Environmental taxation, tax competition and harmonization

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Abstract

This paper studies the tax competition problem in the presence of transboundary pollution. It shows that economic integration causes the firms to adopt the same or less polluting technologies, but it nevertheless increases aggregate emissions and lowers welfare. Second, the paper examines the ramifications of partial tax harmonization policies. It shows that harmonizing commodity taxes above their unrestricted Nash equilibrium value may either increase or lower the equilibrium emission tax. Under the former, firms opt for less polluting technologies, aggregate emissions decrease and welfare improves. On the other hand, if emission tax goes down, firms will choose more polluting technologies, aggregate emissions will increase and welfare deteriorates. Finally, harmonizing the emission tax above its unrestricted Nash equilibrium value, which leads the firms to adopt a less polluting technology, also causes aggregate emissions to decline and overall welfare to increase.

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1 Introduction

A major theme of the tax competition literature, and the earlier fiscal federalism literature on which it is based, has been the potential loss in tax revenues as a result of tax competition. It is generally believed that the integration process will exert a negative influence on the ability of the member states to generate an "adequate" level of tax revenues to finance their social policies.[1] This paper takes a fresh look at the tax competition issue, and the effectiveness of partial fiscal coordination policies, in the presence of another source of economic inefficiency in the economy. In particular, we have in mind cases where, because of transboundary pollution, resource allocation is inefficient even when the economies are closed.

The underlying reasons for the "race to the bottom" concern are simple to grasp. Economic integration entails the dismantling of barriers to free movements of people, capital and goods among states (or countries as the case may be). From the perspective of state governments, this increased mobility may be viewed as an opportunity to move other states' tax bases into one's own. Each state will then try to compete with the others in order to attract the tax bases that are being made mobile. A simple and effective way to achieve this is by lowering one's tax rates. As states try to undercut one another's tax rates, it is not difficult to envisage an end result in which the tax rates, and the corresponding levels of local government services, will be less than optimal.[2]

Applying the logic of tax competition to polluting activities, it is natural to expect some degree of strategic interaction between the states with regards to environmental

[1] The literature also identifies a counter force in terms of a "tax exporting" effect. However, the more serious concerns have been raised about the prospects of less than optimal expenditures on public goods and redistributive policies in Europe. This is reflected in Sinn's [10] warning that "In the end, all countries will settle at an equilibrium where only benefit taxes are charged, and no redistribution policies are carried out" (p. 100).

[2] There is also the possibility of "excessive" tax rates due to tax exporting effect. However, it is the less-than-optimal-tax-rates result which has received the greatest share of attention. See, e.g., Sinn [10], and Edwards and Keen [5]. For recent surveys of the tax competition literature, see Cremer et al. [4], Wilson [12], Wellisch [11] and Haufler [7].

policies. Fredriksson and Millimet [6] have recently tested this proposition empirically for the neighboring states in the U.S. Their results suggest that there is indeed a positive relationship between states' environmental policies. The interesting question, from our perspective, is the form that this interaction takes and its impact on aggregate emissions and welfare. In particular, if the state governments are armed with both emission and output taxes, what role they assign to each instrument. Will it be the case that they will use output taxes for tax competition, keeping emission taxes for the control of emissions?[3] Under this circumstance, there is no *a priori* reason to suspect that tax competition per se should necessarily lead to more pollution. It is true that cutting output taxes (to foster tax competition) tends to lower the consumer price of polluting goods, leading to an increase in their consumption and with it the aggregate emission levels as well. It is equally true, however, that increasing emission taxes to combat pollution would push the consumer prices up, curtail consumption and lower aggregate emissions. This ameliorating effect on aggregate emissions is further enhanced if an increase in emission taxes induces the firms to switch to less polluting techniques of production. The final outcome will ultimately depend on the balance of these conflicting forces.

We consider a simple setting with two identical regions whose inhabitants consume two goods: a non-polluting numeraire good and a polluting consumption good. Every consumer has an endowment of the numeraire good, some of which he consumes, spending the rest to purchase the polluting good and to pay taxes. Production technologies are identical in both states. Pollution (CO_2, SO_2, etc.) is global and a by-product of production. The polluting good may be produced in different ways. Each procedure entails a different resource cost and a different emission level.[4] Emissions are beneficial

[3] In a companion paper, Cremer and Gahvari [3], we show that such a targeting property depends on whether border taxes are origin- or destination-based. For an analysis of this question within the framework of a closed economy, See Cremer and Gahvari [2].

[4] This models situations where a polluting good may be produced through different production techniques, or using different polluting inputs where each particular input entails a different emission level.

in that a higher level of emission reduces the private (per unit) production costs of polluting goods. That is, the production costs of polluting goods are negatively correlated to their emissions. This is to capture the fact that technologies which cut emissions are more expensive to employ. Firms producing the polluting good operate in a competitive environment. The good is produced by an industry that is comprised of a fixed but sufficiently large number of identical firms. It is produced, for a given unit cost of production, by a linear technology subject to constant returns to scale.

The polluting good is produced and consumed in both regions (states). Prior to economic integration, there is no trade between the two states. Upon integration, residents of each state will be able to purchase the polluting good from the foreign as well as the home state. While the physical characteristics of the home- and foreign-produced goods are identical, consumers have a preference for purchasing the home-produced goods. We model this by assuming that consumers experience a certain disutility when they consume one unit of the foreign-produced good. The extent of the disutility differs across consumers. Individuals have otherwise identical quasi-linear preferences.

There are two (distortionary) tax instruments: commodity *and* emission taxes. These are "origin-based". Thus, each state levies a certain tax on each unit of the (polluting) consumption good that its firms produce and sell (regardless of where the purchasers come from). Second, to combat pollution, the state imposes another tax per unit of emissions on (home) firms. Each state rebates its tax revenues to its residents in a lump-sum fashion.

Within this framework, we characterize the emission tax rates in second-best and closed economies and show that no commodity taxes are chosen in either setup. Further, we show that the firms choose an emission technology which is more polluting than the second best. We also show that aggregate emissions exceed their second-best levels as well. Next, we characterize the Nash equilibrium values of commodity and emission

Different abatement techniques also imply that a unit of polluting good is associated with different emission levels.

taxes as the economy opens up. We show that the formula for emission tax remains the same in closed and open economies. On the other hand, there will be a subsidy on the commodity tax.

Our other results include the finding that economic integration causes the firms to adopt the same or less polluting technologies, but it nevertheless increases aggregate emissions and lowers welfare. We then show that the Nash equilibrium value of the emission tax may either increase or decrease as a result of harmonizing the commodity tax above its unrestricted Nash equilibrium value. If the emission tax increases, firms will adopt less polluting technologies. This in turn leads to a lowering of aggregate emissions and improvement in overall welfare. On the other hand, if emission tax goes down, firms will choose more polluting technologies. Under this circumstance, aggregate emissions will increase and welfare deteriorates. Finally, we consider the harmonization of emission taxes above their unrestricted Nash equilibrium value. This would not only lead the adoption of less polluting technologies, but also to a reduction in aggregate emissions and a rise in overall welfare.

2 The model

The residents of two identical states, A and B consume two goods: a polluting good, x, and a nonpolluting numeraire. Every consumer has an endowment of m units of the numeraire good and receives T_j in lump-sum rebate from the government of j in which he resides.

The level of pollution is determined by total (across both states) emissions which are created by the production process. The polluting good is produced from the numeraire according to a linear technology with unit cost $C(e_i)$, where e_i $(i = A, B)$ denotes emission per unit of output in state i. The more polluting a technology, the less costly it is to employ. Thus unit cost is a decreasing function of emission e_i.[5] We assume also

[5]More precisely the assumptions are that $C'(.) < 0$ for all e_i up to some limit $\bar{e}$, and that $C'(\bar{e}) = 0$,

that $C(e_i)$ is continuously differentiable and convex. All firms operate in a perfectly competitive environment.

In a closed economy, consumers can buy the polluting good only from domestic producers. Upon integration, the polluting good may be traded. While the domestic and foreign goods are identical in their physical characteristics and in quality, consumers have a preference for the domestic product. This means that if prices are equal, *all* consumers in a given state buy the domestic product. Individuals, however, differ in their attachment to the home product. Put differently, the price differential required to switch to the foreign product differs across consumers. Individuals are identified by $\theta \in [-1, 1]$, with $|\theta|$ determining a consumer's preference for domestic goods. A resident of B is identified by a negative θ, while a positive θ corresponds to a resident of A. Population size in each state is normalized to one and θ is uniformly distributed.

The utility of a person in $j = A, B$ who purchases the polluting good produced in $i = A, B$ is given by[6]

$$\begin{cases} u_j^j = & m - p_j x_j^j + h(x_j^j) + T_j - \varphi(E), \\ u_i^j = & m - p_i x_i^j + h(x_i^j) - \delta|\theta| x_i^j + T_j - \varphi(E), \quad \text{with } j \neq i, \end{cases} \tag{1}$$

where x_i^j is consumption of the polluting good at the consumer price of p_i, E is the global emission level, and $\delta > 0$ is a "dislike index".[7] Standard assumptions apply to the (continuously differentiable) elements in the utility function: $h'(.) > 0, h''(.) < 0$ and $\varphi'(.) > 0, \varphi''(.) \geq 0$.

Utility maximization yields

$$\begin{cases} h'(x_j^j) = & p_j, \\ h'(x_i^j) = & p_i + \delta|\theta|, \quad \text{with } j \neq i. \end{cases} \tag{2}$$

$i = A, B$. This models situations where a polluting good may be produced through different production techniques, or using different polluting inputs where each particular input entails a different emission level. Different abatement techniques also imply that a unit of polluting good is associated with different emission levels.

[6]All taxes are origin-based. Consequently the price does not depend on the location of the buyer.

[7]If $\delta = 0$, individuals become identical everywhere. Under this circumstance, any deviation in price implies that *no one* will purchase from the state which has a higher price.

The right-hand side of these expression reflect the *net* cost of purchasing one unit of the good. For a resident of j who buys the domestic product, this is simply the consumer price, p_j. On the other hand, the person who buys the foreign product incurs a net (utility) cost of $p_i + \delta|\theta|$. From (2) we obtain the demand for the polluting good as: $x_j^j = x(p_j)$ and $x_i^j = x(p_i + \delta|\theta|)$. Substituting in (1) then yields $u_j^j = u(p_j, T_j, E)$ and $u_i^j = u(p_i + \delta|\theta|, T_j, E)$.

Let $\tilde{\theta}$ denote the "marginal" consumer who is indifferent between buying home- or foreign-produced goods. If the marginal consumer is a resident of A ($\tilde{\theta} > 0$), we have $p_A = p_B + \delta\tilde{\theta}$. Similarly, if $\tilde{\theta}$ is a resident of B ($\tilde{\theta} < 0$), $p_B = p_A - \delta\tilde{\theta}$. In either case

$$\tilde{\theta} = \frac{p_A - p_B}{\delta}. \tag{3}$$

Observe that individuals to the left of $\tilde{\theta}$ buy the from state B while individuals to the right of $\tilde{\theta}$ buy from state A.

Production and sale of one unit of the polluting good generate $p_i - C(e_i)$ in revenues for the government of i. Assuming that governments refund all tax proceedings in a lump-sum way to their residents, we have:

$$T_A(p_A, e_A; p_B) = \begin{cases} [p_A - C(e_A)]\,(1 - \tilde{\theta})x(p_A) & \text{if } p_A \geq p_B, \\ [p_A - C(e_A)]\left[x(p_A) + \int_{\tilde{\theta}}^{0} x(p_A - \delta\theta)d\theta\right] & \text{if } p_A < p_B. \end{cases} \tag{4}$$

$$T_B(p_B, e_B; p_A) = \begin{cases} [p_B - C(e_B)]\left[x(p_B) + \int_{0}^{\tilde{\theta}} x(p_B + \delta\theta)d\theta\right] & \text{if } p_A \geq p_B, \\ [p_B - C(e_B)]\,(1 + \tilde{\theta})x(p_B) & \text{if } p_A < p_B. \end{cases} \tag{5}$$

Similarly, total pollution, E, is given by:

$$E(p_A, p_B, e_A, e_B) = \begin{cases} e_B\left[x(p_B) + \int_{0}^{\tilde{\theta}} x(p_B + \delta\theta)d\theta\right] + e_A(1 - \tilde{\theta})x(p_A) & \text{if } p_A \geq p_B, \\ e_B(1 + \tilde{\theta})x(p_B) + e_A\left[\int_{\tilde{\theta}}^{0} x(p_A - \delta\theta)d\theta + x(p_A)\right] & \text{if } p_A < p_B. \end{cases} \tag{6}$$

In what follows we are mainly interested in symmetric equilibria with $p_A = p_B$ and $\tilde{\theta} = 0$. From that perspective, it is important to note that T_A, T_B and E are continuous at this point. Furthermore, one can easily show that these functions are in fact differentiable at $p_A = p_B$. (See the Appendix).

2.1 Taxes, and the price of the polluting good

Each state has two tax instruments to combat pollution. First, a tax of τ_i on each unit of the polluting good that its firms produce (regardless of the location of the buyer).[8] Second, a tax of t_i per unit of emissions on (home) firms. Given the constant returns to scale assumption, profit maximization is equivalent to maximizing profits *per unit* of output. Consequently a firm in state i chooses e_i to maximize

$$p_i - C(e_i) - t_i e_i - \tau_i,$$

which yields, for $i = A, B$,

$$-C'(e_i) = t_i. \tag{7}$$

Equation (7) determines emissions per unit as a function of the emissions tax rate.[9] Finally, competitive equilibrium requires that the price equals marginal (and average) cost so that

$$p_i = C(e_i) - C'(e_i)e_i + \tau_i. \tag{8}$$

2.2 Welfare

Denote utilitarian welfare for state i by W_i, $(i = A, B)$. The use of a utilitarian measure appears natural in our setting. Given quasi-linear preferences this simply corresponds to total surplus. Define $U_i(.)$ and $F(.)$ by

$$\begin{aligned} U_i(p_i, T_i, E) &\equiv \int_0^1 [m + h(x(p_i)) - p_i x(p_i) + T_i - \varphi(E)]\, d\theta, \\ &= m + h(x(p_i)) - p_i x(p_i) + T_i - \varphi(E), \qquad i = A, B. \qquad (9) \\ \frac{1}{\delta} F(p + \delta\theta) &\equiv \int [h(x(p + \delta\theta)) - (p + \delta\theta)x(p + \delta\theta)]\, d\theta. \qquad (10) \end{aligned}$$

[8] Recall that the tax is origin based.

[9] The second-order condition $C''(e_i) > 0$ is satisfied from the convexity of $C(.)$.

We prove in the Appendix that W_A and W_B have the following characterizations and that they are continuously differentiable everywhere.

$$W_A = \begin{cases} U_A - \tilde{\theta}\left[h(x(p_A)) - p_A x(p_A)\right] + \frac{1}{\delta}\left[F(p_A) - F(p_B)\right] & \text{if } p_A \geq p_B, \\ U_A & \text{if } p_A < p_B. \end{cases} \tag{11}$$

$$W_B = \begin{cases} U_B & \text{if } p_A \geq p_B, \\ U_B + \tilde{\theta}\left[h(x(p_B)) - p_B x(p_B)\right] - \frac{1}{\delta}\left[F(p_A) - F(p_B)\right] & \text{if } p_A < p_B. \end{cases} \tag{12}$$

where T_i and E are given by equations (4)–(5) and (6).

Note that the middle expression in the right-hand side of (11) [when $p_A \geq p_B$,] measures the consumer surplus that state A does not get because some of its residents do not buy the home-produced good. These are the people with a $\theta \in [0, \tilde{\theta})$. Instead, by buying from B, these individuals attain a surplus given by the last expression in the right-hand side. The same interpretation applies to (12) and residents of B.

2.3 First-best benchmark

The first-best (utilitarian) solution is obtained by maximizing worldwide welfare, i.e., the sum of utilities of both states' residents. Given symmetry, one can determine the first-best benchmark by assuming that the two states cooperate fully in their fiscal policies. That is, they set τ_i and t_i taking the welfare of the citizen of *both* states into account. Denote the welfare of state i ($i = A, B$) at *a symmetric allocation* by W_i^S. From (11) or (12), this is given by

$$W_i^S(p_i, T_i, E) = m - p_i x(p_i) + h(x(p_i)) + T_i - \varphi(E). \tag{13}$$

Similarly, state i's budget constraint at a symmetric allocation is obtained from (4) or (5) and requires

$$T_i = \left[p_i - C(e_i)\right] x(p_i). \tag{14}$$

The first-best solution is obtained by maximizing W_i^S subject to (14) and

$$E = 2 e_i x(p_i). \tag{15}$$

We prove in the Appendix that

Proposition 1 *The first-best (utilitarian) allocation, and the supporting prices and tax instruments, are characterized by equations (2), (7), (8), (14), (15), and*

$$\tau_i = 0, \tag{16}$$

$$t_i = -C'(e_i) = 2\varphi'(E). \tag{17}$$

3 Closed borders

We now study the Nash equilibrium when the borders are closed. Governments choose the values of their fiscal instruments (which determine their emission levels) simultaneously and non-cooperatively. A government's objective function is the welfare of its own residents. Consequently, it does not account for the impact of domestic emission on residents of foreign states. This indicates that the problem facing each state is not the same as the "first-best benchmark" problem studied earlier.

Without trade, everyone buys the domestic good and we have $\tilde{\theta} = 0$. Consequently, the government budget constraint continues to be given by equation (14). Turning to the expression for aggregate emissions, it is given by

$$E = e_A x(p_A) + e_B x(p_B). \tag{18}$$

This is obtained from (6) by setting $\tilde{\theta}$ equal to zero.

The best reply of government i is determined by maximizing U_i, as specified in equation (9), with respect to τ_i and t_i, subject to (14) and (18). Solving the problem, setting $\tau_A = \tau_B$ and $t_A = t_B$ yields the symmetric Nash equilibrium. We have

Proposition 2 *(i) The symmetric Nash equilibrium in a closed economy is characterized (for $i = A, B$) by equations (2), (7), (8), (14), (18), and*

$$\tau_i = 0, \tag{19}$$

$$t_i = -C'(e_i) = \varphi'(E). \tag{20}$$

(ii) Per-unit emissions are higher than in the first best so that more polluting technologies are adopted.

(iii) Worldwide emissions exceed their first-best levels.

Condition (19) shows that the equilibrium solution for τ_i is equal to its second-best value of zero. This should not be surprising. With closed borders, there is no tax competition between the states. Hence the optimal tax rule for setting τ_i remains unaffected. On the other hand, the rule for setting emission taxes now differs from the first-best benchmark case. Condition $-C'(e_i) = \varphi'(E)$ in (20) replaces condition $-C'(e_i) = 2\varphi'(E)$ of the first-best. Thus, the environmental tax is set at one half the full marginal social damage of emissions. This reflects our earlier observation that each state, when determining its emissions policy, considers the damage to its own citizens only. Part (ii) follows intuitively from this last observation and the fact that $C(.)$ and $\varphi(.)$ are convex. Part (iii) is ensured by the fact that the price of the polluting good increases and its consumption declines; see the Appendix.

4 Open borders

We now turn to the general specification of the model with the possibility of buying foreign-made products. Trade has important implications for a state's potential tax revenues. With open borders, some residents of one state may find it advantageous to buy from the other state. Consequently, the tax bases of the different states becomes "more elastic". With closed borders, a change in the tax policy (and the induced price variation) affects the demand (taxable transaction) of a given set of taxpayers. With open borders, this may also affect the number of effective taxpayers in the state. Put differently, tax revenues are affected at the extensive as well as at the intensive margin. Formally, the extensive margin is reflected by variations in $\tilde{\theta}$, the index of the marginal consumer; see Section 2. This changes the nature of the strategic interaction between the states. In addition to the global pollution aspect, there is now a potential for tax competition as well.

We determine the symmetric Nash equilibrium of this redefined game. The strategic variables continue to be the tax rates t_i, τ_i which are set simultaneously with each government maximizing welfare of its residents. ¿From the problem of state i, we determine its best-reply functions. Setting $\tau_A = \tau_B$ and $t_A = t_B$ then yields the symmetric Nash equilibrium. We have[10]

Proposition 3 *Denote the absolute value of the elasticity of demand for the polluting good in state* i*,* $i = A, B$*, by* $\varepsilon_i \equiv -x'(p_i)p_i/x(p_i)$*.*[11] *The symmetric Nash equilibrium in an open economy is characterized (for* $i = A, B$*) by equations (2), (7), (8), (14), (18), and*

$$\frac{\tau_i}{p_i} = -\frac{e_i\varphi'(E)}{p_i + \delta\varepsilon_i}, \tag{21}$$

$$t_i = -C'(e_i) = \varphi'(E). \tag{22}$$

Comparing these expression with their closed economy counterparts, (19) and (20), shows the impact of tax competition. While x_i goes untaxed in a closed economy, this is no longer the case when the economy opens up. Indeed, the good is now *subsidized*. The intuition comes from the phenomenon of tax competition. In an attempt to increase its "tax base", state i lowers τ_i to such an extent that it takes a negative value (i.e. assumes a smaller value than its optimal value of zero).[12] Observe also that according to

[10]If individuals have no preference for home-produced goods, i.e. if $\delta = 0$, each state will be able to take over the production of x in its entirety by selling at a price just below the other state's price. Under this circumstance, the price is pushed all the way down to the marginal cost so that $p_i - C(e_i) = \tau_i - C'(e_i)e_i = 0$. Consequently, $\tau_i = C'(e_i)e_i$. Observe also that letting $\delta \to 0$ in (21), we have $\tau_i = -e_i\varphi'(E) = C'(e_i)e_i$.

[11]To be precise, this is the elasticity of demand at a symmetric equilibrium.

[12]Specifically, we have two fiscal and one non-fiscal sources of externality here. As Mintz and Tulkens [9] have pointed out, an increase in the output tax of the home state affects the welfare of the foreign state's residents through a tax-base effect (positive), and a private consumption externality (a negative externality for those residents of the foreign state who buy from the home state). The non-fiscal externality arises because of the global nature of emissions. The increase in the home-state's

equation (21), everything else equal, the higher is ε, the smaller will be the required tax. This relates to, and has the same underpinning, as the well-known "inverse elasticity rule" of optimal commodity taxes in closed economies.

Turning to optimal emission *rule*, it remains the same as under the closed-economy solution. This is due to the availability of τ_i for tax competition. With $p_i = C(e_i) + t_i e_i + \tau_i$, whatever the government of i wants to do via t_i, it can also do through τ_i. Now, whereas changing t_i has a distortionary effect on production decisions, changing τ_i is neutral in this respect. Consequently, there will be no reason for i to want to distort its production decisions for the purpose of tax competition. The marginal social damage of emissions to a citizen of i is thus evaluated the same way by its government whether the economy is closed or open. This result, however, in itself is not sufficient to compare the equilibrium *levels* of emissions (e_i and E) with their values in the absence of trade. This is an even more interesting question to which we now turn.

4.1 Pollution technologies, emissions, and welfare

To address this question, we shall use the property that the closed border solution is effectively a special case (or more precisely the limit) of the open border equilibrium as δ goes to infinity. Intuitively, this property is not surprising: as the dislike for the foreign good intensifies, it effectively ceases to be a viable alternative to the home-produced good. Consequently, all individuals buy from their state of residence and one is back

tax, increases the price of the polluting good and reduces its consumption; leading to a fall in aggregate emissions that benefits the foreign state residents as well. This is also present when the economies are closed.

Observe also that the same fiscal externalities are at work when there are no emissions. Nevertheless, without emissions, $\tau_i = 0$ remains optimal even in an open economy. The reason is that in that setting, the state will not gain by subsidizing x_i which would result in the consumer price being set below marginal cost. With emissions, $\tau_i < 0$ is beneficial (because of tax competition) and does not imply that the consumer price is below marginal cost. To see this, observe from (21)–(22) that we have

$$p_i - C(e_i) = \tau_i + t_i e_i = \frac{\varphi'(E) e_i \delta \epsilon_i}{p_i + \delta \epsilon_i} > 0.$$

in an autarchic world. Formally, this convergence property follows directly from the expressions that characterize the closed- and open-economy equilibria. To see this, first note that except for τ_i, the two sets of equilibria are defined by identical equations, namely, (14), (15) and (22). The τ_i itself is equal to zero in a closed economy and given by (21) in an open economy. The convergence then follows directly from the property that the right-hand side of equation (21) tends to zero as δ tends to infinity.

This property enables us to compare the values of the relevant variables under the two equilibria. We do this by studying their comparative statics properties with respect to δ. Specifically, assuming a constant elasticity of demand for x, we show in the Appendix, that as δ increases, the open-economy equilibrium values of t and E decrease while those of e and p increase. This tells us that the open economy values of t and E exceed, and those of e and p fall short of, their closed-equilibrium values. The interesting aspect of this is that it suggests that *trade makes the states to switch to less polluting technologies (less e) but causes total levels of emissions to increase.* In the special case of a constant marginal social damage of emissions, e remains unchanged but p continues to decrease resulting in an increase in consumption of polluting goods and consequently in aggregate emissions; see the appendix. The appendix also shows that a state's level of welfare will unambiguously decline as the economy opens up.

We summarize the results of this subsection as

Proposition 4 *Assume the elasticity of demand for the polluting good is constant. Economic integration (i) causes the firms to adopt the same or less polluting technologies; (ii) results in price subsidies to polluting goods, (iii) lowers the price of polluting goods; (iv) increases aggregate emissions and (v) lowers welfare.*

5 Harmonization of output taxes

We have seen in the previous sections that the closed and open border solutions are both inefficient and that this is due to a failure of coordination between the states. It is plain that in the context of our model, the first-best can be restored if the states set both instruments, output *and* emission taxes, in a coordinated way. Given symmetry, this can be achieved through a "harmonization" of *both* tax instruments, i.e., by setting their respective values at a common specified level. In practice, however, such a full coordination may be difficult to achieve. Instead, states may have to resort to "partial" harmonization of policies i.e., a harmonization which pertains only to a subset of the policy instruments.[13] In our setting this may occur in two ways. On the one hand, one can harmonize output taxes while emission taxes continue to be set independently. On the other hand, it is possible to think of a harmonization of emissions taxes only. In either case, one may wonder whether such partial harmonization is (necessarily) desirable. In particular, one would like to know if it would have the intended impact on the environment and on welfare. Intuitively, the possible concern is that the neutralization of one variable of fiscal competition, may make competition in the other variable even fiercer and that this may give rise to perverse results. For instance, a harmonization of output taxes could result in a lowering of emission taxes and thus in a switch to more polluting technologies. This section discusses the harmonization of output taxes; the harmonization of emission taxes is considered in the next section.

Assume that output taxes are set at $\tau = \hat{\tau}$ while emission taxes continue to be set non-cooperatively, exactly as in Section 4. The Nash equilibrium is then contingent on $\hat{\tau}$ and we write the solutions as $t^N(\hat{\tau})$, $e^N(\hat{\tau})$, $E^N(\hat{\tau}), p^N(\hat{\tau})$ and $T^N(\hat{\tau})$. Further, denote all unrestricted Nash equilibrium values by the superscript N ($\tau^N, t^N, e^N, E^N, p^N$ and T^N). It is clear that if one were to harmonize τ at its unrestricted Nash equilibrium value, τ^N, t and all the other variables will also take their (unrestricted) Nash equilibrium values.

It is easy to show that the Nash equilibrium value of t_i, conditional on $\tau = \hat{\tau}$, is the

[13]Keen [8] discusses indirect tax harmonization. Cremer and Gahvari [1] discuss partial tax harmonization in the context of tax evasion.

solution to[14]

$$-\left[\frac{p_i - C(e_i)}{\delta\,\varepsilon_i} + \frac{\tau_i}{p_i}\right] + \left[\frac{e_i}{p_i} + \frac{1}{C''(e_i)e_i\varepsilon_i}\right]\left[C'(e_i) + \varphi'(E)\right] = 0. \qquad (23)$$

Observe that when $\hat{\tau} = \tau^N$, the first bracketed expression in the left-hand side of (23) will be zero so that $e^N(\hat{\tau}) = e^N$.

5.1 Harmonization and the environment

To study the effects of harmonization on the environment, we have to first determine how harmonization impacts $t^N(\hat{\tau})$. To this end, differentiate (23) with respect to $\hat{\tau}$ and evaluate the resulting expression at (t^N, τ^N). To simplify the derivations, we will assume that the marginal social damage of emissions is constant. We have (see the Appendix)

$$\frac{dt^N(\hat{\tau})}{d\hat{\tau}} = \frac{-1}{e + \frac{C'(e)e+\delta}{C''(e)e\left[1+(1-\tau/p)\delta\varepsilon/p\right]}}. \qquad (24)$$

It is clear from (24) that $dt^N(\hat{\tau})/d\hat{\tau}$ can take both positive as well as negative values. The numerical example at the end of this section further demonstrates this point. Now with $e^N(\hat{\tau})$ moving in opposite direction to $t^N(\hat{\tau})$, equation (24) indicates that harmonizing τ at "just above" its Nash equilibrium value may have a beneficial as well as a detrimental effect on the choice of a polluting technology.

Next, to determine what happens to aggregate emissions, differentiate $E = 2ex(p)$ with respect to $\hat{\tau}$. After a bit of algebraic manipulation, assuming a constant elasticity of demand, we get

$$\frac{dE^N(\hat{\tau})}{d\hat{\tau}} = 2x(p)\left[\frac{\frac{1}{\delta\varepsilon} - \frac{C'(e)e}{\delta p} - \frac{\tau}{p^2}}{\frac{1}{\delta\varepsilon} + \frac{1}{p} - \frac{\tau}{p^2}}\right]\frac{de^N(\hat{\tau})}{d\hat{\tau}}. \qquad (25)$$

[14]The best-reply function for i is given by equation (A45) in the Appendix when τ_i is set at $\hat{\tau}$. Of course, one must solve (23) in conjunction with equations (2), (7), (8), (14), and (18).

If $\delta = 0$, tax competition pushes the price down to the point such that $p_i - C(e_i) = \hat{\tau} - C'(e_i)e_i = 0$. Consequently, e_i is determined according to $-C'(e_i)e_i = -\hat{\tau}$.

The bracketed expression in the right-hand side of (25) is positive due to the negative signs of τ and $C'(e)$. Consequently, with the effect of harmonization on $e^N(\hat{\tau})$ being ambiguous, equation (25) indicates that the effect of harmonization on E is also ambiguous. Nevertheless we have that e and E always move positively together.

5.2 Harmonization and welfare

So far, we have shown that harmonizing τ_i at a level which is above the unrestricted Nash equilibrium does not necessarily improve the environment. Let us now study the impact of such a policy on overall welfare. This is of course affected by what happens to the environment; but it also depends on other factors, particularly tax distortions. From (13), a state's welfare at a (restricted) symmetric Nash equilibrium with $\tau = \hat{\tau}$ is given by

$$W^S\left(t^N(\hat{\tau}), \hat{\tau}\right) = m - p^N(\hat{\tau})x\left(p^N(\hat{\tau})\right) + h\left(x\left(p^N(\hat{\tau})\right)\right) + T^N(\hat{\tau}) - \varphi\left(E^N(\hat{\tau})\right), \quad (26)$$

where we have dropped the i subscript for simplicity. Differentiating this expression totally with respect to $\hat{\tau}$ yields:

$$\frac{dW^S\left(t^N(\hat{\tau}), \hat{\tau}\right)}{d\hat{\tau}} = \frac{\partial W^S\left(t^N(\hat{\tau}), \hat{\tau}\right)}{\partial\hat{\tau}} + \frac{\partial W^S\left(t^N(\hat{\tau}), \hat{\tau}\right)}{\partial t^N(\hat{\tau})}\frac{dt^N(\hat{\tau})}{d\hat{\tau}}. \quad (27)$$

In the Appendix, we derive the expressions for $\partial W^S/\partial\hat{\tau}$ and $\partial W^S/\partial t^N(\hat{\tau})$. It follows from these expressions and (24) that at $\hat{\tau} = \tau^N$,

$$\frac{dW^S\left(t^N(\hat{\tau}), \hat{\tau}\right)}{d\hat{\tau}} = -x(p)\left\{\frac{\varphi'(E)\left[\frac{p-C(e)}{\delta^2\varepsilon} + \frac{\varphi'(E)e}{\delta p} - \frac{\hat{\tau}}{p^2}\right] - \frac{\hat{\tau}}{\delta\varepsilon e}}{\frac{1}{\delta\varepsilon} + \frac{1}{p} - \frac{\hat{\tau}}{p^2}}\right\}\frac{de^N(\hat{\tau})}{d\hat{\tau}}. \quad (28)$$

Note that the bracketed expression in the right-hand side of (28) is always positive. Equation (28) thus tells us that welfare always moves negatively with e (and E since e and E move together).

5.3 Numerical illustration

Consider a specification of our model with quadratic per unit costs, constant marginal social damage (of total emission) and constant elasticity demand. We thus assume the following functional forms:

$$C(e) = \frac{b(1-e)^2}{2}, \tag{29}$$

$$\varphi(E) = \varphi E, \tag{30}$$

$$h(x) = \frac{x^{1-1/\varepsilon}}{1-1/\varepsilon}. \tag{31}$$

Observe that demand for x is given by

$$x = p^{-\varepsilon}, \tag{32}$$

where ε is the constant elasticity of demand equal (in absolute value).

Given these specifications, we derive the Nash equilibrium allocations under open borders (from equations (2), (7), (8), (14), (18), (21), and (22)). Then, we solve the problem again at a value exceeding its unrestricted Nash equilibrium value (by dropping equations (21)–(22) and replacing them with the set value of τ and (23)). Table 1 reports the equilibrium with and without harmonization for two different values of δ: 0.1 and 0.01. All other parameters take the same values of $b = 0.40, \varphi = 0.25$ and $\varepsilon = 0.20$. In the first example, $t^N(\hat{\tau})$ decreases. The result is a higher value for e and E, and a lower value for welfare. In the second example, $t^N(\hat{\tau})$ increases. The result is a lower value for e and E, and a higher value for welfare.

The results of this section are summarized as

Proposition 5 *Assume states A and B harmonize their commodity tax rates, at $\tau = \hat{\tau}$ which is "just above" its unrestricted Nash equilibrium value, τ^N; they continue to compete in emission taxes. We have, assuming a constant elasticity of demand and a constant marginal social damage of emissions,*

Table 1. Commodity tax harmonization

	$(\delta = 0.1)$						
	τ	t	e	E	W^S	p	$x(p)$
Absent harmonization	-0.07737	0.25000	0.37500	1.20219	3.53635	0.09450	1.60292
With harmonization	-0.07700	0.24907	0.37732	1.20955	3.53544	0.09453	1.60284
	$(\delta = 0.01)$						
	τ	t	e	E	W^S	p	$x(p)$
Absent harmonization	-0.09147	0.25000	0.37500	1.24168	3.52696	0.08040	1.65557
With harmonization	-0.09130	0.25111	0.37223	1.23072	3.52855	0.08099	1.65316

(i) Harmonizing τ_i at $\hat{\tau} > \tau^N$ may increase t_i. Under this circumstance, firms opt for less polluting technologies. Then, aggregate emissions also decrease and welfare improves.

(ii) Harmonizing τ_i at $\hat{\tau} > \tau^N$ may lower t_i. Under this circumstance, firms opt for more polluting technologies. Then, aggregate emissions also increase and welfare deteriorates.

6 Emission tax harmonization

We now turn to the harmonization of taxes at $t = \hat{t}$. Output taxes continue to be set non-cooperatively with each state maximizing its own residents' welfare. We study the restricted Nash equilibrium conditional on $t = \hat{t}$. The equilibrium value of τ_i, denoted $\tau^N(\hat{t})$, is the solution to[15]

$$-\left[\frac{p_i - C(e_i)}{\delta\,\varepsilon_i} + \frac{\tau_i}{p_i}\right] + \frac{e_i}{p_i}\left[C'(e_i) + \varphi'(E)\right] = 0. \tag{33}$$

We also introduce the obvious notations $e^N(\hat{t})$, $E^N(\hat{t})$ and $p^N(\hat{t})$. Observe that when t is unrestricted, the bracketed expression in the left-hand side of (33) will be zero so that τ will also take its unrestricted Nash equilibrium value, τ^N.

[15] The best-reply function for i is given by equation (A46) in the Appendix when t_i is set at $\hat{t}$. Of course, one must solve (33) in conjunction with equations (2), (7), (8), (14), and (18).

If $\delta = 0$, tax competition pushes the price down to the point where $p_i - C(\hat{e}) = \tau + \hat{t}\hat{e} = 0$. Consequently, $\tau = -\hat{t}\hat{e}$.

6.1 Harmonization and the environment

With t_i fixed, per unit emission, e_i, is determined. Consequently, when the emission tax rate is set above its unrestricted Nash equilibrium value, it follows immediately that less polluting technologies will be used. However, the impact of this on E, and thus on the overall environmental quality, is not as obvious. There is another factor here; namely what happens to the aggregate production of the polluting good. This in turn depends on p and thus on the Nash equilibrium level of τ, $\tau^N(\hat{t})$. To study these additional effects, we differentiate (33) with respect to $\hat{t}$ and evaluate the resulting expression at (t^N, τ^N). To simplify the derivations, we continue to assume that the marginal social damage of emissions is constant. We have (see the Appendix)

$$\frac{d\tau^N(\hat{t})}{d\hat{t}} = -e - \frac{C'(e)}{C''(e)\big[1 + (1 - \tau/p)\delta\varepsilon/p\big]}. \tag{34}$$

It is clear that the sign of the above expression is ambiguous. Nevertheless the value of $d\tau^N(\hat{t})/d\hat{t}$ implies that $p^N(\hat{t})$ is increasing in $\hat{t}$ at $\hat{t} = t^N$. To see this, differentiate $p = C(e) - C'(e)e + \tau$ with respect to $\hat{t}$:

$$\frac{dp}{d\hat{t}} = e + \frac{d\tau}{d\hat{t}}.$$

Substituting from (34) in above yields

$$\frac{dp^N(\hat{t})}{d\hat{t}} = -\frac{C'(e)}{C''(e)\big[1 + (1 - \tau/p)\delta\varepsilon/p\big]}, \tag{35}$$

which is positive when evaluated at $\hat{t} = t^N$. This follows from the convexity of $C(.)$ and the fact that $\tau^N < 0$.

Armed with this result, one can easily determine the effect of harmonization on aggregate emissions. Differentiate $E = 2ex(p)$ with respect to $\hat{t}$. After a bit of algebraic manipulation, we get

$$\frac{dE^N(\hat{t})}{d\hat{t}} = -2x(p)\left[\frac{1}{C''(e)} + \frac{e\varepsilon}{p}\frac{dp^N(\hat{t})}{d\hat{t}}\right]. \tag{36}$$

With $dp^N(\hat{t})/d\hat{t} > 0$, the bracketed expression in the right-hand side of (36) will also be positive. Consequently, equation (36) indicates that harmonizing t at just above its unrestricted Nash equilibrium value, reduces aggregate emissions.

6.2 Harmonization and welfare

We have shown that harmonizing the emission tax rate reduces per unit as well as aggregate emissions. Consequently, one would expect that this form of partial harmonization does have a positive impact on overall welfare. We now show that this conjecture is indeed correct.

Use (13) to write a state's welfare, at a symmetric Nash equilibrium with $t = \hat{t}$, as[16]

$$W^S\left(\tau^N(\hat{t}), \hat{t}\right) = m - p^N(\hat{t}) x\left(p^N(\hat{t})\right) + h\left(x\left(p^N(\hat{t})\right)\right) + T^N(\hat{t}) - \varphi\left(E^N(\hat{t})\right). \qquad (37)$$

Differentiate $W^S\left(\tau^N(\hat{t}), \hat{t}\right)$ totally with respect to $\hat{t}$ to get

$$\frac{dW^S\left(\tau^N(\hat{t}), \hat{t}\right)}{d\hat{t}} = \frac{\partial W^S\left(\tau^N(\hat{t}), \hat{t}\right)}{\partial \hat{t}} + \frac{\partial W^S\left(\tau^N(\hat{t}), \hat{t}\right)}{\partial \tau^N(\hat{t})} \frac{d\tau^N(\hat{t})}{d\hat{t}}. \qquad (38)$$

Next, substitute the expressions for $\partial W^S/\partial \hat{\tau}$ and $\partial W^S/\partial t^N(\hat{\tau})$ (derived in the Appendix), and $d\tau^N(\hat{t})/d\hat{t}$ from (34), in (38) and simplify. We have at $\hat{t} = t^N$,

$$\frac{dW^S\left(\tau^N(\hat{t}), \hat{t}\right)}{d\hat{t}} = x(p)\varepsilon\left[\frac{p - C(e)}{\delta\varepsilon} + \frac{\varphi'(E)e}{p}\right]\frac{dp^N(\hat{t})}{d\hat{t}} + \frac{x(p)\varphi'(E)}{C''(e)} > 0. \qquad (39)$$

The sign of (39) follows from the fact that $dp^N(\hat{t})/d\hat{t} > 0$ and the convexity of $C(.)$.

The results of this section are summarized as

Proposition 6 *Assume states A and B harmonize their emission tax rates, at $t = \hat{t}$ which is "just above" its unrestricted Nash equilibrium value, t^N; they continue to compete in commodity taxes. We have, assuming a constant elasticity of demand and a constant marginal social damage of emissions, (i) firms opt for a less polluting technology, (ii) aggregate emissions decline and (iii) overall welfare increases.*

[16] We have again dropped the i subscript for ease in notation.

7 Conclusion

This paper has studied competition in environmental taxes in the presence of a global negative externality. The distinctive feature of the study has been in its differentiation between the choice of polluting technologies and aggregate emissions. The two need not move positively together. Indeed, one main lesson that has emerged is that the possibility of trade causes the firms to adopt the same or less polluting technologies, but it nevertheless increases aggregate emissions and lowers welfare.

Secondly, we studied the ramifications of partial harmonization policies. With commodity tax harmonization, we found that if one were to harmonize the tax on the polluting good at its unrestricted Nash equilibrium value, it will lead the equilibrium emission tax to either increase or decrease. If the emission tax increases, firms will adopt less polluting technologies. In turn, this will lead to a lowering of aggregate emissions and improvement in overall welfare. On the other hand, if emission tax goes down, firms will choose more polluting technologies. Under this circumstance, aggregate emissions will increase and welfare deteriorates.

Turning to harmonization of emission taxes, it is plain that harmonizing emission taxes at a value above their unrestricted Nash equilibrium, would lead the firms to adopt a less polluting technology. Interestingly, we showed that this policy also causes aggregate emissions to decline and overall welfare to increase.

Appendix

Derivation of (4),(5), (6): We have

$$T_A = \begin{cases} \int_{\tilde{\theta}}^{1} \left[p_A - C(e_A)\right] x_A^A d\theta & \text{if } p_A \geq p_B, \\ \int_{\tilde{\theta}}^{0} \left[p_A - C(e_A)\right] x_A^B d\theta + \int_0^1 \left[p_A - C(e_A)\right] x_A^A d\theta & \text{if } p_A < p_B, \end{cases} \tag{A1}$$

$$T_B = \begin{cases} \int_{-1}^{0} \left[p_B - C(e_B)\right] x_B^B d\theta + \int_0^{\tilde{\theta}} \left[p_B - C(e_B)\right] x_B^A d\theta & \text{if } p_A \geq p_B, \\ \int_{-1}^{\tilde{\theta}} \left[p_B - C(e_B)\right] x_B^B d\theta & \text{if } p_A < p_B. \end{cases} \tag{A2}$$

And

$$E = \begin{cases} \int_{-1}^{0} e_B x_B^B d\theta + \int_0^{\tilde{\theta}} e_B x_B^A d\theta + \int_{\tilde{\theta}}^1 e_A x_A^A d\theta & \text{if } p_A \geq p_B, \\ \int_{-1}^{\tilde{\theta}} e_B x_B^B d\theta + \int_{\tilde{\theta}}^0 e_A x_A^B d\theta + \int_0^1 e_A x_A^A d\theta & \text{if } p_A < p_B. \end{cases} \tag{A3}$$

Simplifying yields equations (4)–(5) and (6) in the text.

Differentiability of $T_A(p_A, e_A; p_B), T_B(p_B, e_B; p_A)$ and $E(p_A, p_B, e_A, e_B)$: Partially differentiate equations (4) and (6) with respect to p_A and e_A, and equations (5) and (6) with respect to p_B and e_B. We have

$$\frac{\partial T_A}{\partial p_A} = \begin{cases} (1-\tilde{\theta})x(p_A) - \frac{1}{\delta}\left[p_A - C(e_A)\right] x(p_A) + \left[p_A - C(e_A)\right](1-\tilde{\theta})x'(p_A) & \text{if } p_A \geq p_B, \\ x(p_A) + \int_{\tilde{\theta}}^0 x(p_A - \delta\theta)d\theta + \left[p_A - C(e_A)\right]\left[x'(p_A) - \frac{1}{\delta}x(p_A)\right] & \text{if } p_A < p_B. \end{cases} \tag{A4}$$

$$\frac{\partial T_B}{\partial p_B} = \begin{cases} x(p_B) + \int_0^{\tilde{\theta}} x(p_B + \delta\theta)d\theta + \left[p_B - C(e_B)\right]\left[x'(p_B) - \frac{1}{\delta}x(p_B)\right] & \text{if } p_A \geq p_B, \\ (1+\tilde{\theta})x(p_B) - \frac{1}{\delta}\left[p_B - C(e_B)\right] x(p_B) + \left[p_B - C(e_B)\right](1+\tilde{\theta})x'(p_B) & \text{if } p_A < p_B. \end{cases} \tag{A5}$$

$$\frac{\partial E}{\partial p_A} = \begin{cases} \frac{1}{\delta}e_B x(p_B + \delta\tilde{\theta}) - \frac{1}{\delta}e_A x(p_A) + (1-\tilde{\theta})e_A x'(p_A) & \text{if } p_A \geq p_B, \\ \frac{1}{\delta}e_B x(p_B) + e_A\left[x'(p_A) - \frac{1}{\delta}x(p_A)\right] & \text{if } p_A < p_B. \end{cases} \tag{A6}$$

$$\frac{\partial E}{\partial p_B} = \begin{cases} e_B\left[x'(p_B) - \frac{1}{\delta}x(p_B)\right] + \frac{1}{\delta}e_A x(p_A) & \text{if } p_A \geq p_B, \\ -\frac{1}{\delta}e_B x(p_B) + (1+\tilde{\theta})e_B x'(p_B) + \frac{1}{\delta}e_A x(p_A - \delta\tilde{\theta}) & \text{if } p_A < p_B. \end{cases} \tag{A7}$$

$$\frac{\partial T_A}{\partial e_A} = \begin{cases} -C'(e_A)(1-\tilde{\theta})x(p_A) & \text{if } p_A \geq p_B, \\ -C'(e_A)\left[x(p_A) + \int_{\tilde{\theta}}^0 x(p_A - \delta\theta)d\theta\right] & \text{if } p_A < p_B. \end{cases} \tag{A8}$$

$$\frac{\partial T_B}{\partial e_B} = \begin{cases} -C'(e_B)\left[x(p_B) + \int_0^{\tilde{\theta}} x(p_B + \delta\theta)d\theta\right] & \text{if } p_A \geq p_B, \\ -C'(e_B)(1+\tilde{\theta})x(p_B) & \text{if } p_A < p_B. \end{cases} \tag{A9}$$

$$\frac{\partial E}{\partial e_A} = \begin{cases} (1-\tilde{\theta})x(p_A) & \text{if } p_A \geq p_B, \\ \int_{\tilde{\theta}}^{0} x(p_A - \delta\theta)d\theta + x(p_A) & \text{if } p_A < p_B. \end{cases} \quad \text{(A10)}$$

$$\frac{\partial E}{\partial e_B} = \begin{cases} \int_{0}^{\tilde{\theta}} x(p_B + \delta\theta)d\theta + x(p_B) & \text{if } p_A \geq p_B, \\ (1+\tilde{\theta})x(p_B) & \text{if } p_A < p_B. \end{cases} \quad \text{(A11)}$$

where, in the derivations of (A4)–(A7), we have utilized the following expressions:

$$\begin{aligned} \frac{\partial}{\partial p_A}\int_{\tilde{\theta}}^{0} x(p_A - \delta\theta)d\theta &= \int_{\tilde{\theta}}^{0} x'(p_A - \delta\theta)d\theta - x(p_A - \delta\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial p_A} \\ &= -\frac{1}{\delta}\Big[x(p_A - \delta\theta)\Big]_{\theta=\tilde{\theta}}^{\theta=0} - \frac{1}{\delta}x(p_A - \delta\tilde{\theta}) = -\frac{1}{\delta}x(p_A), \quad \text{(A12)} \\ \frac{\partial}{\partial p_B}\int_{0}^{\tilde{\theta}} x(p_B + \delta\theta)d\theta &= \int_{0}^{\tilde{\theta}} x'(p_B + \delta\theta)d\theta - x(p_B + \delta\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial p_B} \\ &= \frac{1}{\delta}\Big[x(p_B + \delta\theta)\Big]_{\theta=0}^{\theta=\tilde{\theta}} - \frac{1}{\delta}x(p_B + \delta\tilde{\theta}) = -\frac{1}{\delta}x(p_B). \quad \text{(A13)} \end{aligned}$$

Evaluating expressions (A4)–(A11) at $p_A = p_B$ and simplifying, we get an identical expression for the left- and the right-hand derivatives of each of the functions $T_A(.), T_B(.)$ and $E(.)$. They are all continuous and given by, for $i = A, B$,[17]

$$\begin{aligned} \frac{\partial T_i}{\partial p_i} &= x(p_i) - [p_i - C(e_i)]\left[\frac{x(p_i)}{\delta} - x'(p_i)\right], \quad \text{(A14)} \\ \frac{\partial E}{\partial p_i} &= e_i x'(p_i), \quad \text{(A15)} \\ \frac{\partial T_i}{\partial e_i} &= -C'(e_i)x(p_i), \quad \text{(A16)} \\ \frac{\partial E}{\partial e_i} &= x(p_i). \quad \text{(A17)} \end{aligned}$$

Characterizations and differentiability of $W_A(p_A, T_A, E; p_B), W_B(p_B, T_B, E; p_A)$: To simplify the exposition of the proof, first calculate the following expressions based

[17] One can easily show that the same properties hold for all other partial derivatives of $T_A(.), T_B(.)$ and $E(.)$.

on the definition of $F(p)$ in (10).

$$\int_0^{\tilde{\theta}} \left[h\left(x(p_B+\delta\theta)\right) - (p_B+\delta\theta)x(p_B+\delta\theta)\right] d\theta \;=\; \frac{1}{\delta}\left[F(p_A) - F(p_B)\right], \quad \text{(A18)}$$

$$\int_{\tilde{\theta}}^0 \left[h\left(x(p_A-\delta\theta)\right) - (p_A-\delta\theta)x(p_A-\delta\theta)\right] d\theta \;=\; -\frac{1}{\delta}\left[F(p_A) - F(p_B)\right]. \quad \text{(A19)}$$

$$F'(p_i) = h(x(p_i)) - p_i x(p_i), \qquad i = A, B. \quad \text{(A20)}$$

$$\begin{aligned}\frac{\partial}{\partial p_A}\tilde{\theta}\left[h(x(p_A)) - p_A x(p_A)\right] &= \frac{1}{\delta}\left[h(x(p_A)) - p_A x(p_A)\right] + \tilde{\theta}\big[h'(x(p_A))x'(p_A) \\ &- x(p_A) - p_A x'(p_A)\big] = \frac{1}{\delta}F'(p_A) - \tilde{\theta}x(p_A), \quad \text{(A21)} \\ \frac{\partial}{\partial p_B}\tilde{\theta}\left[h(x(p_B)) - p_B x(p_B)\right] &= -\frac{1}{\delta}\left[h(x(p_B)) - p_B x(p_B)\right] + \tilde{\theta}\big[h'(x(p_B))x'(p_B) \\ &- x(p_B) - p_B x'(p_B)\big] = -\frac{1}{\delta}F'(p_B) - \tilde{\theta}x(p_B). \quad \text{(A22)}\end{aligned}$$

Characterizations of $W_A(p_A, T_A, E; p_B), W_B(p_B, T_B, E; p_A)$: Consider the case where $p_A \geq p_B$. We have $\tilde{\theta} \geq 0$ and

$$\begin{aligned} W_A = &\int_0^{\tilde{\theta}} \left[m + h(x(p_B+\delta\theta)) - (p_B+\delta\theta)x(p_B+\delta\theta) + T_A - \varphi(E)\right] d\theta + \\ &\int_{\tilde{\theta}}^1 \left[m + h(x(p_A)) - p_A x(p_A) + T_A - \varphi(E)\right] d\theta, \\ = &\; U_A - \int_0^{\tilde{\theta}} \left[h(x(p_A)) - p_A x(p_A)\right] d\theta + \int_0^{\tilde{\theta}} \left[h(x(p_B+\delta\theta)) - (p_B+\delta\theta)x(p_B+\delta\theta)\right] d\theta, \\ = &\; U_A - \tilde{\theta}\left[h(x(p_A)) - p_A x(p_A)\right] + \frac{1}{\delta}\left[F(p_A) - F(p_B)\right]. \quad \text{(A23)} \\ W_B = &\int_{-1}^0 \left[m + h(x(p_B)) - p_B x(p_B) + T_B - \varphi(E)\right] d\theta, \\ = &\; U_B. \quad \text{(A24)}\end{aligned}$$

where we have made use of equations (A18)–(A19) and the definition of U_i ($i = A, B$) in (9).

Similarly, using (A18)–(A19) and the definition of U_i ($i = A, B$) for the case $p_A < p_B$, we have

$$\begin{aligned}
W_A = & \int_0^1 \quad [m + h(x(p_A)) - p_A x(p_A) + T_A - \varphi(E)]\, d\theta, \\
= & \ U_A. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(A25)} \\
W_B = & \int_{-1}^{\tilde{\theta}} \quad [m + h(x(p_B)) - p_B x(p_B) + T_B - \varphi(E)]\, d\theta + \\
& \int_{\tilde{\theta}}^{0} \quad [m + h(x(p_A - \delta\theta)) - (p_A - \delta\theta) x(p_A - \delta\theta) + T_B - \varphi(E)]\, d\theta, \\
= & \ U_B \quad - \int_{\tilde{\theta}}^{0} [h(x(p_B)) - p_B x(p_B)]\, d\theta + \int_{\tilde{\theta}}^{0} [h(x(p_A - \delta\theta)) - (p_A - \delta\theta) x(p_A - \delta\theta)]\, d\theta, \\
= & \ U_B \quad + \tilde{\theta}\, [h(x(p_B)) - p_B x(p_B)] - \frac{1}{\delta} [F(p_A) - F(p_B)]\,. \qquad\qquad\qquad \text{(A26)}
\end{aligned}$$

Continuous differentiability: Differentiating equation (11) with respect to p_A and equation (12) with respect to p_B, making use of (A21)–(A22), yields

$$\frac{\partial W_A}{\partial p_A} = \begin{cases} \frac{\partial U_A}{\partial p_A} + \frac{1}{\delta} F'(p_A) - \left[\frac{1}{\delta} F'(p_A) - \tilde{\theta} x(p_A)\right] & \text{if } p_A \geq p_B, \\ \frac{\partial U_A}{\partial p_A} & \text{if } p_A < p_B. \end{cases} \tag{A27}$$

$$\frac{\partial W_B}{\partial p_B} = \begin{cases} \frac{\partial U_B}{\partial p_B} & \text{if } p_A \geq p_B, \\ \frac{\partial U_B}{\partial p_B} + \frac{1}{\delta} F'(p_B) - \frac{1}{\delta} F'(p_B) - \tilde{\theta} x(p_B) & \text{if } p_A < p_B. \end{cases} \tag{A28}$$

The equality of left- and right-hand derivatives result follows immediately from the fact that at $p_A = p_B$, $\tilde{\theta} = 0$.[18]

Proof of Proposition 1: The first-order conditions are:

$$\frac{\partial W_i^S}{\partial \tau_i} = \frac{\partial W_i^S}{\partial p_i} = -x(p_i) + \frac{\partial T_i}{\partial p_i}|_{e_i} - \varphi'(E) \frac{\partial E}{\partial p_i}|_{e_i} = 0, \tag{A29}$$

$$\frac{\partial W_i^S}{\partial t_i} = \frac{\partial W_i^S}{\partial p_i} \frac{\partial p_i}{\partial t_i}|_{\tau_i} + \frac{\partial W_i^S}{\partial t_i}|_{p_i} = \frac{\partial W_i^S}{\partial p_i} \frac{\partial p_i}{\partial t_i}|_{\tau_i} + \frac{\partial T_i}{\partial t_i}|_{p_i} - \varphi'(E) \frac{\partial E}{\partial t_i}|_{p_i} = 0. \tag{A30}$$

Simplifying (A29)–(A30), via differentiation of equations (14)–(15), we have

$$\frac{\partial W_i^S}{\partial \tau_i} = -x(p_i) + x(p_i) + [p_i - C(e_i)]\, x'(p_i) - \varphi'(E) 2 e_i x'(p_i) = 0, \tag{A31}$$

$$\frac{\partial W_i^S}{\partial t_i} = \frac{\partial W_i^S}{\partial p_i} \frac{\partial p_i}{\partial t_i}|_{\tau_i} - C'(e_i) \frac{\partial e_i}{\partial t_i} x(p_i) - \varphi'(E) \left[2 \frac{\partial e_i}{\partial t_i} x(p_i)\right] = 0. \tag{A32}$$

[18] One can easily show that the same properties hold for all other partial derivatives of W_i ($i = A, B$).

Further algebraic manipulation of (A31)–(A32) simplifies these equations into:

$$\frac{\partial W_i^S}{\partial \tau_i} = \left\{\tau_i - \left[C'(e_i) + 2\varphi'(E)\right] e_i\right\} x'(p_i) = 0, \tag{A33}$$

$$\frac{\partial W_i^S}{\partial t_i} = \frac{\partial W_i^S}{\partial \tau_i} e_i + \frac{x(p_i)}{C''(e_i)}\left[C'(e_i) + 2\varphi'(E)\right] = 0, \tag{A34}$$

where we have substituted $\tau_i - C'(e_i)e_i$ for $p_i - C(e_i)$.

To prove (17), set $\partial W_i^S / \partial \tau_i = 0$ in (A34) and simplify. Second, to prove $\tau_i = 0$, set $-C'(e_i) = 2\varphi'(E)$ in (A33) and simplify.

Proof of Proposition 2: Part (i). Summarize state i's problem through the Lagrangian

$$\Delta_i = m + h(x(p_i)) - p_i x(p_i) + T_i - \varphi(E), \tag{A35}$$

where $E = e_i x(p_i) + e_j x(p_j)$. Thus the difference with the optimization problem of Proposition 1 is only in the treatment of E. The proof will then be identical to the proof of Proposition 1 except that $\varphi'(E)$ replaces $2\varphi'(E)$ everywhere.

To prove parts (ii)–(iii), consider the system of equations

$$-C'(e) = \alpha\varphi'(E), \tag{A36}$$

$$E = 2ex(p), \tag{A37}$$

$$p = C(e) - C'(e)e, \tag{A38}$$

where α is a positive constant. These equations determine e, E and p as a function of α. These values correspond to the second-best when $\alpha = 2$ and the closed-economy solution when $\alpha = 1$. (Recall that in both cases, $\tau = 0$).

Next, differentiate equations (A36)–(A38) totally with respect to α and simplify.

We have

$$\frac{de}{d\alpha} = \frac{-\varphi'(E)}{C''(e) + 2x(p)\alpha\varphi''(E)\left[1 + C''(e)e^2\varepsilon/p\right]} < 0, \tag{A39}$$

$$\frac{dp}{d\alpha} = \frac{C''(e)e\varphi'(E)}{C''(e) + 2x(p)\alpha\varphi''(E)\left[1 + C''(e)e^2\varepsilon/p\right]} > 0, \tag{A40}$$

$$\frac{dE}{d\alpha} = \frac{-2x(p)\varphi'(E)\left[1 + C''(e)e^2\varepsilon/p\right]}{C''(e) + 2x(p)\alpha\varphi''(E)\left[1 + C''(e)e^2\varepsilon/p\right]} < 0. \tag{A41}$$

Parts (ii)–(iii) follow immediately from the signs of (A39)–(A41) which hold for all values of $\alpha > 0$, given the convexity of $C(.)$ and $\varphi(.)$.

Proof of Proposition 3: To derive the best-reply functions of each state, differentiate equations (11)–(12) with respect to the instrument employed. Thus, let I_i stand for τ_i, t_i or e_i. We have:

$$\frac{\partial W_A}{\partial I_A} = \begin{cases} \frac{\partial U_A}{\partial I_A} + \frac{\partial}{\partial I_A}\left[\frac{1}{\delta}\left[F(p_A) - F(p_B)\right] - \tilde{\theta}\left[h(x(p_A)) - p_A x(p_A)\right]\right] & \text{if } p_A \geq p_B \\ \frac{\partial U_A}{\partial I_A} & \text{if } p_A < p_B. \end{cases} \tag{A42}$$

$$\frac{\partial W_B}{\partial I_B} = \begin{cases} \frac{\partial U_B}{\partial I_B} & \text{if } p_A \geq p_B \\ \frac{\partial U_B}{\partial I_B} + \frac{\partial}{\partial I_B}\left[-\frac{1}{\delta}\left[F(p_A) - F(p_B)\right] + \tilde{\theta}\left[h(x(p_B)) - p_B x(p_B)\right]\right] & \text{if } p_A < p_B. \end{cases} \tag{A43}$$

The first-order conditions are found by setting the above equations equal to zero. Note, however, that in (A42)–(A43), only $\partial U_i/\partial I_i$ $(i = A, B)$ terms matter. Any additional term will vanish at a symmetric equilibrium.

The first-order conditions for state A are then given by,

$$\frac{\partial U_A}{\partial \tau_A} = \frac{\partial U_A}{\partial p_A} = -x(p_A) + \frac{\partial T_A}{\partial p_A} - \varphi'(E)\frac{\partial E}{\partial p_A} = 0, \tag{A44}$$

$$\begin{aligned} \frac{\partial U_A}{\partial t_A} &= \frac{\partial U_A}{\partial p_A}\frac{\partial p_A}{\partial t_A}|_{\tau_A} + \frac{\partial U_A}{\partial t_A}|_{p_A} \\ &= \frac{\partial U_A}{\partial p_A}\frac{\partial p_A}{\partial t_A}|_{\tau_A} + \frac{\partial T_A}{\partial t_A}|_{p_A} - \varphi'(E)\frac{\partial E}{\partial t_A}|_{p_A} = 0. \end{aligned} \tag{A45}$$

At $p_A = p_B, e_A = e_B$, one can simplify equations (A44)–(A45) by substituting from (A14)–(A15) in (A44) and from (A16)–(A17) in (A45). Same conditions hold for state

B and we have:

$$\begin{aligned}\frac{\partial U_i}{\partial \tau_i} &= x(p_i)\varepsilon_i\left\{-\left[\frac{p_i - C(e_i)}{\delta\,\varepsilon_i} + \frac{\tau_i}{p_i}\right] + \frac{e_i}{p_i}\left[C'(e_i) + \varphi'(E)\right]\right\} = 0, && \text{(A46)}\\ \frac{\partial U_i}{\partial t_i} &= \frac{\partial U_i}{\partial \tau_i}e_i + \frac{x(p_i)}{C''(e_i)}\left[C'(e_i) + \varphi'(E)\right] = 0, && \text{(A47)}\end{aligned}$$

where we have substituted ε_i for $-p_i x'(p_i)/x'(p_i)$. Substituting $\partial U_i/\partial \tau_i = 0$ from (A46) into (A47) gives us equation (22). Setting $-C'(e_i) = \varphi'(E)$ in (A46) then yields (21).

Variation in δ, the open-economy equilibrium and proof of Proposition 4: To simplify the calculations, we assume here that the elasticity of demand is constant. Substitute the optimal value of τ from (21) in (8) and insert $-C'(e_i)$ for $\varphi'(E)$. This yields

$$p - \frac{\delta\varepsilon\left[C(e) - C'(e)e\right]}{p} = C(e) - \delta\varepsilon, \tag{A48}$$

where we have dropped the subscript i for simplicity in exposition. Differentiate equations (7), (15), (22) and (A48) totally with respect to δ.[19] We have

$$\begin{aligned}\frac{de}{d\delta} &= \frac{-1}{C''(e)}\frac{dt}{d\delta}, && \text{(A49)}\\ \frac{dp}{d\delta} &= -\frac{p}{\varepsilon e}\left[\frac{1}{2x(p)\varphi''(E)} + \frac{1}{C''(e)}\right]\frac{dt}{d\delta}, && \text{(A50)}\\ \frac{dE}{d\delta} &= \frac{1}{\varphi''(E)}\frac{dt}{d\delta}, && \text{(A51)}\end{aligned}$$

with

$$\frac{dt}{d\delta} = \frac{-\frac{\varepsilon e\varphi'(E)}{p+\delta\varepsilon}}{\left\{\frac{p}{\varepsilon e} + \frac{\delta[C(e)-C'(e)e]}{ep}\right\}\left[\frac{1}{2x(p)\varphi''(E)} + \frac{1}{C''(e)}\right] + \frac{\delta\varepsilon e}{p} - \frac{C'(e)}{C''(e)}} < 0. \tag{A52}$$

The negative sign of (A52) follows from the convexity of $C(e)$ and $\varphi(E)$. Consequently, $de/d\delta > 0, dp/d\delta > 0$ and $dE/d\delta < 0$.

Turning to welfare, differentiate equation (13) totally with respect to δ. We have

$$\frac{dW^S}{d\delta} = \frac{dT}{d\delta} - x(p)\frac{dp}{d\delta} - \varphi'(E)\frac{dE}{d\delta}. \tag{A53}$$

[19] Details of the derivations can be obtained from the authors on request.

Next, differentiate $T = [p - C(e)]x(p)$ totally with respect to δ, substitute the resulting expression in (A53) and simplify. We get

$$\frac{dW^S}{d\delta} = -C'(e)x(p)\frac{de}{d\delta} + \big[p - C(e)\big]x'(p)\frac{dp}{d\delta} - \varphi'(E)\frac{dE}{d\delta}. \tag{A54}$$

Finally, substitute from (A49)–(A51) in (A54). After a bit of simplification, one arrives at

$$\frac{dW^S}{d\delta} = \left[\frac{x(p)\,\tau}{C''(e)e} + \frac{\tau - e\varphi'(E)}{2e\varphi''(E)}\right]\frac{dt}{d\delta} > 0. \tag{A55}$$

The sign of (A55) follows from, among other things, the fact that $\tau < 0$.

In the special case of a constant $\varphi'(E)$, the relationship $t = -C'(e) = \varphi'(E)$ implies that t and e are also constant so that

$$\frac{de}{d\delta} = 0. \tag{A56}$$

Equation (A48) then determines p as a function of δ only. Differentiating (A48) with respect to δ yields

$$\frac{dp}{d\delta} = \frac{\frac{\varepsilon e\varphi'(E)}{p+\delta\varepsilon}}{1 + [C(e) - C'(e)e]\,\delta\varepsilon/p^2} > 0. \tag{A57}$$

In turn, this implies that

$$\frac{dE}{d\delta} = 2ex'(p)\frac{dp}{d\delta} < 0. \tag{A58}$$

To determine the implication for welfare, consider equation (A54) again. Set $de/d\delta = 0$ from (A56), substitute for $dE/d\delta$ from (A58), and set $-C'(e) = \varphi'(E)$ from Proposition 3. We will have

$$\frac{dW^S}{d\delta} = (\tau - e\varphi'(E))x'(p)\frac{dp}{d\delta} > 0. \tag{A59}$$

Derivation of (24): Differentiate (23) with respect to $\hat{\tau}$, evaluate the resulting expression at (t^N, τ^N) and simplify. We get

$$(\frac{\tau}{p^2} - \frac{1}{\delta\varepsilon})\frac{dp}{d\hat{\tau}} + \left\{\frac{C'(e)}{\delta\varepsilon} + C''(e)\big[\frac{e}{p} + \frac{1}{C''(e)e\varepsilon}\big]\right\}\frac{de}{d\hat{\tau}} = \frac{1}{p}. \tag{A60}$$

Next, from differentiating $p = C(e) - C'(e)e + \tau$, we have

$$\frac{dp}{d\hat{\tau}} = -C''(e)e\frac{de}{d\hat{\tau}} + 1. \tag{A61}$$

Substituting from (A61) into (A60) and simplifying yields

$$\frac{de^N(\hat{\tau})}{d\hat{\tau}} = \frac{1}{C''(e)e + \left[\frac{C'(e)}{\delta\varepsilon} + \frac{1}{e\varepsilon}\right]/\left[\frac{1}{\delta\varepsilon} + \frac{1}{p} - \frac{\tau}{p^2}\right]}. \tag{A62}$$

Equation (24) follows immediately from (A62).

Derivation of $\partial W^S(\tau^N, t^N)/\partial\tau$ and $\partial W^S(\tau^N, t^N)/\partial t$: Compare equations (A33)–(A34) with (A46)–(A46). This reveals that

$$\frac{\partial W_i^S}{\partial \tau_i} = \frac{\partial U_i}{\partial \tau_i} + x(p_i)\varepsilon_i\left[\frac{p_i - C(e_i)}{\delta\varepsilon_i} + \frac{e_i}{p_i}\varphi'(E)\right], \tag{A63}$$

$$\frac{\partial W_i^S}{\partial t_i} = \frac{\partial U_i}{\partial t_i} + e_i x(p_i)\varepsilon_i\left[\frac{p_i - C(e_i)}{\delta\varepsilon_i} + \frac{e_i}{p_i}\varphi'(E)\right] + \frac{x(p_i)\varphi'(E)}{C''(e_i)}. \tag{A64}$$

Now at (τ^N, t^N), $\partial U_i/\partial\tau_i = \partial U_i/\partial t_i = 0$. Equations (A63)–(A64) then reduce to

$$\frac{\partial W_i^S}{\partial \tau_i} = x(p_i)\varepsilon_i\left[\frac{p_i - C(e_i)}{\delta\varepsilon_i} + \frac{e_i}{p_i}\varphi'(E)\right], \tag{A65}$$

$$\frac{\partial W_i^S}{\partial t_i} = e_i x(p_i)\varepsilon_i\left[\frac{p_i - C(e_i)}{\delta\varepsilon_i} + \frac{e_i}{p_i}\varphi'(E)\right] + \frac{x(p_i)\varphi'(E)}{C''(e_i)}. \tag{A66}$$

Derivation of (34): Differentiate (33) with respect to $\hat{t}$, evaluate the resulting expression at (t^N, τ^N) and simplify. We get

$$\left(\frac{\tau}{p^2} - \frac{1}{\delta\varepsilon}\right)\frac{dp}{d\hat{t}} - \frac{1}{p}\frac{d\tau}{d\hat{t}} = \frac{e}{p} + \frac{C'(e)}{\delta\varepsilon\, C''(e)}. \tag{A67}$$

Substituting $e + d\tau/dt$ for dp/dt in above and simplifying yields (34).

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