Principle of targeting in environmental taxation: Errata

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In deriving the elements of $\widetilde{\Delta}_w$ for the CES example, the sign of the income term in the Slutsky equation had appeared as negative rather than positive. Specifically, instead of writing $\partial\widetilde{x}/\partial p = \partial x/\partial p + x\partial x/\partial M$ the expression had been written as $\partial\widetilde{x}/\partial p = \partial x/\partial p - x\partial x/\partial M$ (and similarly for $\partial\widetilde{x}/\partial q$ and $\partial\widetilde{y}/\partial q$). This does not change *any* of the paper's results including the point that the example illustrated. Its sole import has been wrong algebraic expressions for the optimal tax rates in this example. While only a few expressions have changed, to set the record straight, I reproduce below the entire subsection with corrected expressions.

5.3 CES preferences

Assume that preferences are of the CES variety given by

$$u = \frac{-1}{\gamma}\left(x^{-\gamma} + y^{-\gamma} + l^{-\gamma}\right) - \varphi\left(E\right),$$

where $l = 1 - L$ is leisure and $E = Ny$ (where I have normalized emissions per unit of output e to one so that producing one unit of y results in one of emissions.) One can then easily determine the demand functions for l, x, and y as follows,

$$\begin{aligned} l &= \frac{1 + \frac{M}{w_n}}{\left(\frac{p}{w_n}\right)^{1-\rho} + \left(\frac{q}{w_n}\right)^{1-\rho} + 1}, & (35)\\ x &= \frac{\left(1 + \frac{M}{w_n}\right)}{\left(\frac{p}{w_n}\right)^{1-\rho} + \left(\frac{q}{w_n}\right)^{1-\rho} + 1}\left(\frac{p}{w_n}\right)^{-\rho}, & (36)\\ y &= \frac{\left(1 + \frac{M}{w_n}\right)}{\left(\frac{p}{w_n}\right)^{1-\rho} + \left(\frac{q}{w_n}\right)^{1-\rho} + 1}\left(\frac{q}{w_n}\right)^{-\rho}. & (37)\end{aligned}$$

where $\rho \equiv 1/\left(1 + \gamma\right)$ denotes the elasticity of substitution.

To calculate second-best taxes, I set $M = 0$ and normalize the wage tax t_w to zero and w to one. Consequently,

$$\begin{pmatrix} t \\ \tau + N\frac{\mathbf{v}_E}{\mu} \end{pmatrix} = \left(\Psi_w - \frac{\mu-\alpha}{\mu}\right)\widetilde{\Delta}_w^{-1}\begin{pmatrix} x \\ y \end{pmatrix}$$
$$= \frac{\frac{\mu-\alpha}{\mu}\left(1+p^{1-\rho}+q^{1-\rho}\right) - \left[tp^{-\rho} + \left(\tau + N\frac{\mathbf{v}_E}{\mu}\right)q^{-\rho}\right]}{\rho}\begin{pmatrix} p \\ q \end{pmatrix}. \quad (38)$$

Rearranging and manipulation yield,

$$\frac{\tau + N\frac{\mathbf{v}_E}{\mu}}{q} = \frac{t}{p} = \frac{\mu-\alpha}{\mu}\frac{1+p^{1-\rho}+q^{1-\rho}}{\rho+p^{1-\rho}+q^{1-\rho}},$$

which one can rewrite as

$$q - \frac{\mu-\alpha}{\mu}\frac{1+p^{1-\rho}+q^{1-\rho}}{\rho+p^{1-\rho}+q^{1-\rho}}q = 1 + N\frac{-\mathbf{v}_E}{\mu}, \quad (39)$$
$$p - \frac{\mu-\alpha}{\mu}\frac{1+p^{1-\rho}+q^{1-\rho}}{\rho+p^{1-\rho}+q^{1-\rho}}p = 1. \quad (40)$$

It follows from these two equations that $N\left(-\mathbf{v}_E/\mu\right)$ appears not only in the expression for q but also in the expression for p and thus t (unless $\rho = 1$). That is, the structure of the tax on non-polluting good is also affected by the emissions.

It is only in the special cases of $\rho = 1$, which corresponds to Cobb-Douglas preferences, that $N\left(-\mathbf{v}_E/\mu\right)$ appears only in q and τ but not in p and t. With $\rho = 1$, it follows from equations (39)–(40) that[1]

$$\tau = \frac{\mu}{\alpha}\left[1 + N\left(\frac{-\mathbf{v}_E}{\mu}\right)\right] - 1,$$
$$t = \frac{\mu}{\alpha} - 1.$$

Proof of (38) in the Appendix: Differentiate equations (36)–(37) with respect to

[1] Observe also that in this simple case with Cobb-Douglas preferences, the "tax differential" between τ and t is $N\left(-\mathbf{v}_E/\alpha\right)$, one of the two different measures of the marginal social damage of emissions. There is a small literature that attempts to compare the optimal tax differential with the "Pigouvian tax" (defined by one or the other measure of the marginal social damage of emissions). See, among others, Bovenberg and de Mooij (1994, 1997), Fullerton (1997), Schöb (1997), Cremer et al. (2001).

M, p, and q to get, when M and t_w are set equal to zero,

$$\begin{aligned}
\frac{\partial x}{\partial M} &= \frac{p^{-\rho}}{1+p^{1-\rho}+q^{1-\rho}}, \\
\frac{\partial x}{\partial p} &= -p^{-1-\rho}\frac{\rho\left(1+q^{1-\rho}\right)+p^{1-\rho}}{\left(1+p^{1-\rho}+q^{1-\rho}\right)^2}, \\
\frac{\partial x}{\partial q} &= -\left(1-\rho\right)\frac{p^{-\rho}q^{-\rho}}{\left(1+p^{1-\rho}+q^{1-\rho}\right)^2}, \\
\frac{\partial y}{\partial M} &= \frac{q^{-\rho}}{1+p^{1-\rho}+q^{1-\rho}}, \\
\frac{\partial y}{\partial p} &= -\left(1-\rho\right)\frac{p^{-\rho}q^{-\rho}}{\left(1+p^{1-\rho}+q^{1-\rho}\right)^2}, \\
\frac{\partial y}{\partial q} &= -q^{-1-\rho}\frac{\rho\left(1+p^{1-\rho}\right)+q^{1-\rho}}{\left(1+p^{1-\rho}+q^{1-\rho}\right)^2}.
\end{aligned}$$

Based on these equations and using the Slutsky equation, derive the derivatives of the compensated demand functions $\widetilde{x}$ and $\widetilde{y}$ with respect to prices p and q. Consequently, one can write the Slutsky matrix as

$$\begin{aligned}
\widetilde{\Delta}_w &= \begin{pmatrix} \frac{\partial \widetilde{x}}{\partial p} & \frac{\partial \widetilde{x}}{\partial q} \\ \frac{\partial \widetilde{y}}{\partial p} & \frac{\partial \widetilde{y}}{\partial q} \end{pmatrix} \\
&= \begin{pmatrix} -p^{-1-\rho}\frac{\rho\left(1+q^{1-\rho}\right)}{\left(1+p^{1-\rho}+q^{1-\rho}\right)^2} & \rho\frac{p^{-\rho}q^{-\rho}}{\left(1+p^{1-\rho}+q^{1-\rho}\right)^2} \\ \rho\frac{p^{-\rho}q^{-\rho}}{\left(1+p^{1-\rho}+q^{1-\rho}\right)^2} & -q^{-1-\rho}\frac{\rho\left(1+p^{1-\rho}\right)}{\left(1+p^{1-\rho}+q^{1-\rho}\right)^2} \end{pmatrix}.
\end{aligned}$$

Simplifying yields

$$\begin{aligned}
\widetilde{\Delta}_w^{-1} &= \frac{1+p^{1-\rho}+q^{1-\rho}}{\rho\left(pq\right)^{-1-\rho}} \times \\
&\quad \begin{pmatrix} -q^{-1-\rho}\left(1+p^{1-\rho}\right) & -p^{-\rho}q^{-\rho} \\ -p^{-\rho}q^{-\rho} & -p^{-1-\rho}\left(1+q^{1-\rho}\right) \end{pmatrix}.
\end{aligned}$$

Now setting $w_n = 1$ and $M = 0$ in equations (36)–(37) results in,

$$\begin{aligned}
x &= \frac{p^{-\rho}}{1+p^{1-\rho}+q^{1-\rho}}, \\
y &= \frac{q^{-\rho}}{1+p^{1-\rho}+q^{1-\rho}}.
\end{aligned}$$

It follows from the above expressions for $\widetilde{\Delta}_w^{-1}, x, y, \partial x/\partial M$, and $\partial y/\partial M$ that, after a bit of algebraic manipulation,

$$\begin{aligned}
\widetilde{\Delta}_w^{-1}\begin{pmatrix} x \\ y \end{pmatrix} &= \frac{-\left(1+p^{1-\rho}+q^{1-\rho}\right)}{\rho}\begin{pmatrix} p \\ q \end{pmatrix}, \\
\Psi_w &= t\left(\frac{\partial x}{\partial M}\right)+\left[\tau+\left(N\frac{\mathbf{v}_E}{\mu}\right)e\right]\left(\frac{\partial y}{\partial M}\right) \\
&= \frac{tp^{-\rho}+\left(\tau+N\frac{\mathbf{v}_E}{\mu}\right)q^{-\rho}}{1+p^{1-\rho}+q^{1-\rho}}.
\end{aligned}$$

where e is set equal to one. Substituting these values in formula (32) leads to (38).