the non-negativity of nominal interest rate, the Friedman re-emerges as a boundary solution. With optimally differentiated commodity taxes, however, complementarity between real cash balances and labor supply is no longer a sufficient condition for the optimality of the Friedman rule. If goods that are Hicksian substitutes for real cash balances are taxed at relatively high rates, deviating from the Friedman rule may be part of an optimal policy even with positively correlated real cash balances and labor supply. The same is true if goods that are Hicksian complements to real cash balances are taxed at relatively low rates.

The presence of tax evasion has important consequences for the desirability of the Friedman rule. The reason is that although the agents who are high-skilled but pretend to be low-skilled and true low-skilled agents have the same before-tax reported labor income, they will nevertheless have different disposable incomes. A situation that cannot happen without tax evasion. Specifically, the "mimickers" conceal a larger amount of income which results in their having a larger disposable income than the true low-skilled agents. This difference breaths a new life into the redistributive power of monetary growth rate and the Friedman rule does not generally hold as an interior solution if preferences are separable in labor supply and goods. Moreover, because of tax evasion, the labor supply of a high-skilled mimicker can exceed that of a low-skilled agent. This implies that complementarity of real cash balances and labor supply may even weaken the case for the optimality of the Friedman rule as a boundary solution.

We conclude by pointing out that while not formally discussed in the paper, our results are robust on three fronts: the number of types in the economy, modeling income tax evasion as a risky activity subject to audits, and the possibility of commodity tax evasion; see Gahvari and Micheletto (2013).

Appendix

Derivation of (7)–(10): Formally, given any vector $\left(\tau^c, \tau^d, \theta, e_{t+1}, z_t, I_t\right)$, an individual of type j chooses c_t, d_{t+1}, x_t and a_t to maximize

$$u_t^j = u\left(c_t, d_{t+1}, x_t, \frac{I_t + a_t}{w_t^j}\right), \quad j = h, \ell, \tag{A1}$$

subject to the per-period budget constraints

$$p_t\left[\left(1+\tau^c\right)c_t + s_t\right] + m_t = p_t\left[z_t + a_t - f\left(a_t\right)\right], \tag{A2}$$

$$p_{t+1}\left(1+\tau^d\right)d_{t+1} = p_t s_t\left(1+i_{t+1}\right) + m_t + e_{t+1}, \tag{A3}$$

where s_t is the level of real savings chosen by the agent. Observe that θ does not explicitly appear in the problem above; it does so implicitly through its effect on i_{t+1}.

Next combine equations (A2)–(A3) into a single intertemporal budget constraint for the young. To this end, substitute $z_t + a_t - f\left(a_t\right) - \left(1+\tau^c\right)c_t - m_t/p_t$ for s_t from (A2) into (A3) to get:

$$\begin{aligned}
&p_{t+1}\left(1+\tau^d\right)d_{t+1} = p_t\left[z_t + a_t - f\left(a_t\right) - \left(1+\tau^c\right)c_t - m_t/p_t\right]\left(1+i_{t+1}\right) + m_t + e_{t+1} = \\
&p_{t+1}\left[z_t + a_t - f\left(a_t\right) - \left(1+\tau^c\right)c_t - m_t/p_t\right]\left(1+r\right) + m_t + e_{t+1} = \\
&p_{t+1}\left[z_t + a_t - f\left(a_t\right) - \left(1+\tau^c\right)c_t - x_t - \beta a_t\right]\left(1+r\right) + p_t x_t + \beta p_t a_t + e_{t+1},
\end{aligned}$$

where in deriving the last equality we have substituted $p_t\left(x_t + \beta a_t\right)$ for m_t from equation (6). Divide the above expression by $p_{t+1}\left(1+r\right)$ and substitute $1/\left(1+i_t\right)$ for $p_t/p_{t+1}\left(1+r\right)$ to arrive at

$$\frac{\left(1+\tau^d\right)d_{t+1}}{1+r} = \left[z_t + a_t - f\left(a_t\right) - \left(1+\tau^c\right)c_t - x_t - \beta a_t\right] + \frac{x_t + \beta a_t}{1+i_t} + \frac{e_{t+1}}{p_{t+1}\left(1+r\right)}.$$

Rearranging the terms and simplifying leads to

$$\left(1+\tau^c\right)c_t + \frac{\left(1+\tau^d\right)d_{t+1}}{1+r} + \frac{i_{t+1}}{1+i_{t+1}}x_t = z_t + a_t - f\left(a_t\right) - \frac{i_{t+1}}{1+i_{t+1}}\beta a_t + \frac{e_{t+1}}{p_{t+1}\left(1+r\right)}. \tag{A4}$$

The problem of a young j-type, who is facing the sextuple $\left(\tau^c, \tau^d, \theta, e_{t+1}, z_t, I_t\right)$, is to choose c_t, d_{t+1}, x_t, and a_t in order to maximize (A1) subject to (A4). The first-order conditions for this problem are

$$\frac{\partial u_t^j / \partial d_{t+1}}{\partial u_t^j / \partial c_t} = \frac{1+\tau^d}{\left(1+\tau^c\right)(1+r)}, \tag{A5}$$

$$\frac{\partial u_t^j / \partial x_t}{\partial u_t^j / \partial c_t} = \frac{i_{t+1}}{\left(1+\tau^c\right)\left(1+i_{t+1}\right)}, \tag{A6}$$

$$\frac{\partial u_t^j / \partial L_t}{\partial u_t^j / \partial c_t} = -\frac{\left[1-f'\left(a_t\right)-\beta i_{t+1}/\left(1+i_{t+1}\right)\right] w_t^j}{1+\tau^c}, \tag{A7}$$

where (A7) holds when $a \neq 0$ is optimal for an agent.[44] Conditions (A5)–(A7), along with the individual's intertemporal budget constraint (A4), yield equations (7)–(10).

Derivation of (31): Substitute for M_t from equation (1) into (4), divide by $N_t p_t$, and substitute for m_t^j from (6) into the resulting equation to get:

$$\begin{aligned} \pi^h \frac{e_{t+1}^h}{p_t} + \pi^\ell \frac{e_{t+1}^\ell}{p_t} &= \theta\left(\pi^h \frac{m_t^h}{p_t} + \pi^\ell \frac{m_t^\ell}{p_t}\right) \\ &= \theta\left[\pi^h\left(x_t^h + \beta a_t^h\right) + \pi^\ell\left(x_t^\ell + \beta a_t^\ell\right)\right]. \end{aligned}$$

Then, using the relationship between e_{t+1}^j and b_{t+1}^j given by (14), rewrite the above equation in terms of b_{t+1}^j, and substitute $\left(1+i_{t+1}\right)$ for $(1+r)\, p_{t+1}/p_t$:

$$\left(1+i_{t+1}\right)\left(\pi^h b_{t+1}^h + \pi^\ell b_{t+1}^\ell\right) = \theta\left[\pi^h\left(x_t^h + \beta a_t^h\right) + \pi^\ell\left(x_t^\ell + \beta a_t^\ell\right)\right].$$

The steady version of this equation is,

$$(1+i) \sum_{j=\ell,h} \pi^j b^j = \theta \sum_{j=\ell,h} \pi^j \left(x^j + \beta a^j\right).$$

Dividing this equation by $(1+i)$ and substituting for $(1+i)$ from (21) in it yields (31).

[44] If $a = 0$ is an optimal choice for an agent, it might be the case that

$$\frac{\partial u_t^j / \partial L_t}{\partial u_t^j / \partial c_t} < -\frac{\left[1-f'\left(a_t\right)-\beta i_{t+1}/\left(1+i_{t+1}\right)\right] w_t^j}{1+\tau^c}.$$

This can happen if an agent faces a positive marginal income tax rate $T'\left(I_t\right)$ and at the same time $0 < T'\left(I_t\right) < \beta i_{t+1}/\left(1+i_{t+1}\right)$. We are ignoring this possibility because its occurrence is predicated on the optimality of violating the FR. See also footnote 28 above.

Proof of Proposition 3: The mechanism designer's problem can be described by means of the Lagrangian:

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{j=\ell,h} \delta^j v^j + \lambda \left(v^h - v^{h\ell} \right) + \eta \left[\sum_{j=\ell,h} \pi^j b^j - \frac{1+g}{1+r} \frac{\theta}{1+\theta} \sum_{j=\ell,h} \pi^j \left(x^j + \beta a^j \right) \right] \\
& + \mu \left[\sum_{j=\ell,h} \pi^j \left(I^j - z^j + \tau^c c^j + \frac{\tau^d}{1+r} d^j \right) - \bar{R} \right] + \gamma i.
\end{aligned}
$$

Given the redundancy of one of the redistributive instruments b^h and b^ℓ, it is sufficient to carry out our optimization with respect to only b^h or b^ℓ. This follows because optimization with respect to θ and either b^h or b^ℓ will determine the other b^h or b^ℓ.[45] Without any loss of generality, we will choose b^h. Then, the first-order conditions of this problem are:

$$
\frac{\partial \mathcal{L}}{\partial I^h} = \left(\delta^h + \lambda \right) \frac{\partial v^h}{\partial I^h} - \eta \frac{1+g}{1+r} \frac{\theta}{1+\theta} \pi^h \left(\frac{\partial x^h}{\partial I^h} + \beta \frac{\partial a^h}{\partial I^h} \right) + \mu \pi^h \left(1 + \tau^c \frac{\partial c^h}{\partial I^h} + \frac{\tau^d}{1+r} \frac{\partial d^h}{\partial I^h} \right) = 0, \tag{A8}
$$

$$
\frac{\partial \mathcal{L}}{\partial I^\ell} = \delta^\ell \frac{\partial v^\ell}{\partial I^\ell} - \lambda \frac{\partial v^{h\ell}}{\partial I^\ell} - \eta \frac{1+g}{1+r} \frac{\theta}{1+\theta} \pi^\ell \left(\frac{\partial x^\ell}{\partial I^\ell} + \beta \frac{\partial a^\ell}{\partial I^\ell} \right) + \mu \pi^\ell \left(1 + \tau^c \frac{\partial c^\ell}{\partial I^\ell} + \frac{\tau^d}{1+r} \frac{\partial d^\ell}{\partial I^\ell} \right) = 0, \tag{A9}
$$

$$
\frac{\partial \mathcal{L}}{\partial z^h} = \left(\delta^h + \lambda \right) \frac{\partial v^h}{\partial y^h} - \eta \frac{1+g}{1+r} \frac{\theta}{1+\theta} \pi^h \left(\frac{\partial x^h}{\partial y^h} + \beta \frac{\partial a^h}{\partial y^h} \right) - \mu \pi^h \left(1 - \tau^c \frac{\partial c^h}{\partial y^h} - \frac{\tau^d}{1+r} \frac{\partial d^h}{\partial y^h} \right) = 0, \tag{A10}
$$

$$
\frac{\partial \mathcal{L}}{\partial z^\ell} = \delta^\ell \frac{\partial v^\ell}{\partial y^\ell} - \lambda \frac{\partial v^{h\ell}}{\partial y^\ell} - \eta \frac{1+g}{1+r} \frac{\theta}{1+\theta} \pi^\ell \left(\frac{\partial x^\ell}{\partial y^\ell} + \beta \frac{\partial a^\ell}{\partial y^\ell} \right) - \mu \pi^\ell \left(1 - \tau^c \frac{\partial c^\ell}{\partial y^\ell} - \frac{\tau^d}{1+r} \frac{\partial d^\ell}{\partial y^\ell} \right) = 0, \tag{A11}
$$

$$
\frac{\partial \mathcal{L}}{\partial b^h} = \left(\delta^h + \lambda \right) \frac{\partial v^h}{\partial y^h} + \eta \pi^h \left[1 - \frac{1+g}{1+r} \frac{\theta}{1+\theta} \left(\frac{\partial x^h}{\partial y^h} + \beta \frac{\partial a^h}{\partial y^h} \right) \right] + \mu \pi^h \left(\tau^c \frac{\partial c^h}{\partial y^h} + \frac{\tau^d}{1+r} \frac{\partial d^h}{\partial y^h} \right) = 0, \tag{A12}
$$

[45] If the money disbursements to skilled and unskilled workers were set according to $e^j_{t+1} = \theta m^j_t$, then the optimization with respect to θ will suffice. Under this circumstance $b^j = \theta m^j / (1+i) = \theta \left(x^j + \beta a^j \right) / (1+i)$ is fully determined once θ is determined.

$$\frac{\partial \mathcal{L}}{\partial \tau^c} = \sum_j \delta^j \frac{\partial v^j}{\partial \tau^c} + \lambda \left(\frac{\partial v^h}{\partial \tau^c} - \frac{\partial v^{h\ell}}{\partial \tau^c} \right) - \eta \frac{1+g}{1+r} \frac{\theta}{1+\theta} \sum_j \pi^j \left(\frac{\partial x^j}{\partial \tau^c} + \beta \frac{\partial a^j}{\partial \tau^c} \right)$$
$$+ \mu \sum_j \pi^j \left(c^j + \tau^c \frac{\partial c^j}{\partial \tau^c} + \frac{\tau^d}{1+r} \frac{\partial d^j}{\partial \tau^c} \right) = 0, \tag{A13}$$
$$\frac{\partial \mathcal{L}}{\partial \tau^d} = \sum_j \delta^j \frac{\partial v^j}{\partial \tau^d} + \lambda \left(\frac{\partial v^h}{\partial \tau^d} - \frac{\partial v^{h\ell}}{\partial \tau^d} \right) - \eta \frac{1+g}{1+r} \frac{\theta}{1+\theta} \sum_j \pi^j \left(\frac{\partial x^j}{\partial \tau^d} + \beta \frac{\partial a^j}{\partial \tau^d} \right)$$
$$+ \mu \sum_j \pi^j \left(\tau^c \frac{\partial c^j}{\partial \tau^d} + \frac{d^j}{1+r} + \frac{\tau^d}{1+r} \frac{\partial d^j}{\partial \tau^d} \right) = 0, \tag{A14}$$
$$\frac{\partial \mathcal{L}}{\partial \theta} = \sum_j \delta^j \frac{\partial v^j}{\partial \theta} + \lambda \left(\frac{\partial v^h}{\partial \theta} - \frac{\partial v^{h\ell}}{\partial \theta} \right) + \mu \sum_j \pi^j \left(\tau^c \frac{\partial c^j}{\partial \theta} + \frac{\tau^d}{1+r} \frac{\partial d^j}{\partial \theta} \right)$$
$$- \eta \frac{1+g}{1+r} \left[\frac{1}{(1+\theta)^2} \sum_j \pi^j \left(x^j + \beta a^j \right) + \frac{\theta}{1+\theta} \sum_j \pi^j \left(\frac{\partial x^j}{\partial \theta} + \beta \frac{\partial a^j}{\partial \theta} \right) \right] + \gamma \frac{1+r}{1+g} = 0, \tag{A15}$$

where comparing (A10) with (A12) reveals that $\mu = -\eta$.

By way of substituting for i from (21) in (13),

$$q^x = \frac{(1+r)(1+\theta) - (1+g)}{(1+r)(1+\theta)} = 1 - \frac{1+g}{(1+r)(1+\theta)}.$$

Differentiating with respect to θ yields

$$\frac{\partial q^x}{\partial \theta} = \frac{1+g}{1+r} \frac{1}{(1+\theta)^2}. \tag{A16}$$

Using $\partial x^j / \partial \theta = \left(\partial x^j / \partial q^x \right) \left(\partial q^x / \partial \theta \right)$ and $\partial a^j / \partial \theta = \left(\partial a^j / \partial q^x \right) \left(\partial q^x / \partial \theta \right)$, one can then derive the following expressions,

$$\begin{aligned} \frac{\partial x^j}{\partial \theta} &= \frac{1+g}{1+r} \frac{1}{(1+\theta)^2} \frac{\partial x^j}{\partial q^x}, \\ \frac{\partial a^j}{\partial \theta} &= \frac{1+g}{1+r} \frac{1}{(1+\theta)^2} \frac{\partial a^j}{\partial q^x}. \end{aligned}$$

Similarly,

$$\begin{aligned} \frac{\partial c^j}{\partial \theta} &= \frac{1+g}{1+r} \frac{1}{(1+\theta)^2} \frac{\partial c^j}{\partial q^x}, \\ \frac{\partial d^j}{\partial \theta} &= \frac{1+g}{1+r} \frac{1}{(1+\theta)^2} \frac{\partial d^j}{\partial q^x}. \end{aligned}$$

Next, differentiate v^j and v^{jk}, as specified by equations (29) and (36), with respect to z^j, b^j, z^k, b^k. We have,

$$\frac{\partial v^j}{\partial z^j}|_{\theta,\tau^c,\tau^d,b^j,I^j} = \frac{\partial v^j}{\partial b^j}|_{\theta,\tau^c,\tau^d,z^j,I^j} = \frac{\partial v^j}{\partial y^j}|_{q^x,q^c,q^d,I^j} \equiv \alpha^j, \tag{A17}$$

$$\frac{\partial v^{jk}}{\partial z^k}|_{\theta,\tau^c,\tau^d,b^k,I^k} = \frac{\partial v^{jk}}{\partial b^k}|_{\theta,\tau^c,\tau^d,z^k,I^k} = \frac{\partial v^{jk}}{\partial y^k}|_{q^x,q^c,q^d,I^k} \equiv \alpha^{jk}. \tag{A18}$$

Similarly, differentiate v^j and v^{jk} with respect to θ, using (A16) and Roy's identity, to get

$$\frac{\partial v^j}{\partial \theta}|_{b^j,\tau^c,\tau^d,z^j,I^j} = \frac{\partial v^j}{\partial q^x}|_{q^c,q^d,y^j,I^j}\frac{\partial q^x}{\partial \theta} = -\alpha^j\left(x^j+\beta a^j\right)\frac{1+g}{1+r}\frac{1}{(1+\theta)^2}, \tag{A19}$$

$$\frac{\partial v^{jk}}{\partial \theta}|_{b^k,\tau^c,\tau^d,z^k,I^k} = \frac{\partial v^{jk}}{\partial q^x}|_{q^c,q^d,y^k,I^k}\frac{\partial q^x}{\partial \theta} = -\alpha^{jk}\left(x^{jk}+\beta a^{jk}\right)\frac{1+g}{1+r}\frac{1}{(1+\theta)^2}. \tag{A20}$$

Moreover, with $\partial q^c/\partial \tau^c = 1$ and $\partial q^d/\partial \tau^d = (1+r)^{-1}$, we also have,

$$\frac{\partial v^j}{\partial \tau^c}|_{\theta,\tau^d,b^j,z^j,I^j} = \frac{\partial v^j}{\partial q^c}|_{q^x,q^d,y^j,I^j}\frac{\partial q^c}{\partial \tau^c} = -\alpha^j c^j \tag{A21}$$

$$\frac{\partial v^{jk}}{\partial \tau^c}|_{\theta,\tau^d,b^k,z^k,I^k} = \frac{\partial v^{jk}}{\partial q^c}|_{q^x,q^d,y^k,I^k}\frac{\partial q^c}{\partial \tau^c} = -\alpha^{jk} c^{jk}, \tag{A22}$$

$$\frac{\partial v^j}{\partial \tau^d}|_{\theta,\tau^c,b^j,z^j,I^j} = \frac{\partial v^j}{\partial q^d}|_{q^x,q^c,y^j,I^j}\frac{\partial q^d}{\partial \tau^d} = -\alpha^j d^j\frac{1}{1+r}, \tag{A23}$$

$$\frac{\partial v^{jk}}{\partial \tau^d}|_{\theta,\tau^c,b^k,z^k,I^k} = \frac{\partial v^{jk}}{\partial q^d}|_{q^x,q^c,y^k,I^k}\frac{\partial q^d}{\partial \tau^d} = -\alpha^{jk} d^{jk}\frac{1}{1+r}. \tag{A24}$$

Finally, use the result $\mu = -\eta$ and equations (A17)–(A24) to simplify and reduce the first-order conditions (A8)–(A15) into the following seven equations:

$$\left(\delta^h+\lambda\right)\frac{\partial v^h}{\partial I^h} + \mu\pi^h\left[\frac{1}{1+r}\frac{(1+g)\,\theta}{1+\theta}\left(\frac{\partial x^h}{\partial I^h}+\beta\frac{\partial a^h}{\partial I^h}\right)+1+\tau^c\frac{\partial c^h}{\partial I^h}+\frac{\tau^d}{1+r}\frac{\partial d^h}{\partial I^h}\right]=0, \tag{A25}$$

$$\delta^\ell\frac{\partial v^\ell}{\partial I^\ell} - \lambda\frac{\partial v^{h\ell}}{\partial I^\ell} + \mu\pi^\ell\left[\frac{1}{1+r}\frac{(1+g)\,\theta}{1+\theta}\left(\frac{\partial x^\ell}{\partial I^\ell}+\beta\frac{\partial a^\ell}{\partial I^\ell}\right)+1+\tau^c\frac{\partial c^\ell}{\partial I^\ell}+\frac{\tau^d}{1+r}\frac{\partial d^\ell}{\partial I^\ell}\right]=0, \tag{A26}$$

$$\left(\delta^h+\lambda\right)\alpha^h + \mu\pi^h\left[\frac{1}{1+r}\frac{(1+g)\,\theta}{1+\theta}\left(\frac{\partial x^h}{\partial y^h}+\beta\frac{\partial a^h}{\partial y^h}\right)-1+\tau^c\frac{\partial c^h}{\partial y^h}+\frac{\tau^d}{1+r}\frac{\partial d^h}{\partial y^h}\right]=0, \tag{A27}$$

$$\delta^\ell \alpha^\ell - \lambda \alpha^{h\ell} + \mu \pi^\ell \left[\frac{1}{1+r} \frac{(1+g)\,\theta}{1+\theta} \left(\frac{\partial x^\ell}{\partial y^\ell} + \beta \frac{\partial a^\ell}{\partial y^\ell} \right) - 1 + \tau^c \frac{\partial c^\ell}{\partial y^\ell} + \frac{\tau^d}{1+r} \frac{\partial d^\ell}{\partial y^\ell} \right] = 0, \tag{A28}$$

$$- \sum_j \delta^j \alpha^j c^j + \lambda \left(\alpha^{h\ell} c^{h\ell} - \alpha^h c^h \right) + \mu \sum_j \pi^j \left\{ c^j + \tau^c \frac{\partial c^j}{\partial q^c} + \frac{1}{1+r} \left[\tau^d \frac{\partial d^j}{\partial q^c} + \frac{(1+g)\,\theta}{1+\theta} \left(\frac{\partial x^j}{\partial q^c} + \beta \frac{\partial a^j}{\partial q^c} \right) \right] \right\} = 0, \tag{A29}$$

$$- \sum_j \delta^j \alpha^j d^j + \lambda \left(\alpha^{h\ell} d^{h\ell} - \alpha^h d^h \right) + \mu \sum_j \pi^j \left\{ d^j + \left[\tau^c \frac{\partial c^j}{\partial q^d} + \frac{\tau^d}{1+r} \frac{\partial d^j}{\partial q^d} + \frac{(1+g)\,\theta}{(1+\theta)\,(1+r)} \left(\frac{\partial x^j}{\partial q^d} + \beta \frac{\partial a^j}{\partial q^d} \right) \right] \right\} = 0, \tag{A30}$$

$$- \sum_j \delta^j \alpha^j \left(x^j + \beta a^j \right) + \lambda \left[\alpha^{h\ell} \left(x^{h\ell} + \beta a^{h\ell} \right) - \alpha^h \left(x^h + \beta a^h \right) \right] + \gamma \frac{(1+r)^2\,(1+\theta)^2}{(1+g)^2} + \mu \sum_j \pi^j \left\{ \left(x^j + \beta a^j \right) + \tau^c \frac{\partial c^j}{\partial q^x} + \frac{\tau^d}{1+r} \frac{\partial d^j}{\partial q^x} + \frac{1}{1+r} \frac{(1+g)\,\theta}{1+\theta} \left(\frac{\partial x^j}{\partial q^x} + \beta \frac{\partial a^j}{\partial q^x} \right) \right\} = 0. \tag{A31}$$

Let $\widetilde{c}^j$, $\widetilde{d}^j$, $\widetilde{x}^j$ and $\widetilde{a}^j$ denote the compensated versions of c^j, d^j, x^j and a^j. Use the Slutsky equation to rewrite equations (A29)–(A31). Rearranging the terms and using (21),

$$- \sum_j \delta^j \alpha^j c^j + \lambda \left(\alpha^{h\ell} c^{h\ell} - \alpha^h c^h \right) + \mu \sum_j \pi^j \left[\tau^c \frac{\partial \widetilde{c}^j}{\partial q^c} + \frac{\tau^d}{1+r} \frac{\partial \widetilde{d}^j}{\partial q^c} + \frac{(1+g)\,\theta}{(1+\theta)\,(1+r)} \left(\frac{\partial \widetilde{x}^j}{\partial q^c} + \beta \frac{\partial \widetilde{a}^j}{\partial q^c} \right) \right] + \mu \sum_j \pi^j c^j \left\{ 1 - \left[\tau^c \frac{\partial c^j}{\partial y^j} + \frac{\tau^d}{1+r} \frac{\partial d^j}{\partial y^j} + \frac{(1+g)\,\theta}{(1+\theta)\,(1+r)} \left(\frac{\partial x^j}{\partial y^j} + \beta \frac{\partial a^j}{\partial y^j} \right) \right] \right\} = 0, \tag{A32}$$

$$
-\sum_j \delta^j \alpha^j d^j + \lambda\left(\alpha^{h\ell} d^{h\ell} - \alpha^h d^h\right) + \mu \sum_j \pi^j \left[\tau^c \frac{\partial \widetilde{c}^j}{\partial q^d} + \frac{\tau^d}{1+r}\frac{\partial \widetilde{d}^j}{\partial q^d} + \frac{(1+g)\,\theta}{(1+\theta)(1+r)}\left(\frac{\partial \widetilde{x}^j}{\partial q^d} + \beta \frac{\partial \widetilde{a}^j}{\partial q^d}\right)\right]
$$

$$
+\mu \sum_j \pi^j d^j \left\{1 - \left[\tau^c \frac{\partial c^j}{\partial y^j} + \frac{\tau^d}{1+r}\frac{\partial d^j}{\partial y^j} + \frac{(1+g)\,\theta}{(1+\theta)(1+r)}\left(\frac{\partial x^j}{\partial y^j} + \beta \frac{\partial a^j}{\partial y^j}\right)\right]\right\} = 0, \tag{A33}
$$

$$
-\sum_j \delta^j \alpha^j \left(x^j + \beta a^j\right) + \lambda\left[\alpha^{h\ell}\left(x^{h\ell} + \beta a^{h\ell}\right) - \alpha^h \left(x^h + \beta a^h\right)\right]
$$

$$
+\mu \sum_j \pi^j \left[\tau^c \frac{\partial \widetilde{c}^j}{\partial q^x} + \frac{\tau^d}{1+r}\frac{\partial \widetilde{d}^j}{\partial q^x} + \frac{(1+g)\,\theta}{(1+\theta)(1+r)}\left(\frac{\partial \widetilde{x}^j}{\partial q^x} + \beta \frac{\partial \widetilde{a}^j}{\partial q^x}\right)\right] + \gamma\,(1+i)^2
$$

$$
+\mu \sum_j \pi^j \left(x^j + \beta a^j\right) \left\{1 - \left[\tau^c \frac{\partial c^j}{\partial y^j} + \frac{\tau^d}{1+r}\frac{\partial d^j}{\partial y^j} + \frac{(1+g)\,\theta}{(1+\theta)(1+r)}\left(\frac{\partial x^j}{\partial y^j} + \beta \frac{\partial a^j}{\partial y^j}\right)\right]\right\} = 0. \tag{A34}
$$

Next multiply equation (A27) by c^h and equation (A28) by c^ℓ and add them together; similarly, multiply equation (A27) by d^h and equation (A28) by d^ℓ and add them together, and multiply equation (A27) by $x^h + \beta a^h$ and equation (A28) by $x^\ell + \beta a^\ell$ and add them together. We get

$$
\mu \sum_j \pi^j c^j \left\{1 - \left[\tau^c \frac{\partial c^j}{\partial y^j} + \frac{\tau^d}{1+r}\frac{\partial d^j}{\partial y^j} + \frac{(1+g)\,\theta}{(1+\theta)(1+r)}\left(\frac{\partial x^j}{\partial y^j} + \beta \frac{\partial a^j}{\partial y^j}\right)\right]\right\} =
$$

$$
\left(\delta^h + \lambda\right)\alpha^h c^h + \delta^\ell \alpha^\ell c^\ell - \lambda \alpha^{h\ell} c^\ell, \tag{A35}
$$

$$
\mu \sum_j \pi^j d^j \left\{1 - \left[\tau^c \frac{\partial c^j}{\partial y^j} + \frac{\tau^d}{1+r}\frac{\partial d^j}{\partial y^j} + \frac{(1+g)\,\theta}{(1+\theta)(1+r)}\left(\frac{\partial x^j}{\partial y^j} + \beta \frac{\partial a^j}{\partial y^j}\right)\right]\right\} =
$$

$$
\left(\delta^h + \lambda\right)\alpha^h d^h + \delta^\ell \alpha^\ell d^\ell - \lambda \alpha^{h\ell} d^\ell, \tag{A36}
$$

$$
\mu \sum_j \pi^j \left(x^j + \beta a^j\right) \left\{1 - \left[\tau^c \frac{\partial c^j}{\partial y^j} + \frac{\tau^d}{1+r}\frac{\partial d^j}{\partial y^j} + \frac{(1+g)\,\theta}{(1+\theta)(1+r)}\left(\frac{\partial x^j}{\partial y^j} + \beta \frac{\partial a^j}{\partial y^j}\right)\right]\right\} =
$$

$$
\left(\delta^h + \lambda\right)\alpha^h \left(x^h + \beta a^h\right) + \delta^\ell \alpha^\ell \left(x^\ell + \beta a^\ell\right) - \lambda \alpha^{h\ell} \left(x^\ell + \beta a^\ell\right). \tag{A37}
$$

Equations (37)–(39) are obtained substituting from (A35)–(A37) into (A32)–(A34), simplifying terms and taking into account that $\gamma\,(1+i)^2 = \gamma$ (since $\gamma = 0$ for $i > 0$).

Proof of (44): "Solving" (40)–(41) for τ^c and τ^d, one gets

$$\tau^c = \frac{\lambda \alpha^{h\ell}}{\mu} \frac{\left(c^\ell - c^{h\ell}\right) \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^d} - \left(d^\ell - d^{h\ell}\right) \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^c}}{\sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^c} \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^d} - \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^d} \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^c}}, \tag{A38}$$

$$\frac{\tau^d}{1+r} = \frac{\lambda \alpha^{h\ell}}{\mu} \frac{\left(d^\ell - d^{h\ell}\right) \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^c} - \left(c^\ell - c^{h\ell}\right) \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^d}}{\sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^c} \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^d} - \sum_j \pi^j \frac{\partial \widetilde{c}^j}{\partial q^d} \sum_j \pi^j \frac{\partial \widetilde{d}^j}{\partial q^c}}, \tag{A39}$$

Substituting in (42) and simplifying leads to (44).

Proof of Lemma 1: Consider the problem of a j-type individual choosing c, d, and x to maximize $u\left(c, d, x, (I+a)/w^j\right)$ subject to the budget constraint

$$q^c c + q^d d + q^x x = y + a - f(a) - \beta q^x a,$$

conditional on a given value for a. This optimization problem yields

$$c = c\left(y, I, w^j; a\right); \quad d = d\left(y, I, w^j; a\right); \quad x = x\left(y, I, w^j; a\right),$$

where for ease in notation we have suppressed q^c, q^d, q^x from the list of arguments. This allows one, through the composite commodity theorem, to define a new utility function in the (y, I) space:

$$U\left(y, I, w^j; a\right) \equiv u\left(c\left(y, I, w^j; a\right), d\left(y, I, w^j; a\right), x\left(y, I, w^j; a\right), (I+a)/w^j\right).$$

Observe that normality of c, d, and x in $u(\cdot)$ ensures normality of y in $U(\cdot)$. In turn, this ensures that the "single-crossing" property is satisfied for $U(\cdot)$.[46]

Next define the marginal rate of substitution between observable disposable income, y, and before-tax reported labor income, I, for an agent of type j as,

$$\begin{aligned} MRS_{yI}\left(y, I, w^j; a\right) &\equiv -\frac{\partial U\left(y, I, w^j; a\right)/\partial I}{\partial U\left(y, I, w^j; a\right)/\partial y} \\ &= -\frac{1+\tau^c}{w^j} \frac{u_L\left(c\left(y, I, w^j; a\right), d\left(y, I, w^j; a\right), x\left(y, I, w^j; a\right), (I+a)/w^j\right)}{u_c\left(c\left(y, I, w^j; a\right), d\left(y, I, w^j; a\right), x\left(y, I, w^j; a\right), (I+a)/w^j\right)}. \end{aligned} \tag{A40}$$

[46] The single-crossing or "agent monotonicity" condition requires that the marginal rate of substitution between consumption and income, y and I in this case, to be decreasing in wage so that at any (y, I) bundle, the high-ability agent will have a flatter indifference curve than the low-ability agent. In this way, they can cross only once. See, e.g., Salanié (2011).

Observe that the normality of y also implies that $MRS_{yI}\left(y, I, w^j; a\right)$ is increasing in a. This happens both because an increase in a, for a given I and w^j, implies a higher labor supply, and because it implies a larger disposable income, $y + a - f(a) - \beta q^x a$.

Finally, from equation (A7),

$$MRS_{yI}\left(y, I, w^j; a\right) = 1 - f'(a) - \beta q^x.$$

Hence a low-skilled agent when faced with the quadruple $(q^c, q^d, q^x, y^\ell, I^\ell)$, implied by the mechanism $\left(\tau^c, \tau^d, \theta, b^h, z^h, z^\ell, I^h, I^\ell\right)$, chooses a to satisfy,

$$MRS_{yI}\left(y^\ell, I^\ell, w^\ell; a\right) = 1 - f'(a) - \beta q^x. \tag{A41}$$

On the other hand, the $h\ell$-mimicker chooses a such that

$$MRS_{yI}\left(y^\ell, I^\ell, w^h; a\right) = 1 - f'(a) - \beta q^x. \tag{A42}$$

Denote the solution to (A41) by a^ℓ and the solution to (A42) by $a^{h\ell}$. It follows from (A41)–(A42) that

$$MRS_{yI}\left(y^\ell, I^\ell, w^\ell; a^\ell\right) + f'\left(a^\ell\right) = MRS_{yI}\left(y^\ell, I^\ell, w^h; a^{h\ell}\right) + f'\left(a^{h\ell}\right). \tag{A43}$$

At the same time, the single-crossing property implies that for the same value of a, $MRS_{yI}\left(y^\ell, I^\ell, w^\ell; a\right) > MRS_{yI}\left(y^\ell, I^\ell, w^h; a\right)$; or

$$MRS_{yI}\left(y^\ell, I^\ell, w^\ell; a^\ell\right) + f'\left(a^\ell\right) > MRS_{yI}\left(y^\ell, I^\ell, w^h; a^\ell\right) + f'\left(a^\ell\right). \tag{A44}$$

Substituting from (A43) for the left-hand side of (A44),

$$MRS_{yI}\left(y^\ell, I^\ell, w^h; a^{h\ell}\right) + f'\left(a^{h\ell}\right) > MRS_{yI}\left(y^\ell, I^\ell, w^h; a^\ell\right) + f'\left(a^\ell\right).$$

Now with $MRS_{yI}\left(y, I, w^j; a\right)$ increasing in a as shown earlier and $f'(a)$ increasing in a, due to convexity of $f(a)$, it follows from the above inequality that $a^{h\ell} > a^\ell$.

References

[1] Abel, Andrew, 1987. Optimal monetary growth. Journal of Monetary Economics 19, 437–450.

[2] Allingham, Michael G. and Agnar Sandmo, 1972. Income tax evasion: a theoretical analysis. Journal of Public Economics 1, 323–338.

[3] Arbex, Marcelo and Nurlan Turdaliev, 2011. Optimal monetary and audit policy with imperfect taxation. Journal of Macroeconomics 33, 327–340.

[4] Atkinson, Anthony B. and Joseph E. Stiglitz, 1972. The structure of indirect taxation and economic efficiency. Journal of Public Economics 1, 97–119.

[5] Atkinson, Anthony B. and Joseph E. Stiglitz, 1976. The design of tax structure: direct versus indirect taxation. Journal of Public Economics 6, 55–75.

[6] Blomquist, Sören, Christiansen, Vidar, and Luca Micheletto, 2011. Public provision of private goods, self-selection and income tax avoidance. UCFS working paper 2011:6, Department of Economics, Uppsala University.

[7] Boadway, Robin, Marchand, Maurice, and Pierre Pestieau, 1994. Towards a theory of the direct-indirect tax mix. Journal of Public Economics 55, 71–88.

[8] Chari, Varadarajan V., Christiano, Lawrence, and Patrick Kehoe, 1991. Optimal fiscal and monetary policy: some new results. Journal of Money, Credit and Banking 23, 519–539.

[9] Chari, Varadarajan V., Christiano, Lawrence, and Patrick Kehoe, 1996. Optimality of the Friedman rule in economies with distorting taxes. Journal of Monetary Economics 37, 203–233.

[10] Cremer, Helmuth and Firouz Gahvari, 1993. Tax evasion and optimal commodity taxation. Journal of Public Economics 50, 261–275.

[11] Cremer, Helmuth and Firouz Gahvari, 1995. Tax evasion and the optimum general income tax. Journal of Public Economics 60, 235–249.

[12] Cremer, Helmuth and Firouz Gahvari, 1997. In–kind transfers, self–selection and optimal tax policy. European Economic Review 41, 97–114.

[13] da Costa, Carlos E. and Iván Werning, 2008. On the optimality of the Friedman rule with heterogeneous agents and nonlinear income taxation. Journal of Political Economy 116, 82–112.

[14] Diamond, Peter A., 1965. National debt in a neoclassical growth model. American Economic Review 41, 1126–1150.

[15] Edwards, Jeremy, Keen, Michael and Matti Tuomala, 1994. Income tax, commodity taxes and public good provision: a brief guide, FinanzArchiv 51, 472–487.

[16] Erceg, Christopher J., Henderson, Dale W., and Andrew T. Levin, 2000. Optimal monetary policy with staggered wage and price contracts. Journal of Monetary Economics 46, 281–313.

[17] Friedman, Milton, 1969. The optimum quantity of money. In: Milton Friedman, ed., The Optimum Quantity of Money and Other Essays. Aldine Publishing Company, Chicago.

[18] Gahvari, Firouz, 1988. Lump-sum taxation and the superneutrality and optimum quantity of money in life cycle growth models. Journal of Public Economics 36, 339–367.

[19] Gahvari, Firouz, 2007. The Friedman rule: old and new. Journal of Monetary Economics 54 (2), 581–589.

[20] Gahvari, Firouz, 2012. Friedman rule in a model with endogenous growth and cash-in-advance constraint, Journal of Money, Credit, and Banking 44, 787–823.

[21] Gahvari, Firouz and Luca Micheletto, 2013. The Friedman rule in an overlapping-generations model with nonlinear taxation and income misreporting. Unpublished paper.

[22] Hamada, Koichi, 1972. Lifetime equity and dynamic efficiency on the balanced growth path. Journal of Public Economics 1, 373–396.

[23] Ireland, Peter, 1996. The role of countercyclical monetary policy. Journal of Political Economy 104, 704–723.

[24] Khan, Aubhik, King, Robert G., and Alexander L. Wolman, 2003. Optimal monetary policy. Review of Economic Studies 70, 825–860.

[25] McCallum, Bennett T., 1983. The role of overlapping-generations models in monetary economics. Carnegie-Rochester Conference Series on Public Policy 18, 9–44.

[26] Mirrlees, James A., 1971. An exploration in the theory of optimum income taxation. Review of Economic Studies 38, 175-208.

[27] Morse, Susan, Karlinsky, Stewart, and Joseph Bankman, 2009. Cash businesses and tax evasion. Stanford Law & Policy Review 20, 37–67.

[28] Niehans, Jürg, 1978. The theory of money. The Johns Hopkins University Press, Baltimore and London.

[29] Obstfeld, Maurice and Kenneth Rogoff, 1983. Speculative hyperinflations in maximizing models: Can we rule them out? Journal of Political Economy 91, 675–687.

[30] Obstfeld, Maurice and Kenneth Rogoff, 1985. Ruling out divergent speculative bubbles. Journal of Monetary Economics 17, 1986, 349–362.

[31] Pestieau, Pierre, 1974. Optimal taxation and discount rate for public investment in a growth setting. Journal of Public Economics 3, 217-235.

[32] Phelps, Edmund S., 1961. The golden rule of accumulation. American Economic Review 51, 638–643.

[33] Phelps, Edmund S., 1973. Inflation in the theory of public finance. Swedish Journal of Economics 75, 67–82.

[34] Salanié, Bernard, 2011. The Economics of Taxation. Second edition. The MIT Press, Cambridge, Massachusetts.

[35] Samuelson, Paul A., 1958. An exact consumption-loan model of interest with or without the social contrivance of money. Journal of Political Economy 66, 467–482.

[36] Sandmo, Agnar, 1974. A note on the structure of optimal taxation. American Economic Review 64, 701–706.

[37] Schmitt-Grohé, Stephanie and Martín Uribe, 2004. Optimal fiscal and monetary policy under imperfect completion. Journal of Macroeconomics 26, 183–209.

[38] Shaw, Jonathan, Slemrod, Joel, and John Whiting, 2011. Administration and Compliance. In: Stuart Adam, Timothy Besley, Richard Blundell, Stephen Bond, Robert Chote, Malcolm Gammie, Paul Johnson, Gareth Myles, and James Poterb, eds, Dimensions of Tax Design: the Mirrlees Review, 1100–1162. Oxford University Press: Oxford.

[39] Shaw, Ming-Fu, Chang, Juin-Jen, and Ching-Chong Lai, 2006. (Non)optimality of the Friedman rule and optimal taxation in a growing economy with imperfect competition. Economics Letters 90, 412–420.

[40] Scheinkman, Jose A., 1980. Discussion in: John H. Kareken and Neil Wallace, eds., Models of Monetary Economies (Federal Reserve Bank of Minneapolis).

[41] Stiglitz, Joseph E., 1982. Self-Selection and Pareto Efficient Taxation. Journal of Public Economics 17, 213–240.

[42] Usher, Dan, 1986. Tax evasion and the marginal cost of public funds. Economic Inquiry 24(4), 563–86.

[43] van der Ploeg, Frederick and George S. Alogoskoufis, 1994. Money and endogenous growth. Journal of Money, Credit, and Banking 26 (4), 771–791.

[44] Weiss, Laurence M., 1980. The effects of money supply on economic welfare in the steady state. Econometrica 48, 565–576.