Uncertain altruism and the provision of long term care*

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February 2013, revised February 2017

*Helmuth Cremer and Pierre Pestieau gratefully acknowledge financial support from the Chaire "*Marché des risques et creation de valeur*" of the FdR/SCOR. We thank Craig Brett, Chiara Orsini, Grégory Ponthière, Motohiro Sato and Erik Schokkaert for their comments. We are also grateful to two referees and the editor, Sören Blomquist, for their insightful and constructive remarks.

Abstract

When family assistance is uncertain, benefits cannot be conditioned on family aid. We study the role of private and public LTC insurance in this environment and compare the properties and optimality of the topping up versus opting out public insurance schemes. Under topping up, the required LTC is less than full insurance and should be provided publicly unless private insurance market for dependency is fair. With an opting out scheme, there will be three possible equilibria depending on the children's degree of altruism. These imply: full LTC insurance with no aid from children, less than full insurance just enough to induce aid, and full insurance with aid. Fair private insurance can support only the first equilibrium. Opting out policies are self-targeted and dominate topping up schemes when the degree of children's altruism is sufficiently large. However, when the degree of altruism is small the dominance goes in the opposite direction.

JEL classification: H2, H5.

Keywords: Long term care, uncertain altruism, private insurance, public insurance, topping up, opting out.

1 Introduction

Long-term care (LTC) is the provision of assistance and services to people who, because of disabling illnesses or conditions, have limited ability to perform basic daily activities such as bathing, cleaning, and cooking. It is a problem mainly, though not exclusively, for the elderly. In recent years, as people have come to live longer, the demand for LTC services by the elderly population has grown substantially—a trend likely to further increase and intensify in future years. There are two related reasons for this. First, LTC needs start to rise exponentially from around the age of 80 years old; second, the number of persons aged 80 years and above are growing faster than any other segment of the population. As a consequence, in most countries, the number of dependent elderly is expected to more than double by 2050. This will exacerbate the current pressures on the demand for LTC services and lead to new challenges for these countries and their governments.[1]

There are, currently, three institutions that finance and provide LTC services: the family, the market, and the state. The majority of the dependent population receiving long-term care at home rely exclusively on assistance from family members, mainly women; this is often referred to as "informal care". This avenue for LTC provision is, unfortunately, facing a number of formidable challenges: drastic changes in family values, increasing number of childless households, mobility of children, and growing rate of market activity on the part of women (particularly those aged 50–65). As a consequence, the number of dependent elderly who will be unable to count on the assistance of family members is likely to increase. This creates a pressing demand on the other two institutions, the market and the state, to offer either a substitute or a complement to what the family has thus far been providing by way of long term care.

The aim of this paper is to highlight and study the challenge posed by the idea

[1] For surveys on LTC and for more details on these estimates, see Cremer, Pestieau and Ponthière (2012) and Grabowski et al. (2012).

that family solidarity is uncertain. There are multifold reasons for this. First, there are pure demographic factors such as widowhood and the absence, or the loss, of children. Divorce and migration can also be put in this category. Other causes are conflicts within the family, or financial problems incurred by children, that prevent them from helping their parents. Whatever the reason, the possibility of "solidarity default" requires people to take appropriate steps such as self-insuring, purchasing private insurance, and relying on some public insurance or assistance scheme. What makes the problem particularly daunting and ripe for government intervention is the fact that there exists no good insurance mechanism to protect individuals against the default of family altruism.[2]

We study the role of private and public insurance programs in a world in which family assistance is uncertain. We do this by modeling the behavior and welfare of one single generation of parents over their life cycle. When they are young; they work, consume, and save for their retirement. In retirement, they face a probability of becoming dependent. This probability is exogenously determined and parents cannot affect it through their behavior (either when they are young or when they become old). If they become dependent, parents face yet another uncertainty. They may or may not receive assistance from their children. Many factors affect the children's behavior. Some causes of altruism default are purely exogenous but others can be influenced by the parents. Investment in the children's education and inculcating values in them through one's own behavior are such mechanisms.[3]

An important feature of our study is that we do not rule out private insurance markets by fiat. Indeed, we allow for the possibility of parents insuring themselves against becoming dependent. Plainly, however, moral hazard problems preclude the development of insurance markets against the default of altruism *per se* (as opposed to becoming dependent). For the same reasons, the government cannot condition its

[2]The consequences of uncertain altruism for old age retirement have been studied by Chabbakatri *et al.* (1993) and Leroux and Pestieau (2014).

[3]On this, see on this Kotlikoff and Spivak (1981) or Cox and Stark (2005).

assistance to the old on the default of altruism; only on age-old dependency.[4] Within this framework, we provide answers to two broadly-defined questions.

One question is the general need for insurance and how it should be provided: privately or publicly. We study the conditions under which private savings will or will not be enough for the three states of the world parents face in retirement (autonomy and dependency with or without assistance from children). When insurance is required, we examine if parents can rely on private insurance markets to secure the extra resources they need in case of dependency (because of the possibility of altruism default on the part of the children). We also discuss the circumstances that call for the government to step in and provide the needed assistance.

The second broad question we address concerns the nature of public assistance. One possibility is for the government to provide all dependent parents with monetary help while allowing them to top this up as they see fit. Another possibility is for the government to provide every dependent parent a "minimal" care facility whenever they ask for it. If this is deemed insufficient, the parents will have to opt out and use their own resources, and their children's, to purchase whatever home care services they need (without any help from the government). The dependent parents consume either one or the other. We examine and compare the properties of these two schemes.

In searching a role for the government, we confine ourselves to scenarios wherein the cost of financing the LTC program is borne by the potential beneficiaries themselves (and not by their children or future generations). In this way, one can zero-in on the insurance reasons for such programs rather than compounding this with issues that arise from wealth transfers across generations. We thus model the behavior and welfare of one single generation of parents over their life cycle. Our interest in their children is

[4]However, the government has a basic advantage over the private sector in that it can induce "self-targeting". That is, it can devise an opting-out policy with the property that only the old who suffer from the default of altruism opt for this insurance. Exploiting this possibility, and investigating its properties, is an important feature of our paper.

limited only to their role in providing assistance to their parents. As a consequence, the welfare of the grown-up children does not figure in government's objective function; only the expected utility of parents. Nor do the children pay any taxes to finance the LTC program (otherwise, they become a "costless" source of taxation to provide benefits for their parents).[5]

In answering the first set of questions we have asked, we find that the scheme the government adopts, topping up or opting out, has an important bearing on the question of who should provide LTC; the market or the state. Specifically, under the topping up scheme, if the probability of altruism is high there is no need for insurance regardless of who provides it. All assistance is provided through one's children and private savings. At lower probabilities, LTC insurance is called for; albeit one that is less than full. Moreover, the amount of insurance varies negatively with the probability of altruism. If private insurance markets for dependency are fair, private insurance will suffice (although public assistance is just as good). At higher than fair insurance premiums, on the other hand, public assistance dominates private insurance.

With an opting out scheme, the *degree* of altruism assumes an important role. This arises because under opting out, the government does not have to worry that the insurance intended for altruism default is automatically provided to all dependent parents (as it would under opting out). There is no leakage of benefits to the parents who are helped by their altruistic children, and thus indirectly the altruistic children themselves.[6]

More specifically, three types of equilibria emerge depending on the degree of children's altruism. If the degree of altruism is "small" or "very large", the optimal solution is for the government to provide full LTC insurance for everyone. With a small degree of altruism, all children opt for the government plan providing no assistance of their own to

[5] In Appendix C, we explore the implications of including the children's utility in social welfare. Interestingly, our qualitative results do not change. Some of the expression are affected but the changes are minor.

[6] The reason for it becomes clear after we discuss the properties of the two schemes in Sections 2–4. See, in particular, our discussion of this issue in subsection 4.5 including footnote 27.

their parents. With the very large degree of altruism, altruistic children do not consider the government's full insurance plan good enough and opt out of it. Instead of what the government offers, they provide their own assistance. The only option open to the parents of non-altruistic children is of course the government assistance. Interestingly, when the children's degree of altruism is "moderate," lying somewhere between small and very large, the best strategy for the government is to provide less than full LTC insurance. This will be just small enough to entice the altruistic children to substitute their own assistance for the government's.

As to the question of private versus public insurance, we show that the two equilibria which entail assistance from altruistic children—arising when the children's degree of altruism is moderate and very large—can be supported only through the public opting out scheme. Private insurance markets cannot do the job even if they are fair. On the other hand, when children have a low degree of altruism so that the equilibrium is one without assistance, fair private insurance markets are just as good as public insurance.

Finally, comparing topping up and opting out policies, we show that opting out always dominates when children are sufficiently altruistic. This is because under opting out, public LTC can be targeted to the parents whose children turn out not to be altruistic. However, for lower degrees of altruism, this more precise targeting scheme comes at a price, namely that public LTC is distorted downward (to ensure continued aid from altruistic children). This makes the comparison ambiguous. But when the degree of altruism is small, the dominance goes in the opposite direction.

2 The model with topping up

Consider a setup wherein (i) in period 0, the government formulates and announces its tax/transfer policy; (ii) in period 1, a young working parent and a child appear on the scene and the parent decides on his present and future consumption levels; in period 2, the parent has grown old and is retired, and the child who has turned into a working adult decides if he wants to help his parents. We will not be concerned with what will happen to the grown-up child when he turns old, nor with any future generations. Two uncertain events give rise to the problem that we study. One concerns the health of the parent in old age. He may be either "dependent" or "independent". Denote the probability of dependency by π; naturally, the parent will be independent with probability $1-\pi$. We assume that π is exogenously given. The second source of uncertainty concerns the help that the parent might get from his children *if* dependent. Denote the probability that a child is altruistic towards his parents by p and the probability that he is not by $1-p$. We shall assume that p is also exogenously fixed.

Parents have preferences over consumption when young, c, and consumption when old, d; there is no disutility associated with working. We assume, for simplicity, that the parents' preferences are quasilinear in consumption when young. With p fixed, labor supply is also predetermined as there is no other usage for the parents' time.

Government's policy consists of levying a tax at rate of τ on the parents' wage, w, to finance benefits g provided in case of dependency. For simplicity we shall refer to g as the LTC insurance. It may consist of a cash transfer used to finance market care provided at home or represent the cost of public care provided at home. In the topping up case and with the simple specification of preferences we use this distinction does not matter. The crucial point is that either kind of transfer can easily be topped up.

Labor supply is fixed and equal to $\overline{T}$. As far as private insurance is concerned, it is easier to rule it out initially for modeling purposes. Having established the need for insurance, we examine if it can be decentralized through private insurance markets.

Denote the level of assistance an altruistic child would give his parents by a, savings by s, and set the rate of interest on savings at zero. At this point (up to Section 3) we assume that g is non exclusive in the sense that it can be topped up by a and s. One can then represent the parent's preferences by means of the expected utility,

$$EU = w(1-\tau)\overline{T} - s + (1-\pi)U(s) + \pi\left[pH(s+g+a) + (1-p)H(s+g)\right], \quad (1)$$

where H and U are Bernoulli utilities if the parent does and does not become dependent. We shall assume that the grown-up children of dependent parents too have quasilinear preferences represented by

$$u = y - a + \beta H\left(s + g + a\right), \tag{2}$$

where y denotes their income with β being the degree of their altruism towards their parents.[7] Those who are not altruistic towards their parents have a $\beta = 0$.

To determine the government's policy we start by studying the last stage of our decision making process. This is when the grown-up children decide on the extent of their help to their parents, if any.

2.1 Stage 3: The child's choice

The altruistic child allocates an amount a of his income y to assist his dependent parent (given the parent's savings s and the government's provision of g). Its optimal level, A, is found through the maximization of equation (2). The first-order condition with respect to a is, assuming an interior solution,[8]

$$-1 + \beta H'\left(s + g + a\right) = 0.$$

It follows from this condition that A satisfies

$$s + g + A = \left(H'\right)^{-1}\left(\frac{1}{\beta}\right) \equiv m\left(\beta\right). \tag{3}$$

Finally, differentiating (3) yields

$$\frac{\partial m}{\partial \beta} = \frac{-1}{\beta^2 H''\left(s + g + a\right)} > 0,$$

[7] Throughout the paper we assume that β, and more generally individual preferences are common knowledge.

[8] A sufficient condition for an interior solution is to have

$$\frac{du}{da}|_{a=0} = -1 + \beta H'\left(s + g\right) > 0,$$

or

$$H'\left(s + g\right) > \frac{1}{\beta}.$$

where the sign follows from concavity of $H(\cdot)$.[9]

2.2 Stage 2: The parent's choice

Recall that the parent may experience two states of nature when retired: dependency with probability π and autonomy with probability $(1-\pi)$. Substituting for A from (3) in the parent's expected utility function (1), we have

$$EU = w(1-\tau)\overline{T} - s + (1-\pi)U(s) + \pi\left[pH(m(\beta)) + (1-p)H(s+g)\right]. \quad (4)$$

Maximizing EU with respect to s, and assuming an interior solution, the optimal value of s satisfies

$$(1-\pi)U'(s) + \pi(1-p)H'(s+g) = 1. \quad (5)$$

Denote the solution to equation (5) by $s(p;g)$. Substituting $s(p;g)$ for s in (5), the resulting relationship holds for all values of p and g. Differentiating this relationship partially with respect to p and g yields,

$$\frac{\partial s}{\partial p} = \frac{\pi H'(s+g)}{(1-\pi)U''(s) + \pi(1-p)H''(s+g)} < 0, \quad (6)$$

$$\frac{\partial s}{\partial g} = \frac{-\pi(1-p)H''(s+g)}{(1-\pi)U''(s) + \pi(1-p)H''(s+g)} < 0. \quad (7)$$

Thus a parent's savings move negatively with the probability of altruism and with the government's level of assistance.

[9] Differentiating (3) results in

$$\frac{\partial m(\beta)}{\partial \beta} = \frac{\partial(s+g+a^*)}{\partial \beta}.$$

But we also have, from differentiating $H'(s+g+a^*) = 1/\beta$ with respect to β, that

$$H''(s+g+a^*)\frac{\partial(s+g+a^*)}{\partial \beta} = \frac{-1}{\beta^2}.$$

2.3 Stage 1: The optimal policy

The government chooses τ and g to maximize EU, as optimized by the parents in stage 2, subject to its budget constraint

$$\tau w\overline{T} = \pi g. \tag{8}$$

Substituting for τ from (8) into the parents' optimized value of EU, the government chooses g to maximize

$$\begin{aligned} \pounds \equiv w\overline{T} - \pi g - s\left(p; g\right) + \left(1 - \pi\right) U\left(s\left(p; g\right)\right) + \\ \pi\left[pH\left(m\left(\beta\right)\right) + \left(1 - p\right) H\left(s\left(p; g\right) + g\right)\right]. \end{aligned} \tag{9}$$

Differentiating $\pounds$ with respect to g yields, using the envelope theorem,

$$\frac{d\pounds}{dg} = \pi\left[\left(1 - p\right) H'\left(s\left(p; g\right) + g\right) - 1\right]. \tag{10}$$

There are two possible outcomes depending on the sign of $d\pounds/dg$ at $g = 0$.

Let $s\left(p\right) \equiv s\left(p; 0\right)$. One possible outcome arises if

$$\left(1 - p\right) H'\left(s\left(p\right)\right) - 1 \leq 0. \tag{11}$$

This condition holds in the neighborhood of $p = 1$.[10] Under this condition, $g = \tau = 0$ and no LTC insurance is called for. The consumption levels provided through one's own savings, and help from altruistic children, are sufficient (albeit, if $p < 1$, ex post there will be some people with "insufficient" means).[11]

The more interesting outcome occurs when

$$\left(1 - p\right) H'\left(s\left(p\right)\right) - 1 > 0. \tag{12}$$

[10] The condition is satisfied at $p = 1$ and by continuity in the neighborhood of $p = 1$.

[11] One explanatory reason mentioned for the LTC insurance puzzle is family solidarity (as well as the more well-known annuity puzzle); see Pestieau and Ponthière (2011). Our result suggests that even less than certain family solidarity may explain the puzzle.

This condition is necessarily satisfied for some p as long as $H'(s(0)) > U'(s(0))$.[12] In this case, there will be an interior solution for g, and τ, characterized by

$$H'(s(p;g)+g) = \frac{1}{1-p} > 1. \tag{13}$$

Consequently, there is less than full insurance.[13] Substituting from (13) into (5), it is also the case that

$$U'(s(p;g)) = 1. \tag{14}$$

The optimal value of g in this second outcome is found as the solution to equation (13) (or equation (14)). We denote this optimal level by $G(p)$.

Interestingly too, it follows from equations (13) and (14) that the optimal value of publicly-provided LTC insurance $G(p)$, and its implementing tax rate τ, decrease with p. Specifically, differentiating (13) with respect to p yields

$$\frac{dG}{dp} = \frac{H'(s+G)}{(1-p)H''(s+G)} < 0. \tag{15}$$

The intuition behind this result is easily understood. Providing more g for dependent parents who do not get help from their children is also accompanied by providing more g for dependent parents who do receive aid from their children. Now while parents bear all the cost of providing more g in terms of a loss in their present day consumption, the beneficiary of more g also includes the altruistic children who would have otherwise provided the extra g to their parents. Put differently, g will only provide extra benefits to

[12] Observe that

$$\frac{d}{dp}\left[(1-p)H'(s(p)) - 1\right] = -H'(s(p)) + (1-p)H''(s(p))s'(p),$$

which is negative at $p = 1$. Consequently, as p becomes smaller than one, $(1-p)H'(s(p)) - 1$ initially increases so that at some point *might* change its sign. Condition $H'(s(0)) > 1$ ensures that this will happen. In turn, from (5), this condition is equivalent to assuming that $H'(s(0)) > U'(s(0))$. The need for this assumption arises because with $H''(s(p)) < 0$ and $s'(p) < 0$, the sign of the above derivative while initially negative is in general indeterminate so that it may not always remain negative. Even if it does, there is no guarantee that $(1-p)H'(s(p)) - 1$ necessarily changes sign.

[13] Full insurance requires the equality of $H'(\cdot)$ with the marginal utility of current consumption which, with quasilinear preferences, is equal to one.

parents when children are not altruistic which occurs with probability p. Consequently a higher p will result in a lower level of g.

Finally, with our findings that $dG/dp < 0$ and $G = 0$ at high values of p, there must exist a $p = \widehat{p}$ at which the switch from one outcome to the other occurs. This is when condition (11) holds as an equality:

$$H'(s(\widehat{p})) = \frac{1}{1-\widehat{p}}. \tag{16}$$

Observe also that at this price, we have from equation (14) that

$$U'(s(\widehat{p})) = 1. \tag{17}$$

2.4 The question of private insurance

Assume $0 \leq p < \widehat{p}$ so that dependent parents need additional assistance beyond their own savings when assistance is not forthcoming from their children. The question is if they can secure this through private insurance markets. To answer this question, denote the amount of insurance the parent may buy against old-age dependency, if it is optimal to have private insurance markets, by θ and its unit price by q. First, observe that allowing private insurance markets does not change the consumption level of a parent receiving assistance from his children; it remains at $m(\beta)$.[14] Second, let e denote the government's assistance *above* the privately-purchased insurance, θ, so that $G = \theta + e$. Reconsidering the parents' optimization problem, one can show that savings depends on $\theta + e$ and not its division between public and private insurance.[15]

[14] Let e denote the government's assistance *above* the privately-purchased insurance, θ. The children's optimization level results in the first-order condition $-1 + \beta H'(s+\theta+e+a) = 0$, so that the total consumption of the parent, $s+\theta+e+a$, does not change.

[15] Expected utility of the parent is now written as

$$\begin{aligned} EU = w(1-\tau)\overline{T} - q\theta - s + (1-\pi)U(s) + \\ \pi\left[pH(m(\beta)) + (1-p)H(s+\theta+e)\right]. \end{aligned}$$

This leads to the following first-order condition with respect s,

$$(1-\pi)U'(s) + \pi(1-p)H'(s+\theta+e) = 1.$$

Now consider the expected utility of the parent, if private insurance markets are operative. Fix the total amount of LTC insurance provided both privately and publicly, for the given p, at its optimal level derived for when all insurance is provided by the government. That is set, $\theta + e = G$. Under this circumstance, parents' savings would continue to be what it was when all LTC insurance is provided publicly. We thus have

$$EU = w(1-\tau)\overline{T} - q\theta - S + (1-\pi)U(S) + \pi\left[pH(m(\beta)) + (1-p)H(S+G)\right], \tag{18}$$

where capital letters S and G denote the fixed optimal value of s and g.[16] With the parents' purchasing θ, the government must finance only $e = G - \theta$ in assistance to dependent parents. This changes the government's budget constraint to

$$\tau w\overline{T} = \pi(G-\theta). \tag{19}$$

Substitute for τ from (19) into (18). Upon simplification, we have

$$\begin{aligned} EU = w\overline{T} - \pi G + (\pi - q)\theta - S + (1-\pi)U(S) + \\ \pi\left[pH(m(\beta)) + (1-p)H(S+G)\right]. \end{aligned} \tag{20}$$

At $\theta = 0$, we have the optimal solution with insurance being provided solely by the government. The question is if having $\theta > 0$ benefits or harms the parents. To answer this question, differentiate (20) with respect to θ. This yields

$$\frac{\partial EU}{\partial \theta} = \pi - q. \tag{21}$$

The result in (21) makes perfect sense. Both the private sector and the government provide insurance against dependency and not lack of altruism. The per unit cost through the private sector is q and through the government, implicitly, π. If the private

Comparing this equation with (5) indicates this point.

[16] Both S and G are functions of p.

insurance price is "fair" then $q = \pi$ and one is just as good as another. Under this circumstance, private markets suffice and there is no additional need for government intervention. On the other hand, if $q > \pi$, providing assistance through private markets can only hurt the parents. Insurance should be provided only publicly.

The main results of this section are summarized in the following proposition.

Proposition 1 *Consider a topping up scheme. Let $s(p)$ solve*

$$(1-\pi) U'(s(p)) + \pi (1-p) H'(s(p)) = 1.$$

Define $\widehat{p}$ as the solution to

$$H'(s(p)) = \frac{1}{1-p}.$$

(i) If

$$\widehat{p} \leq p \leq 1,$$

there is no need for insurance regardless of who provides it. All assistance is provided through one's children and private savings. In other words, even uncertain family solidarity may explain the LTC insurance puzzle.

(ii) If

$$0 \leq p < \widehat{p},$$

there will be need for LTC insurance.

(a) If $q = \pi$, i.e., if private insurance markets for dependency are fair, private insurance will suffice (although public assistance is just as good). On the other hand, if $q > \pi$, public assistance dominates private insurance.

(b) The optimal amount of LTC insurance g is characterized by

$$H'(s(p; g) + g) = \frac{1}{1-p} > 1,$$

and there is less than full insurance.

(c) The amount of LTC insurance g varies negatively with the probability of receiving aid from one's children, p.

3 Opting out

We now assume that, if made dependent, parents will need a particular good/service, x. Specifically, one can think of purchasing home care services using one's own, and one's children's, resources versus government provision of a minimum facility. The crucial point is that one consumes either one or the other. One cannot top up what the government provides. Thus unlike in the topping up case the specific nature of the transfer provided by the government is very relevant here. In particular when topping up is not possible it is most natural to think about an in-kind transfer like formal care provided in an institution.

Denote government's provision by z. Clearly, $z \geq s$; otherwise z will be of no use to the parents. When providing such a facility, the government taxes away the recipient's resources (savings and any private insurance that he may have purchased). Consequently, $x = s + a$ if LTC insurance is provided by the altruistic children and $x = z$ if it is provided by the government. Clearly, the dependent parents with non-altruistic children end up with a consumption level $x = z$. What the consumption level of parents with altruistic children will be is not clear; however. That depends on the behavior of these children. Whereas under a topping up scheme an altruistic child decides on how much to *supplement* the government's provision; under an opting out scheme, he would have to decide *between* his own assistance versus that of the government's. Thus the parents end up either with some assistance from their altruistic children or no assistance at all. As in the previous section, we start with stage 3 and the children's decision.

3.1 Stage 3: The child's choice

The optimal level of family assistance a^* is found from the maximization of

$$u = y - a + \beta H(s + a).$$

To have an interior solution for a, it must be the case that $(\partial u/\partial a)\,|_{a=0} = -1+\beta H'(s) > 0$; or $\beta > 1/H'(s)$. We assume this is the case. The value of a^* is then given by

$$s + a^* = \left(H'\right)^{-1}\left(\frac{1}{\beta}\right) \equiv m(\beta), \tag{22}$$

where, as with the topping-up case, $m'(\beta) > 0$. Observe that while the optimal value of a is now given by $a^* = m(\beta) - s$, as opposed to $A = m(\beta) - (s+g)$ in the topping up scenario of the previous section, the actual consumption level of the dependent parent who he is being assisted by his children remains the same. Put differently, under OO, the child fully makes up for what the government would have under TU.

Clearly, the altruistic child chooses a^* if and only if it gives him more utility than the option of no assistance. However, whereas in the topping up case $A = m(\beta) - (s+g)$ moved continuously with g; here a^* is independent of z and the child faces a discontinuous choice. This is settled by comparing his utility if he chooses his own assistance,

$$u = y - a^* + \beta H(s + a^*), \tag{23}$$

versus his utility if he goes with government's assistance for his parents,

$$u = y + \beta H(z). \tag{24}$$

The two yield the same utility level if

$$H(z) = H(s + a^*) - \frac{a^*}{\beta}, \tag{25}$$

where $a^* = m(\beta) - s$. Denote the solution to equation (25) by $\widehat{z}(\beta)$. If the z offered by the government falls short of $\widehat{z}(\beta)$, the altruistic child provides his parents with a^* in aid; if z exceeds $\widehat{z}(\beta)$, the child opts for no assistance. The crucial difference with the topping up case is that there the possibility of altruistic children not providing help can be ruled out by assuming that $\beta > 1/H'(s+g)$. Here neither $\beta > 1/H'(s)$ nor $\beta > 1/H'(z)$ is sufficient for this purpose. No matter how large β is if z exceeds $\widehat{z}(\beta)$, altruistic children will not provide any assistance to their parents.

Observe that $\widehat{z}(\beta)$ is an increasing function of β. To see this, substitute $\widehat{z}(\beta)$ for z in (25) to get

$$-a^* + \beta H(s + a^*) - \beta H(\widehat{z}) = 0.$$

Differentiating, using the envelope theorem, yields

$$\frac{d\widehat{z}(\beta)}{d\beta} = \frac{H(s + a^*) - H(\widehat{z})}{\beta H'(\widehat{z})} > 0.$$

Observe also that if an altruistic child opts to help his parents, he must be providing them with a higher consumption level than what they can get from the government. That is, it must be the case that $s + a^* > z$.[17] An implication of this is that parents who receive aid from their children will never ask the government for help; the program is self-targeted.

3.2 Stage 2: The parent's choice

Assume first that altruistic children come to the help of their parents. With $s + a^* = m(\beta)$, the expected utility of the parent is

$$EU = w(1-\tau)\overline{T} - s + (1-\pi)U(s) + \pi[pH(m(\beta)) + (1-p)H(z)]. \quad (26)$$

The parent's first-order condition with respect to s then results in,[18]

$$U'(s) = \frac{1}{1-\pi}. \quad (27)$$

Note that, unlike the topping up case, the solution for s is independent of the probability of altruism, p, and government's level of assistance, z. We denote the solution to equation (27) by s^*.

[17] This follows because when the altruistic child decides to help his parent, it follows from (23)–(24) that

$$H(s+a) - H(z) > a/\beta > 0.$$

[18] An interior solution is guaranteed by assuming that $U'(0) > 1/(1-\pi)$.

Next consider the case where altruistic children do not help their parents. The parent's expected utility is now given by

$$EU = w(1-\tau)\overline{T} - s + (1-\pi)U(s) + \pi H(z). \tag{28}$$

This leads to the first-order condition,

$$U'(s) = \frac{1}{1-\pi}.$$

This is identical to the optimal condition for s when altruistic children help their parents, equation (27). We thus have the same solution, s^*.

3.3 Stage 1: The optimal policy

The determination of optimal policy is a more complicated undertaking under the opting out scheme as compared to the topping up scheme. The government chooses its policy anticipating what the children and parents do. The complicating factor here is the discontinuous manner with which the government can, through its policy choice, induce altruistic children to move from providing a given level of assistance to none, and vice versa. A determining factor in this is the value of $\widehat{z}$ which depends on β. Another factor is how z affects the parents' expected utility when they receive assistance from their children as compared to when they do not. We study the relevance and the implications of these factors in the next section.

4 Optimal policy under the opting out scheme

Two observations help in figuring out if the government may want to opt for an equilibrium with or an equilibrium without assistance from children (as well as the corresponding levels of the assistance). The first observation is that, for a given level of z, parents enjoy a higher expected utility with children's assistance than without.[19] This

[19] For the same z, the expected utility of parents will be higher by

$$\pi p \{[H(m(\beta)) - H(z)] + (z - s^*)\} > 0,$$

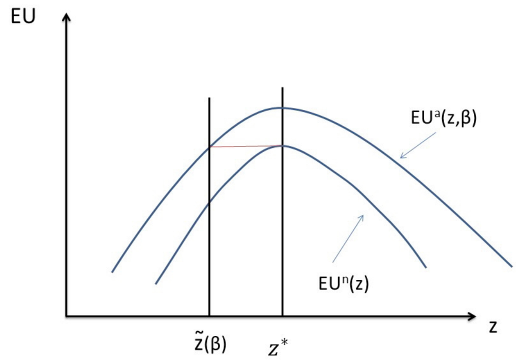

Figure 1: Parents' expected utility with and without probabilistic aid from children.

follows because help from children is "free" but help from the government comes at the cost of taxing the parents. The second observation is that the value of z that maximizes the expected utility of the parent conditional on the parent's receiving aid from his children is the same as the value of z that maximizes the expected utility of the parent conditional on the parent's not receiving aid from his children.

4.1 Altruistic children help

With targeted assistance, and help from altruistic children, the government's budget constraint changes to

$$\tau w\overline{T} + \pi (1-p)\, s = \pi (1-p)\, z, \tag{29}$$

under an equilibrium with assistance from altruistic children as compared to no assistance.

as only parents who are not helped by their children ask for assistance (and pay the additional taxes).[20] Substitute for $\tau w\overline{T}$ from (29) into (26) to get

$$EU^a(z,\beta) \equiv w(1-\tau)\overline{T} - s^* + (1-\pi)U(s^*) + \pi[pH(m(\beta)) + (1-p)H(z)]$$
$$= w\overline{T} - \pi(1-p)z + \pi(1-p)s^* - s^* +$$
$$(1-\pi)U(s^*) + \pi[pH(m(\beta)) + (1-p)H(z)], \tag{30}$$

where s^* is the solution to (27). Thus $EU^a(z,\beta)$ denotes the expected utility of a parent conditional on receiving probabilistic aid from his children when the government provision is z and the children's degree of altruism is β. Differentiating $EU^a(z,\beta)$ partially with respect to z yields

$$\frac{\partial EU^a(z,\beta)}{\partial z} = \pi(1-p)\left[H'(z) - 1\right]. \tag{31}$$

Assuming $\partial EU^a(z,\beta)/\partial z|_{\tau=0} > 0$, or equivalently $H'(s^*) > 1$,[21] the value of z that maximizes $EU^a(z,\beta)$ is characterized by[22]

$$H'(z^*) = 1. \tag{32}$$

4.2 No help from altruistic children

With targeted assistance but no help from altruistic children, the government's budget constraint is

$$\tau w\overline{T} = \pi(z-s), \tag{33}$$

as it has to provide z for everyone who is dependent. Substituting for $\tau w\overline{T}$ from (33) into (28) yields

$$EU^n(z) \equiv w(1-\tau)\overline{T} - s^* + (1-\pi)U(s^*) + \pi H(z)$$
$$= w\overline{T} - \pi z + \pi s^* - s^* + (1-\pi)U(s^*) + \pi H(z), \tag{34}$$

[20] Recall that when providing public care, the government taxes away the recipient's resources (savings and any private insurance that he may have purchased).

[21] Given that $U'(s^*) = 1/(1-\pi) > 1$, this condition is necessarily satisfied as long as $H'(s^*) > U'(s^*)$.

[22] Except for $p = 1$ at which $\tau = z = 0$ and the government does not have an optimization problem.

where s^* is, as previously, the solution to (27). Thus $EU^n(z)$ denotes the expected utility of a parent who receives no aid from his children when the government provision is z. Differentiating $EU^n(z)$ with respect to z yields

$$\frac{dEU^n(z)}{dz} = \pi\left[H'(z) - 1\right].$$

Consequently, $z = z^*$ also maximizes $EU^n(z)$.

4.3 Three possible equilibria

Figure 1 shows the graphs for $EU^a(z, \beta)$ and $EU^n(z)$ in light of the above observations (for a given value of β). The figure also shows the value of $z < z^*$ at which the parents' expected utility when receiving probabilistic assistance from their children, $EU^a(z, \beta)$, is equal to $EU^n(z^*)$, the parents' expected utility at $z = z^*$ when they do not receive assistance. Denote this value by $\widetilde{z}(\beta)$; it is defined by $EU^a(\widetilde{z}(\beta), \beta) = EU^n(z^*)$ or

$$(1-p)\left[H(\widetilde{z}(\beta)) - \widetilde{z}(\beta)\right] = p\left[s^* - H(m(\beta))\right] + \left[H(z^*) - z^*\right].$$

The optimal value of z depends on the location of $\widehat{z}(\beta)$ in relation to $\widetilde{z}(\beta)$ and z^*. If $\widehat{z}(\beta)$ is to the left of $\widetilde{z}(\beta)$, the optimal solution for z is z^* with the solution entailing no aid from children; see Figure 2. If $\widehat{z}(\beta)$ is between $\widetilde{z}(\beta)$ and z^*, the optimal solution for z is $\widehat{z}(\beta)$; see Figure 3. By restricting z to $\widehat{z}(\beta)$, which is just low enough to entice the children to help, the government increases the parents' expected utility as compared to providing z^*.[23] Finally, if $\widehat{z}(\beta)$ is to the right of z^*, the government provides z^*; but in this case children also provide assistance (probabilistically); see Figure 4.

4.4 The degree of altruism and the equilibrium type

We have already seen that there are three possible equilibria: (a) z^* with no aid from children, (b) $\widehat{z}(\beta)$, and (c) z^* with probabilistic aid from children. We now prove

[23] We assume that the tie-breaker is to provide assistance whenever altruistic children are indifferent between providing and not providing assistance.

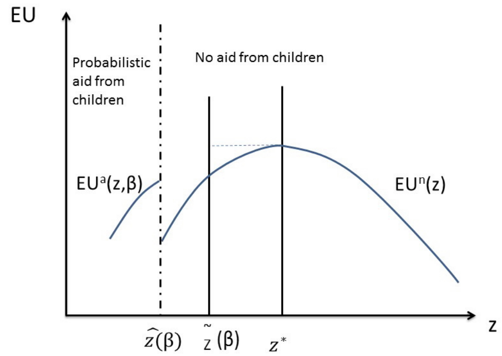

Figure 2: Equilibrium with no aid from children.

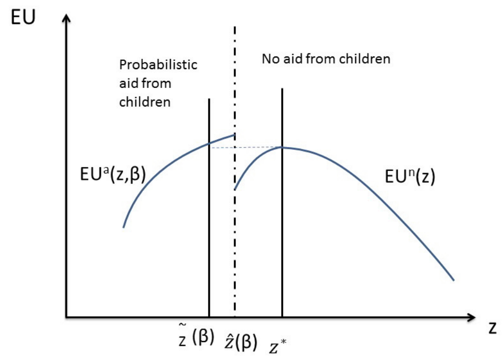

Figure 3: Constrained equilibrium with probabilistic aid from children.

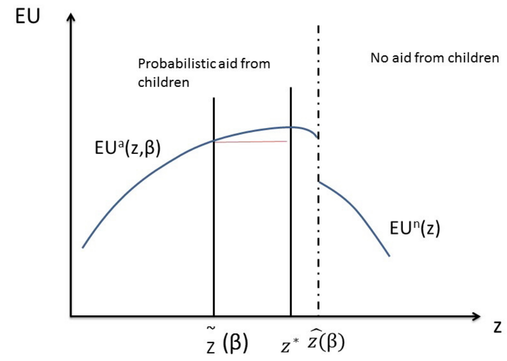

Figure 4: Unconstrained equilibrium with probabilistic aid from children.

in Appendix A that the type of equilibrium that emerges depends on the value of β, the children's degree of altruism towards their parents. The equilibrium is of type (a) for "low" values of β, of type (b) for "moderate" values of β, and of type (c) for "very large" values of β. These regimes as well as the main properties obtained in each case are illustrated in Figure 5. Observe that the z^* solution with probabilistic assistance from children occurs only if children exhibit an extremely high degree of altruism towards their parents. Specifically, we must have $\beta > 1$, so that children care about their dependent parents more than the parents care about themselves. One may refer to such children as "super altruistic".[24]

4.5 Full and less-than-full LTC insurance

Introduce g to denote the *net* contribution of the government to the dependent parent who seeks help. This is what the government provides net of the parent's savings, s; it

[24]On the other hand, one could also argue that Alzheimer patients do not care much about what happens to them but their children want them to be cared for in a dignified and decent manner. We remain agnostic about this possibility and do not a priori rule out $\beta > 1$.

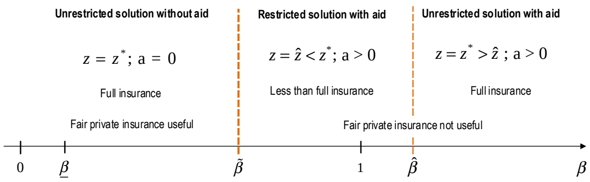

Figure 5: Solutions under opting out.

is the LTC insurance. Hence[25]

$$g \equiv z - s. \tag{35}$$

Substituting from (35) into (32), while setting $s = s^*$, the optimal g is characterized by

$$H'(s^* + g^*) = 1,$$

and we have full LTC insurance whenever the equilibrium is z^*, with or without aid from children.[26]

On the other hand, when equilibrium is the constrained $\widehat{z}(\beta)$, the solution entails less than full insurance. To see this, note that $\widehat{z}(\beta) > s^*$ so that there is positive LTC insurance. However, given that $\widehat{z}(\beta) < z^*$, we have

$$H'(\widehat{z}(\beta)) > H'(z^*) = 1.$$

Finally observe that, in contrast to the topping up scheme, the optimality of a positive LTC insurance no longer rests on the probability of altruism default being high (small p). With z^*, $\widehat{z}(\beta)$, and s^* not varying with p, the optimal level of LTC insurance g (whether $z^* - s^*$ or $\widehat{z}(\beta) - s^*$), is independent of p. The difference arises because

[25] If there is private insurance of the amount θ, the dependent parent will have $s + \theta + a$ if receiving assistance from his children, and z with the default of altruism. However, in this case θ is also taxed away so that $g = z - s - \theta$.

[26] The z^* solution with aid from children implies that children do not consider full insurance as "sufficient enough" and go on to substitute their own higher level of help for it.

under opting out, the government does not have to worry that the insurance intended for altruism default is automatically provided to all dependent parents. With no leakage of benefits to the parents who are helped by their altruistic children, and thus indirectly the altruistic children themselves, the probability of altruism does not matter.[27] The main results of this section are summarized in the following proposition.

Proposition 2 *Consider an opting-out LTC scheme. Altruistic children have a degree of altruism towards their parents equal to β. Let $\underline{\beta}$, defined by equation (A2), denote the minimum value of β below which no children helps his parents. Let $\widehat{z}(\beta)$, defined by equation (25), denote the maximum value of public assistance z consistent with children, whose degree of altruism towards their parents is β, providing assistance to their parents. Let z^* denote the amount of z that entail full LTC insurance defined by $H'(z^*) = 1$.*

(i) The expected utility of the parents who receive probabilistic aid from children, $EU^a(\widehat{z}(\beta), \beta)$, has the following properties: (a) It is increasing at $\beta = \underline{\beta}$; (b) it assumes a value equal to the expected utility of the parents who do not receive assistance from their children and are given z^* in public assistance at $\beta = \widetilde{\beta} > \underline{\beta}$, and (c) it attains its maximal value at $\widehat{\beta}$.

(ii) An opting out LTC scheme has three types of equilibria depending on the children's degree of altruism towards their parents: (a) For all $\underline{\beta} < \beta < \widetilde{\beta}$, the equilibrium is z^* with no assistance from children; (b) for all $\widetilde{\beta} < \beta < \widehat{\beta}$, the equilibrium is $\widehat{z}(\beta)$ so that probabilistic aid from children is forthcoming; and (c) for all $\beta > \widehat{\beta}$, the equilibrium is z^* with probabilistic assistance from children. This last equilibrium occurs only if $\beta > \widehat{\beta} > 1$ and children are "super altruistic."

[27] Except, of course, if $p = 1$ and no public LTC insurance is required. At $p < 1$, the calculus of LTC provision is as follows. Under topping up, providing one more dollar worth of g entails a marginal benefit of $\pi(1-p)H'(\cdot)$ and a marginal cost of π for a net benefit of $\pi[(1-p)H'(\cdot) - 1]$; see equations (9) and (10). Under opting out, there are two possibilities. With probabilistic aid from children, one extra dollar expenditure on z has a marginal benefit of $\pi(1-p)H'(\cdot)$ and a marginal cost of $\pi(1-p)$ for a net benefit of $\pi(1-p)[H'(\cdot) - 1]$; see equations (30) and (31) Without aid from altruistic children, the marginal benefit is $\pi H'(\cdot)$ and the marginal cost π resulting in a net benefit of $\pi[H'(\cdot) - 1]$; see equation (34). It is only in the topping up case that the sign of the net marginal benefit depends on p.

(iii) The optimal amount of LTC insurance, $g^* = z - s^*$, is always positive.. There is full insurance with equilibria z^* with and without probabilistic assistance from children; there is less than full insurance with the $\widehat{z}(\beta)$ equilibrium.

5 Opting out and private insurance

5.1 Solutions with no aid from children: $\underline{\beta} < \beta < \widetilde{\beta}$

Consider again the expected utility of the parent, if private insurance markets are operative, and fix the level of z at its optimal level, z^*. We have

$$EU = w(1-\tau)\overline{T} - s^* - q\theta + (1-\pi)U(s^*) + \pi H(z^*).$$

However, if parents buy θ in private insurance, given that the government will tax these resources away as well when providing z, we have

$$\tau w\overline{T} = \pi(z^* - s^* - \theta). \qquad (36)$$

Substitute for τ from (36) into the expression for EU and simplify to get

$$EU = w\overline{T} - \pi(z^* - s^*) + (\pi - q)\theta - s^* + (1-\pi)U(s^*) + \pi H(z^*). \qquad (37)$$

At $\theta = 0$, we have the optimal solution when insurance is being provided solely by the government. To see if having $\theta > 0$ benefits or harms the parents, differentiate (37) with respect to θ. This results in

$$\frac{\partial EU}{\partial \theta} = \pi - q.$$

If the private insurance price is fair, we will have $q = \pi$. The private insurance will then be as good as public insurance. Otherwise, if $q > \pi$, public insurance will dominate. Intuitively, when no child helps his parents, public assistance is provided to all dependent parents. This makes its implicit price, when provided publicly, to be π again.

5.2 Solutions with probabilistic aid from children: $\beta > \widetilde{\beta}$

Consider again the expected utility of the parent, if private insurance markets are operative, and fix the level of z at its optimal level. Denote this by ζ where ζ stands for $\widehat{z}$ or z^* depending on whether we have an equilibrium with less than or full insurance. As previously, the consumption level of parents receiving assistance from their children remains the same at $m(\beta)$. Parents' savings will also remain at s^*. We have

$$EU = w(1-\tau)\overline{T} - q\theta - s^* + (1-\pi)U(s^*) + \pi\left[pH(m(\beta)) + (1-p)H(\zeta)\right],$$

where

$$\tau w\overline{T} = \pi(1-p)(\zeta - s^* - \theta). \tag{38}$$

Substitute for τ from (38) into the expression for EU. We have, upon simplification,

$$\begin{aligned} EU = {} & w\overline{T} - \pi(1-p)(\zeta - s^*) + [\pi(1-p) - q]\theta - s^* \\ & + (1-\pi)U(s^*) + \pi\left[pH(m(\beta)) + (1-p)H(\zeta)\right]. \end{aligned} \tag{39}$$

Again, at $\theta = 0$, we have the optimal solution when insurance is being provided solely by the government. Differentiating (39) with respect to θ yields

$$\frac{\partial EU}{\partial \theta} = \pi(1-p) - q. \tag{40}$$

If the private insurance price is fair, $q = \pi$ so that $\partial EU/\partial\theta = -\pi p < 0$. Interestingly, then, even fair private insurance markets now cannot do as good a job as public insurance. Intuitively, to the extent that private insurance markets may exist, they do for the possibility of dependency and not the default of altruism. Thus the lowest rate they can offer is $q = \pi$. The opting out scheme, on the other hand, allows the government to effectively provide assistance against the default of altruism and that at an implicit price of $\pi(1-p)$.[28] This is because only the parents who do not receive aid from their

[28] Fair insurance markets can duplicate this rate only if they can sell insurance against the default of altruism which is not possible.

children request government's assistance even though all dependent parents can do so. Clearly, if $q > \pi$, providing assistance through private markets will be even worse.

The main results of this section are summarized in the following proposition.

Proposition 3 *Consider the opting-out LTC scheme described in Proposition 2:*

(i) If $\underline{\beta} < \beta < \widetilde{\beta}$ so that the equilibrium is z^* with no aid from children, the LTC insurance can be provided privately or publicly if private insurance is fair and publicly otherwise.

(ii) If $\beta > \widetilde{\beta}$ so that the equilibrium entails probabilistic aid from children, no private insurance is used even if insurance markets are fair. All LTC insurance, whether less-than-full or full, should be provided publicly.

6 Opting out versus topping up

So far we have assumed from the outset that the LTC policy was either of the topping up or the opting out type. To complete the picture we shall now examine the choice between the two types of policies. Can we say that one approach always dominates the other or does the comparison depend on the parameters of the model? It turns out that the answer hinges on the value the degree of altruism, β, takes.

Start with high values and specifically when $\beta > \widehat{\beta}$ when children are sufficiently altruistic so that it an opting out scheme results in an unconstrained solution with probabilistic aid. It turns out that this solution always dominates a feasible topping up scheme. To see this, denote the optimal solutions under opting out and topping up schemes (with assistance from altruistic children), by superscripts OO and TU. From equations (13)–(14), the optimal level of government provision is characterized by

$$H'\left(s\left(p;g^{TU}\right)+g^{TU}\right)=\frac{1}{1-p}>1,$$
$$U'\left(s\left(p;g^{TU}\right)\right)=1.$$

Similarly, from equations (27), (32), and (35),

$$H'\left(s^{OO}+g^{OO}\right)=1,$$
$$U'\left(s^{OO}\right)=\frac{1}{1-\pi}>1,$$

where $s^{OO}=s^{*}$. Comparing the equations for $H'(\cdot)$ under the two schemes, and the equations for $U'(\cdot)$, it follows from the concavity of $H(\cdot)$ and $U(\cdot)$ that

$$s\left(p;g^{TU}\right)+g^{TU}<s^{OO}+g^{OO},$$
$$s\left(p;g^{TU}\right)>s^{OO}.$$

These results are interesting. They tell us that, assuming optimal government's policy, parents who are not helped by their children have a smaller consumption level if dependent and a higher consumption level if independent, under the topping up scheme as compared to the opting out scheme.

In turn, these inequalities imply

$$g^{TU}<g^{OO}.$$

Net assistance to dependent parents who receive no help from their children is always smaller under the topping up scheme as compared to opting out scheme. Finally, this tells us that one can always implement the optimal topping up policy under opting out with some resources left over. This follows because under an opting out scheme, altruistic children will make up for the $g^{OO}-g^{TU}$ that the government is not providing. Consequently,

$$EU^{TU}<EU^{OO}\quad\text{when}\quad\beta>\widehat{\beta}.$$

Intuitively, the dominance of OO when children are sufficiently altruistic, does not come as a surprise. As shown in the previous section, public LTC can be targeted under OO to the parents whose children's turn out not to be altruistic. In other words, under OO, the public system can effectively provide insurance against the failure of altruism, while this is not possible with under a TU scheme. And, when children are sufficiently altruistic, this can be done without deterring the altruistic children form providing care.

The above argument goes through as long as the degree of altruism is sufficiently high as to ensure an unrestricted solution. For an intermediate degree of altruism $\widetilde{\beta} < \beta < \widehat{\beta}$, the more precise targeting comes at a price, namely that public LTC is distorted downward (to ensure continued aid from altruistic children). This makes the comparison ambiguous; the dominance result will by continuity go through for $\beta < \widehat{\beta}$ but only sufficiently close. The exact characterization of the frontier between TU and OO in this is rather tedious and is not very telling.

Interestingly, the dominance goes exactly the other way when the children's degree of altruism is small, namely when $\beta < \widetilde{\beta}$. The important point to remember is that when $\beta < \widetilde{\beta}$, the OO solution implies foregoing all informal care and the "better targeting" becomes irrelevant. One can then show that the OO solution will be dominated by a TU scheme in which LTC insurance is set at $g = z^* - s^*$. The proof follows from the following two lemmas that are proved in the Appendix.

Lemma 1 *Under a TU policy with $g = z^* - s^*$, parents can set their saving s equal to s^*. Under this circumstance, as long as $\beta < \widetilde{\beta} < 1$, altruistic children will not assist their dependent parents and this policy would yield exactly the same outcome as that under OO.*

Lemma 1 shows that it is possible to replicate the OO solution under a TU scheme by setting $g = z^* - s^*$, if parents choose $s = s^*$.

Lemma 2 *Under a TU policy with $g = z^* - s^*$, parents choose a value for s that differs from s^*. Given that s^* is feasible, this choice increases the parents' expected utility.*

Lemma 2 tells us that since the optimal value of s, conditional on $g = z^* - s^*$, is not s^*, as compared to OO, an one can do strictly better with a TU by setting $g = z^* - s^*$.

It follows, a fortiori, that if g^{TU} differs from $z^* - s^*$, the government can do even better. Consequently, we have

$$EU^{TU} > EU^{OO} \quad \text{if} \quad \beta < \widetilde{\beta}.$$

Proposition 4 summarizes the main results of this section.

Proposition 4 *When comparing a TU policy with an OO policy, we have:*

(i) If $\beta > \widehat{\beta}$, so that the OO equilibrium is z^* with probabilistic aid from children, as one can always implement the optimal topping up policy under opting out with some resources left over (as the children will make up for the g^{TU} that will not be provided under an opting out scheme to the parents who are being assisted by their children). Consequently, OO strictly dominates TU and we have,

$$EU^{TU} < EU^{OO} \quad \textit{if} \quad \beta > \widehat{\beta}.$$

(ii) If $\beta < \widetilde{\beta}$, so that the OO equilibrium is z^* without any aid from children, one can replicate the OO solution by providing $g = g^{OO} - s^{OO}$ on a topping up basis. Consequently, TU dominates OO and we have,

$$EU^{TU} > EU^{OO} \quad \textit{if} \quad \beta < \widetilde{\beta}.$$

7 Concluding remarks

The idea of altruism default is not new. Yet there does not exist much work on it in economics and particularly in the area of long-term care where one expects it to have multiple implications. Even though the family continues to remain the prime source of LTC provision in the US and elsewhere, its role has been on the decline. Among the contributing factors in this trend are mobility of the children, increasing frequency of childless families, fading family norms, and higher labor force participation of middle-aged women. This phenomenon in conjunction with rising levels of life expectancy and an aging population pose challenging questions for policy makers concerning LTC provision. The current paper has studied the role of LTC insurance programs in a world in which family assistance is uncertain. And the main lesson that has emerged is that the type of insurance program the government may offer, "topping up" or "opting out," affects not only the desirability of private versus public insurance but also the desirability of insurance in general and its optimal size (whether private or public).

We have shown that under a topping up scheme, if the probability of altruism is high there is no need for insurance; public or private. At lower probabilities, LTC insurance is called for, albeit one that is less than full. The amount of insurance varies negatively with the probability of altruism. If private insurance markets for dependency are fair, private insurance will suffice (although public assistance is just as good). At higher than fair insurance premiums, public assistance dominates private insurance.

Studying an opting out scheme, the paper has shown the possibility of three equilibria depending on the children's degree of altruism towards their parents. With a "small" degree of altruism, the optimal solution calls for full LTC insurance for everyone. Under this circumstance, even altruistic children provide no assistance to their parents. The required insurance may be provided privately if private insurance markets are fair.

Other types of equilibria emerge under "moderate" and "very large" degree of altruism. These equilibria can be supported only through the public opting out scheme. Private insurance markets cannot do the job even if they are fair. With the very large degree of altruism, the optimal solution is again one of full LTC insurance. Nevertheless altruistic children do not consider the public full insurance plan good enough and opt out of it; they provide their own assistance instead. Of course, the parents of non-altruistic children have no other option but public assistance. With the moderate degree of altruism, the optimal solution is one of less than full LTC insurance. This will be just small enough to entice the altruistic children to substitute their own assistance for the government's.

The most interesting feature of the opting out scheme is that it provides, indirectly, insurance against not only dependency but also the default of altruism. It does so by creating incentives for self-targeting; ensuring that only dependent parents who are not helped by their children seek help from the government. This feature cannot be replicated by private insurance markets. A word of caution is called for; however. The system works in part because the government has the ability to tax away the dependent elderly's resources (savings and any private insurance that they may have purchased). This a strong assumption. It is not at all clear that one's private assets are publicly observable to be taxed away at no cost. Furthermore the targeting benefits of an opting out policy are relevant only when children are sufficiently altruistic. For low levels of altruism, when the opting out solution involves no informal care, topping up policies are a dominant option.

In thinking about fruitful extensions of this work, a number of avenues come to mind. One is to allow for altruism default to be endogenously determined. Another is to introduce heterogeneity in the model. Cremer *et al.* (2014) take a first step in that direction. However, they consider only opting out policies and concentrate on the simplest solution regime, namely the unrestricted solution with aid and they focus on wealth heterogeneity. To introduce heterogeneity in a more meaningful way, one would have to adopt a multi dimensional setting where individuals differ both in wealth and in probability of altruism. And while we are not aware of any hard evidence on this issue, one might expect the correlation between these two characteristics to be negative. More educated individuals will tend to have better educated children and this is likely to increase their mobility and thus to result in a lower probability of altruism. Similarly, it is not clear that investing time and/or money in children's education will increase this probability. Quite the opposite; the relationship between investment in children and the probability of receiving informal care is likely to be rather complex and possibly U-shaped. Children who feel neglected may not be very inclined to help their dependent parents, but highly educated children are likely to move away to pursue their career so that they may not be in a position to provide informal care.

Another issue we have neglected is that of myopia which is a serious problem when it comes to LTC; it is well documented that individuals tend to underestimate the probability of dependency or its cost; see Cremer and Roeder (2013). These authors study the design of LTC policy with myopic agents in an optimal tax setting but where informal care is ignored. They show that the expression characterizing the optimal policy then includes "Pigouvian" terms aimed at "correcting" individuals' insurance and savings (self-insurance) decisions which are based on their misperceived loss probabilities. In our paper, this misperception can be expected to be even more significant as individuals may also overestimate the probability of receiving informal care: When private insurance is available, they would then tend to buy insufficient coverage and in the absence of private insurance, they would under-invest in self-insurance (namely savings). This would obviously strengthen the case for public intervention. In particular the result that fair private insurance is equivalent to TU public coverage would no longer be true. One can also expect that overestimation of p results in a higher levels of g under TU than otherwise optimal. However, the impact on the OO is more complicated to assess; it is effectively not clear *a priori* how this might affect the choice between TU and OO. And if the misperception affects both p and π (and/or if parents have a self-control problem and discount the future too heavily) the analysis would be yet more complex, even in our stylized model.

In this paper, we have proceeded as if children live in a different community and that we are not concerned about their welfare. In Appendix C we study how the results would change if children's utility were accounted for. In that case informal care no longer comes as a "free lunch" and its crowding out has some benefits. We show that this increases the level of public LTC in the TU case but leaves the OO policy unaffected. The major lesson that emerges from this exercise is that quite surprisingly, the inclusion of children's utility presents no major challenge to our results.

Appendix

A The degree of altruism and the equilibrium type: proofs and definitions of the benchmarks

Substitute $\widehat{z}(\beta)$ for z in (30), the expression for $EU^a(z,\beta)$, to get

$$EU^a(\widehat{z}(\beta),\beta) = w\overline{T} + \pi(1-p)s^* - s^* + (1-\pi)U(s^*) - \\ \pi(1-p)\widehat{z}(\beta) + \pi[pH(m(\beta)) + (1-p)H(\widehat{z}(\beta))]. \tag{A1}$$

This shows the expected utility of parents at the maximum value of z beyond which their children, with a degree of altruism equal to β, will not provide them any assistance. For a given value of β, $EU^a(\widehat{z}(\beta),\beta)$ is represented in the figures by the point of intersection between the graphs of $EU^a(z,\beta)$ and $\widehat{z}(\beta)$. Clearly, by setting $z = \widehat{z}(\beta)$, the government can ensure the parents an expected utility equal to $EU^a(\widehat{z}(\beta),\beta)$. The other alternative for the government is to set $z = z^*$. If $\widehat{z}(\beta) < z^*$ there will be no aid forthcoming from the children so that the comparison would be between $EU^a(\widehat{z}(\beta),\beta)$ and $EU^n(z^*)$ (which is independent of β). On the other hand, if $\widehat{z}(\beta) > z^*$, there will be probabilistic aid from the children and the comparison would be between $EU^a(\widehat{z}(\beta),\beta)$ and $EU^a(z^*,\beta)$ (which also depends on β).

Recall from the discussion of children's choice in subsection 3.1 that $\beta > 1/H'(s^*)$ to ensure the possibility of assistance from children. Let

$$\underline{\beta} \equiv \frac{1}{H'(s^*)}. \tag{A2}$$

Starting at $\beta = \underline{\beta}$, we have $\widehat{z}(\underline{\beta}) = s^* < z^*$ and $EU^a(\widehat{z}(\underline{\beta}),\underline{\beta}) = EU^n(\widehat{z}(\underline{\beta})) < EU^n(z^*)$. The solution is thus z^* with no assistance from children. Continuity implies that the solution remains the same in the neighborhood of $\beta = \underline{\beta}$.

Next differentiate $EU^a(\widehat{z}(\beta),\beta)$ with respect to β:

$$\frac{d}{d\beta}EU^a(\widehat{z}(\beta),\beta) = \pi\left\{(1-p)\left[H'(\widehat{z}(\beta)) - 1\right]\frac{d\widehat{z}(\beta)}{d\beta} + pH'(m(\beta))\frac{dm(\beta)}{d\beta}\right\}.$$

As long as $\widehat{z}(\beta) < z^*$, $H'(\widehat{z}(\beta)) > H'(z^*) = 1$ and the above expression is positive so that $EU^a(\widehat{z}(\beta), \beta)$ is increasing in β. The solution will thus change only when $EU^a(\widehat{z}(\beta), \beta)$ equals $EU^n(z^*)$ with $\widehat{z}(\beta) < z^*$. This is when $\widehat{z}(\beta) = \widetilde{z}(\beta)$. Denote the solution to this equation by $\beta = \widetilde{\beta}$; it is given by

$$(1-p)\left[H\left(\widehat{z}\left(\widetilde{\beta}\right)\right) - \widehat{z}\left(\widetilde{\beta}\right)\right] + pH\left(m\left(\widetilde{\beta}\right)\right) = H(z^*) - z^* + ps^*. \tag{A3}$$

Hence z^* with no assistance from children is the optimal solution for all $\underline{\beta} < \beta < \widetilde{\beta}$.

Starting at $\beta = \widetilde{\beta}$, the solution changes to $\widehat{z}(\beta)$. It will remain so in the neighborhood of $\beta > \widetilde{\beta}$ because of continuity and the fact that $EU^a(\widehat{z}(\beta), \beta)$ is increasing in β at $\beta = \widetilde{\beta}$. As long as $\widehat{z}(\beta) < z^*$, $EU^a(z^*, \beta)$ is unattainable and the solution remains $EU^a(\widehat{z}(\beta), \beta)$. Now as β increases, both $EU^a(\widehat{z}(\beta), \beta)$ and $EU^a(z^*, \beta)$ increase. Nevertheless since $\widehat{z}(\beta)$ is increasing in β, at some point $\widehat{z}(\beta) = z^*$ so that $EU^a(\widehat{z}(\beta), \beta) = EU^a(z^*, \beta)$. Let $\widehat{\beta}$ denote the solution to $\widehat{z}(\beta) = z^*$. The question is what happens if $\beta > \widehat{\beta}$.

We now prove that $EU^a(z^*, \beta) - EU^a(\widehat{z}(\beta), \beta) > 0$ whenever $\beta > \widehat{\beta}$ so that for these values of β we will have the z^* solution which entails probabilistic assistance from children. We have

$$EU^a(z^*, \beta) - EU^a(\widehat{z}(\beta), \beta) = \pi(1-p)\left\{\widehat{z}(\beta) - H(\widehat{z}(\beta)) - [z^* - H(z^*)]\right\}.$$

Thus β affects $EU^a(z^*, \beta) - EU^a(\widehat{z}(\beta), \beta)$ only through z and not directly. But this function has a minimum value of zero at $\beta = \widehat{\beta}$ and will be positive otherwise.[29]

Finally, we show that $\widehat{\beta} > 1$. Since in this unrestricted solution with (probabilistic) aid altruistic children consider government provision insufficient and substitute their

[29] First- and second-order derivatives of $EU^a(z^*, \beta) - EU^a(\widehat{z}(\beta), \beta)$ with respect to β are

$$\frac{\partial}{\partial\beta}\left[EU^a(z^*, \beta) - EU^a(\widehat{z}(\beta), \beta)\right] = \pi(1-p)\left[1 - H'(\widehat{z}(\beta))\right]\frac{\partial\widehat{z}(\beta)}{\partial\beta},$$

$$\frac{\partial^2}{\partial\beta^2}\left[EU^a(z^*, \beta) - EU^a(\widehat{z}(\beta), \beta)\right] = \pi(1-p)\left\{-H''(\widehat{z}(\beta))\left[\frac{\partial\widehat{z}(\beta)}{\partial\beta}\right]^2 + \frac{\partial^2\widehat{z}(\beta)}{\partial\beta^2}\left[1 - H'(\widehat{z}(\beta))\right]\right\}.$$

At $\widehat{z}(\beta) = z^*$, the first-order derivative is equal to zero and the second-order derivative is positive.

own help, it must be the case that $a^* + s^* > z^*$. The concavity of $H(\cdot)$ then implies $H'(a^* + s^*) < H'(z^*)$. Now we have, from the first-order condition (22) for altruistic children providing help, $H'(a^* + s^*) = 1/\beta$. Additionally, we know that $H'(z^*) = 1$. Consequently, $1/\beta < 1$ so that $\beta > 1$ must apply which in turn implies $\widehat{\beta} > 1$.

B Proofs of Lemma 1 and Lemma 2

Lemma 1: Observe first that because g is provided on a topping up basis, parents do not have to forego their saving to qualify for public care. Now, with $g = z^* - s^*$ and $s = s^*$, children will not help their parents for $\beta < \widetilde{\beta} < 1$. This follows because $g + s = z^* - s^* + s^* = z^*$. Hence $H'(s + g) = H'(z^*) = 1 < 1/\beta$ which is the condition for altruistic children not helping; see equation (3) and footnote 7. Then, with children not helping, the parents' consumption level under TU when dependent is $g + s = z^*$ (with z^* being their consumption level under OO). Furthermore, with $s = s^*$, their first-period consumption would remain the same as it was under opting out. To see this, observe that the parents' first-period consumption levels under OO and TU are respectively equal to $w(1-\tau)\overline{T} - s^* = w\overline{T} - \pi(z^* - s^*) - s^*$ and $w(1-\tau)\overline{T} - s = w\overline{T} - \pi g - s = w\overline{T} - \pi(z^* - s^*) - s^*$.

Lemma 2 Consider a TU policy with $g = z^* - s^*$. As in Section 2.2 in the paper, assuming an interior solution for children's aid, the optimal value of s, which we denote by s^A, is found as the solution to equation (5). It immediately follows from this equation that

$$U'\left(s^A\right) < \frac{1}{1-\pi} = U'(s^*),$$

so that $s^A > s^*$.

The second possibility arises if the altruistic children do not help. Under this circumstance, at $g = z^* - s^*$, we have

$$EU = w\overline{T} - \pi(z^* - s^*) - s + (1-\pi)U(s) + \pi H(s + z^* - s^*).$$

Maximizing EU in above with respect to s, results in the first-order condition

$$(1-\pi)\, U'\left(s^{NA}\right) + \pi H'\left(s^{NA} + z^* - s^*\right) = 1.$$

It follows from this equation that,

$$U'\left(s^{NA}\right) < \frac{1}{1-\pi} = U'\left(s^*\right),$$

so that $s^{NA} > s^*$.

C Including children's utility in social welfare

C.1 Topping up

When children's utility is accounted for in welfare, the Lagrangian associated with the government's topping up problem can be written as[30]

$$\begin{aligned} \pounds = {} & wT - s - \pi g + (1-\pi)U(s) + \pi\left[pH(m(\beta) + (1-p)H(s+g)\right] \\ & + \pi p\left[y - m(\beta) + g + s + \theta\beta\pi p H(m(\beta)\right] + (1-\pi p)\, y. \end{aligned}$$

In this expression the second line represents utility of the children. Observe that the altruistic term is premultiplied by θ measuring the extent of "laundering out". When $\theta = 1$, we have a pure utilitarian approach advocated by some authors but this amounts to double counting this term. To avoid this problem other authors have suggested setting θ at zero, which is referred to as "laundering out" the altruistic terms; see Hammond (1987). This has by now become the standard approach in the literature on altruism and intergenerational transfers. We shall follow this tradition but we have not simply deleted the term altogether because it turns out that under the level of θ is of no relevance, which in itself is interesting.

With the available instruments, there is no way to affect the consumption of children when parents are healthy or do not benefit from their children assistance. Consumption

[30] The rest of the game remains the same as before.

of children in the case of assistance, which is equal to $y - m(\beta) + g + s(g)$ can be modified directly through g and indirectly though s which is set by parents but depends on g as explained in Subsection 2.2. The FOC is given by

$$\frac{\partial \pounds}{\partial g} = \pi \left[\pi(1-p)H'(s+g) - 1 \right] + \pi p \left[1 + \frac{\partial s}{\partial g} \right], \tag{A4}$$

where from equation (7) $-1 < \partial s/\partial g < 0$ so that the last term on the RHS of (A4) is positive. This in turn implies that the range of p's for which $g > 0$ is larger when children's utility is accounted for. To see this observe that when evaluated at $\widehat{p}$ defined by (16), equation (A4) is positive since the first term on the RHS is by definition equal to zero. Similarly, at g^{TU}, (A4) is positive which means that including children in welfare leads to a larger level of g under TU. This property is also shown by the following expression obtained by solving the FOC to equation (13):

$$H'(s+g) = \frac{1}{1-p} - \frac{p}{(1-p)} \left[1 + \frac{\partial s}{\partial g} \right].$$

It is not surprising because when children's utility is accounted for crowding out of informal care no longer necessarily has a negative impact on welfare. Observe that laundering (the level of θ) does not play any role in these results.

C.2 Opting out

Her we restrict our attention to the case where $\theta = 0$, to avoid double-counting the altruistic term. Under OO the consumption of children helping their parents cannot be modified and thus the problem is the same as that without taking into account their welfare. Their consumption is equal to $y - a$ when there is aid and y when $a = 0$. Consequently, for a given regime (type of equilibrium), the opting out policy is not affected.[31] This also implies that the critical level $\widehat{\beta}$ separating the unconstrained

[31] When $\beta > \widehat{\beta}$, this result does not depend on θ. In that case care provided by altruistic children is not affected by z. However, for lower levels of β the double-counting will be relevant so that z increases with θ.

solution with aid from the constrained solution with aid is not affected either. However, the level of $\widetilde{\beta}$ defining the frontier between solution with and without care will increase because the no aid solution becomes more attractive.

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