On the Optimal Linkage of Social Security Benefits to Payroll Taxes*

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Abstract

This paper employs a three period overlapping generations model to investigate (i) the labor supply effects of the linkage between the benefits of a *pay-as-you-go* social security program and the payroll taxes that finance them and (ii) the nature of the optimal linkage. The main result of the paper is that, for a given *statuary* tax rate, the weights that must be placed on earnings of different periods (in benefit calculation) depend on population and productivity growth rates only. This result implies that the optimal *net* tax rates are not uniform over the life cycle unless the economy is on its steady state golden rule path. Moreover, if the economy is on the golden rule path, the optimal net tax rates are not only uniform but zero. The paper also demonstrates that, if preferences are additively separable, as more weight is placed on earnings when young labor supply by the young increases while labor supply by the middle-aged decreases.

1 Introduction

In the pay-as-you-go social security systems of the US and elsewhere, benefits are usually linked to the recipients' payroll tax payments through a benefit formula. Many studies treat payroll taxes just like other income taxes on labor when studying their effects on labor supply and savings or when calculating the deadweight loss of taxation. Yet this treatment is problematic in that the direct link between the tax payments and benefits are, in part, at an *individual* level and well-understood. This type of linkage is very different from, say, income tax payments wherein the benefits one may receive from the government in return, in terms of public goods or welfare payments, are linked to the tax revenues only at an *aggregate* economy-wide level through the government's budget constraint. Which is precisely why individuals treat both the income tax rates they face and government goods as fixed and independent of each other. This is of course not to say that the aggregate relationship between payroll taxes and benefits are not present in a pay-as-you-go-system. They are and to some extent embedded in the benefit formula.[1] However, what is relevant is that the individual link is there to be noticed by rational taxpayers.

To be sure, many economists have long recognized the link between payroll taxes and social security benefits. See, among others, Browning (1975, 1985), Blinder et al. (1980), Burkhauser and Turner (1985), Feldstein and Samwick (1992), Diamond and Gruber (1999), Liebman, Luttmer, Erzo and Seif (2009) and Liebman, Luttmer, and Erzo (2011). These papers correctly point out that if taxpayers perceive this link, they will not consider the *statuary* tax rate to be the *effective* (or *net*) tax rate. In calculating the effective tax rates they face over their life cycles, taxpayers will adjust the legislated rate to take account of the benefits they receive. Some calculate the effective tax rate,[2] while others provide empirical and experimental evidence for

[1] Through a "replacement factor".

[2] Computing effective marginal tax rates for a cross-section of workers in the US in

taxpayers taking the benefit linkage into account.[3] Kaplow (2015), while recognizing the problem, argues that one might "rationalize" the ignoring of the link on the basis of myopia.[4]

These studies also recognize, quite correctly of course, that because tax contributions of different periods are treated equally for the purpose of calculating social security benefits, while the legislated rate remains constant over time, effective tax rates decline with age. (This follows because the benefit received for a dollar paid in tax will be discounted to a greater extent, and will thus have a smaller present value, the earlier it is paid). However, none of these papers examine the question of the optimal linkage of payroll taxes and social security benefits. Nor do they model the impact of changing the benefit formula on labor supply. These two issues are what the current paper focuses on.

To address these questions, we employ a three-period overlapping generations model à la Samuelson (1958) and Diamond (1965) *focusing on steady-states alone*. We derive the optimal weights assigned to the contributions of different periods and show that they are in general non-uniform. Interestingly, one would want to assign a higher weight to the earnings of earlier years than later years in the benefit formula as long as net population-plus-productivity growth rate (the "productivity augmented" generalization of the Samuelson's biological rate of interest) is positive. This reduces the net payroll tax rates of earlier years. Nevertheless, one still wants the net tax rates to decline with age as long as net population-plus-productivity growth

1983, Burkhauser and Turner (1985) find that the social security tax has in fact been a subsidy for all cohorts except those under twenty five. Browning (1985) computes an effective tax in much the same manner as Burkhauser and Turner. However, he discounts future benefits at a higher rate, thus reporting higher effective tax rates than the ones reported by Burkhauser and Turner (1985).

Feldstein and Samwick (1992) provide a more elaborate and detailed formulation for such calculations.

[3] Liebman, Luttmer, Erzo and Seif (2009) provide empirical evidence, and Liebman, Luttmer, and Erzo (2011) experimental evidence.

[4] Diamond and Gruber (1999) note that most of the literature focuses on effects of the level of Social Security Wealth ignoring the effect of the marginal Social Security benefit rate.

rate exceeds the rate of return on capital. Consequently, the optimal net tax rates over time are zero if and only if the economy is on its *golden rule* path. Regarding the labor supply responses, we prove that one can in fact increase the young's labor supply (who are more numerous in the economy) by assigning higher weights to the earlier period contributions in calculating social security benefits.

Our findings point out to a costless reform of the social security system. Admittedly, this constitutes a limited reform that does not touch upon the many substantial issues confronting the system.[5] Yet, this will be a simple reform at essentially no cost.[6] And it is crucially important to realize that social security reform cannot succeed without convincing the public that the reform will not harm the basic tenet of the program. Nor can a reform succeed if it does not adequately address the entitlement sensibilities of the public. It is natural that the program's history and the recipients' or would-be recipients' contributions to it would lead to formation of such feelings.

2 The model

The model is a three-period version of the standard overlapping generations model of Samuelson (1958) and Diamond (1965) in the steady state. There are many (identical) persons in each generation. Individuals work in the first two periods of their lives (when young and middle-aged) and retire in the last (when old). Consumption in the last period is provided from savings of the first two periods plus social security retirement benefits. Each person derives utility from consuming a consumption good, c, in all periods of his life, and leisure, ℓ, in the first two periods. The utility function is assumed to be strictly quasi-concave and twice differentiable in all its arguments. It

[5] We touch on some of the pertinent issues in our Concluding Remarks.

[6] If taxpayers are not rational and completely ignore the linkage, the reform will have no impact. So it either enhances the efficiency of the system or at the extreme will have no impact!

is represented by

$$u = u(c_1, \ell_1, c_2, \ell_2, c_3), \tag{1}$$

where subscripts denote periods.

Each person is endowed with one unit of leisure each period. The population grows at the constant rate of n. The economy is in a steady state with a productivity growth (Harrod-neutral) at the constant rate of g. The (gross of tax) wage of the current young, w, and interest rate, r, are assumed to be determined exogenously (rather than endogenously through a neoclassical production function). One can justify this assumption by an appeal to the international mobility of factors. It helps prevent one to get sidetracked into a discussion of "short term" versus "long term" equilibria [as in Hu (1979)], and as to what may happen to the welfare of individuals on the transitional path to a new steady state.[7] These issues are not germane to the focus of our study.[8] The interest rate may be greater, equal to, or smaller than the population growth rate. No distinction between wage rates for individuals of different ages is made because labor is assumed to be homogeneous.

There is a *pay-as-you-go* social security program in place. The government taxes the wages of the young and the middle-aged at some fixed legislated rate and distributes the proceeds to the old. The social security program is the only tax/expenditure policy in effect.

From the perspective of a young person, he pays a tax at the rate of θ on the wages he earns during the first two periods of his life when he works. He then receives retirement benefits, b, from the government when he is old

[7]Of course, one can just simply ignore the transitional issues and focus on steady-state welfare maximization as many in the literature do; see Samuelson (1975). One can also justify the steady-state welfare maximization criterion by postulating a social welfare function defined over (undiscounted) average utilities of all future generations. A criterion that poses some interesting philosophical questions in terms of the inclusion of the utility of unborn children, using average versus sum of utilities, and the extent to which the utilities of future generations should be discounted.

[8]It is possible to introduce a neoclassical production technology, and allow wages and interest rates to be determined endogenously depending on the pension system and the benefit rules in place. However, while in this case the main qualitative messages of the paper continue to hold, derivations of analytical results become tedious in that setting.

and does not work. The benefits are calculated according to the formula

$$b = \mu[\alpha w(1-\ell_1) + (1-\alpha)w(1+g)(1-\ell_2)], \tag{2}$$

where α and $1-\alpha$ are the weights placed on the earnings of the first two periods. The factor μ stands for the so-called "replacement rate" that ensures the solvency of the system. This factor equates the social security disbursements on to its tax revenues. Thus while the worker treats it as given, it is determined by the government endogenously to balance its budget.

The formula given by (2) is a simplified version from the actual benefit formula in use. In particular, the two differ in two ways. First, (2) simplifies the current formula by substituting a single replacement factor for its three tier factors.[9] This will make the modeling and the presentation simpler without much bearing on the substantive issues we are raising. Second, equation (2) generalizes the current formula by assigning different weights to the earnings of different periods. In the current system, earnings of different periods have "identical" but wage-indexed weights. The different weights in our specification ignore wage indexing on purpose. One of our aims is to find out if wage-indexing is in fact required.

Each young person faces a lifetime budget constraint given by

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} = w(1-\ell_1)(1-\theta) + \frac{w(1+g)(1-\ell_2)(1-\theta)}{1+r} + \frac{b}{(1+r)^2}. \tag{3}$$

Assuming that the young know the benefit formula, one may substitute for

[9] In the current US system, first an average of monthly contributions is calculated. This is the so-called average indexed monthly earnings (AIME). Indexing brings the earnings of prior months up to current wage levels. The average is based on the highest 420 indexed monthly wages. A person who elects to begin receiving retirement benefits at his/her normal retirement age, would then receive a monthly benefit called the "primary insurance amount" (PIA). It is equivalent to 90% of the first x dollars of AIME, plus 32% of the next y dollars of AIME plus 15% of any remaining amount. [x and y are referred to as"break points" and are wage-indexed. For a person turning 66 in 2015, $x = \$856$ and $y = \$5,157$]. This is done to make the system more redistributive towards low lifetime earners.

b from (2) into (3). The lifetime budget constraint will then be rewritten as

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} = w(1-\ell_1)(1-\hat{\theta}_1) + \frac{w(1+g)(1-\ell_2)(1-\hat{\theta}_2)}{1+r}, \quad (4)$$

where

$$\hat{\theta}_1 \equiv \theta - \frac{\mu\alpha}{(1+r)^2} \quad (5a)$$

$$\hat{\theta}_2 \equiv \theta - \frac{\mu(1-\alpha)}{1+r}, \quad (5b)$$

denote the *effective* or *net* wage tax rates of the two periods as *seen* by the young. Observe that according to our formula

$$\hat{\theta}_1 - \hat{\theta}_2 = \frac{\mu}{(1+r)^2}\left[(1-\alpha)(1+r) - \alpha\right].$$

Now the current system wage-indexed identical weights implies $\alpha(1+g) = 1-\alpha$ so that $\alpha = 1/(2+g)$ and $1-\alpha = (1+g)/(2+g)$. Hence, under the current system, $\hat{\theta}_1 - \hat{\theta}_2 = \mu\left[(1+g)(1+r) - 1\right]/(1+r)^2(2+g) > 0$. That is, the current benefit formula implies that effective social security tax rates decline by age.

Each young person chooses his present and future consumption by maximizing the utility function (1) subject to the budget constraint (4). The first-order conditions are given by

$$\frac{\partial u/\partial \ell_1}{\partial u/\partial c_1} = w(1-\hat{\theta}_1), \quad (6a)$$

$$\frac{\partial u/\partial c_2}{\partial u/\partial c_1} = \frac{1}{1+r}, \quad (6b)$$

$$\frac{\partial u/\partial \ell_2}{\partial u/\partial c_1} = \frac{w(1+g)(1-\hat{\theta}_2)}{1+r}, \quad (6c)$$

$$\frac{\partial u/\partial c_3}{\partial u/\partial c_1} = \frac{1}{(1+r)^2}. \quad (6d)$$

The solution to equations (4) and (6a)–(6d) determines the demand functions for c_1, ℓ_1, c_2, ℓ_2 and c_3 as functions of w, r, g, $\hat{\theta}_1$ and $\hat{\theta}_2$.

Next observe that the values of $\hat{\theta}_1$ and $\hat{\theta}_2$ are effectively set by the government via its choice of the three instruments: θ, α, and μ. With w, r

and g being exogenously fixed, to close the system, one needs to specify how θ, α, and μ are determined. However, as we noted earlier, μ is determined to balance the government's pay-as-you-go constraint. Thus, effectively, the government has only two degrees of freedom in its choice: θ and α.

We now show formally how, given the values of θ and α, the value of the replacement factor μ is determined. Let N_1, N_2 and N_3 denote the number of young, middle-aged and old individuals in the economy. To meet its pay-as-you-go budget constraint for the aggregate economy, the government must set the benefits it pays out to the old equal to the tax revenues it collects from the young. With b denoting the benefit a current young person will receive when old, and a productivity growth rate of g, the benefit that a current old person would receive is equal to $b/(1+g)^2$. Consequently, the pay-as-you-go budget constraint is given by

$$N_3 \frac{b}{(1+g)^2} = \theta[N_1(1-\ell_1) + N_2(1-\ell_2)]w.$$

Given the constant population growth rate of n, the above constraint can be simplified to[10]

$$b = \theta w(1+g)^2(1+n)\left[(1+n)(1-\ell_1) + (1-\ell_2)\right]. \tag{7}$$

Then constraint (7), in conjunction with equation (2), determines μ. Specifically, we have

$$\mu = \theta(1+g)^2\,(1+n)\,\frac{(1+n)\,(1-\ell_1) + (1-\ell_2)}{\alpha(1-\ell_1) + (1-\alpha)(1+g)(1-\ell_2)}. \tag{8}$$

Finally, equation (8) then along with equation (4), (5a)–(5b) and (6a)–(6d) allow us to rewrite c_1, ℓ_1, c_2, ℓ_2, and c_3 as functions of θ and α (instead of $\hat{\theta_1}, \hat{\theta_2}$ and μ). Substituting the equilibrium values of c_1, ℓ_1, c_2, ℓ_2, and c_3,

[10]Note that while the government's budget constraint can be reduced to (7), each individual in his optimization problem takes b to be given by equation (2). This is the Nash-type behavior assumption: Each person takes the actions of the others, and thus the government budget constraint for the aggregate economy, as given. It is the standard assumption in taxation theory. See, among others, Diamond (1970).

written as functions of α and θ, in (1) yields the following indirect utility function

$$v(\alpha,\theta) \equiv u\left(c_1(\alpha,\theta), \ell_1(\alpha,\theta), c_2(\alpha,\theta), \ell_2(\alpha,\theta), c_3(\alpha,\theta)\right). \tag{9}$$

The welfare properties of our model thus depends on the behavior of $v(\alpha,\theta)$.

3 "Optimality" of the pay-as-you-go system

Choosing θ determines the *size* of the social security program. However, the simple model we have set up is not comprehensive enough to address this question. The determination of the optimal size of the social security system depends on a host of consideration that goes far beyond our objective in this paper.[11] Our aim in this paper is a more limited one. We want to show how we can improve the system by choosing the weights assigned to the contributions of different years *given the size of the program.* In particular, we want to show that the current practice of assigning equal weights, adjusted only by wage indexing, is not the "correct" way of going about this.

Nevertheless it will be instructive to start our discussion by showing that our model subscribes to the well-known proposition that whether or not a pay-as-you-go social security program increases or lowers the steady-state welfare depends on the relationship between the rate of return to real capital in the economy on the one hand, and the sum of the population and productivity growth rates on the other. This property holds regardless of how the benefit formula's earning weights are determined.

Proposition 1 *The pay-as-you-go social security system of Section 2 enhances steady-state welfare if* $(1+g)(1+n) > 1+r$ *and reduces it if* $(1+g)(1+n) < 1+r$.

[11] See the Concluding Remarks below.

Proof. First, differentiate $v(\alpha, \theta)$ partially with respect to θ and factor $\partial u/\partial c_1$ out to get,

$$\frac{\partial v}{\partial \theta} = \frac{\partial u}{\partial c_1}\left[\frac{\partial c_1}{\partial \theta} + \frac{\partial u/\partial \ell_1}{\partial u/\partial c_1}\frac{\partial \ell_1}{\partial \theta} + \frac{\partial u/\partial c_2}{\partial u/\partial c_1}\frac{\partial c_2}{\partial \theta} + \frac{\partial u/\partial \ell_2}{\partial u/\partial c_1}\frac{\partial \ell_2}{\partial \theta} + \frac{\partial u/\partial c_3}{\partial u/\partial c_1}\frac{\partial c_3}{\partial \theta}\right]. \tag{10}$$

Then substitute for $\frac{\partial u/\partial \ell_1}{\partial u/\partial c_1}, \frac{\partial u/\partial c_2}{\partial u/\partial c_1}, \frac{\partial u/\partial \ell_2}{\partial u/\partial c_1}, \frac{\partial u/\partial c_3}{\partial u/\partial c_1}$ from (6a)–(6d) into (10). This yields,

$$\frac{\partial v}{\partial \theta} = \frac{\partial u}{\partial c_1}\left[\frac{\partial c_1}{\partial \theta} + w(1-\hat{\theta}_1)\frac{\partial \ell_1}{\partial \theta} + \frac{1}{1+r}\frac{\partial c_2}{\partial \theta} + \frac{w(1+g)(1-\hat{\theta}_2)}{1+r}\frac{\partial \ell_2}{\partial \theta} + \frac{1}{(1+r)^2}\frac{\partial c_3}{\partial \theta}\right]. \tag{11}$$

Second, substitute for b from the government's budget constraint (7) into (3). A bit of algebraic manipulation yields

$$\begin{aligned} c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} &= w(1-\ell_1)\left\{1 - \left[1 - \frac{(1+g)^2(1+n)^2}{(1+r)^2}\right]\theta\right\} \\ &+ \frac{w(1+g)(1-\ell_2)}{1+r}\left\{1 - \left[1 - \frac{(1+g)(1+n)}{1+r}\right]\theta\right\}. \end{aligned} \tag{12}$$

Equation (12) indicates the constraint that must be satisfied by the young *in per capita terms*, so that the government's budget constraint for the *aggregate economy* is satisfied. We shall refer to this equation as the *per capita young constraint.*[12] Partially differentiate equation (12) with respect to θ and rearrange the terms,

$$\begin{aligned} &\frac{\partial c_1}{\partial \theta} + \frac{1}{1+r}\frac{\partial c_2}{\partial \theta} + \frac{1}{(1+r)^2}\frac{\partial c_3}{\partial \theta} + w\left(1-\hat{\theta}_1\right)\frac{\partial \ell_1}{\partial \theta} + \frac{w(1+g)}{1+r}\left(1-\hat{\theta}_2\right)\frac{\partial \ell_2}{\partial \theta} = \\ &- w(1-\ell_1)\left[1 - \frac{(1+g)^2(1+n)^2}{(1+r)^2}\right] - \frac{w(1+g)(1-\ell_2)}{1+r}\left[1 - \frac{(1+g)(1+n)}{1+r}\right]. \end{aligned} \tag{13}$$

[12]It is important to keep in mind that (12) differs from the young's budget constraint given by (4). The latter equation was derived by substituting for b from (2) into (3).

Third, substitute from (13) into (11)

$$\begin{aligned}\frac{\partial v}{\partial \theta} &= -w\frac{\partial u}{\partial c_1}\left\{(1-\ell_1)\left[1-\frac{(1+g)^2(1+n)^2}{(1+r)^2}\right]+\frac{(1+g)(1-\ell_2)}{1+r}\left[1-\frac{(1+g)(1+n)}{1+r}\right]\right\} \\ &= w\frac{\partial u}{\partial c_1}\left[\frac{(1+g)(1+n)}{1+r}-1\right]\left\{\left[1+\frac{(1+g)(1+n)}{1+r}\right](1-\ell_1)+\frac{(1+g)(1-\ell_2)}{1+r}\right\}.\end{aligned} \tag{14}$$

It follows from (14) that $\partial v/\partial\theta$ has the same sign as $(1+g)(1+n)-(1+r)$.

■

Proposition 1 tells us that if $(1+g)(1+n) < (1+r)$, having a pay-as-you-go social security system is not a "good deal" in our model as far as steady-state welfare is concerned. Under this circumstance, one may surmise that the determination of α would be a superfluous exercise. Nevertheless, there is no advantage in restricting our discussion only to this case. As observed earlier, endogenizing the choice of θ, with the possibility that optimality may require no social security program, requires a more elaborate and general model. For instance, even in this circumstance, it may be the case that for, say, political considerations the government may want to institute a social security program of a certain size. The problem of interest for us is one of *constrained* welfare maximization. That is, how to improve the system through the choice of α given a predetermined positive value for θ.

4 Optimal weights

Given θ, the optimal weights are found by differentiating $v(\alpha,\theta)$ with respect to α and setting the resulting equation equal to zero. Proposition 2 describes our result.

Proposition 2 *Assume that the labor supply response of the young to an increase in the weight associated with their earnings, $\partial\ell_1/\partial\alpha$, is of the same sign as that of the middle-aged to their weight, $\partial\ell_2/\partial(1-\alpha)$. The optimal weights associated with the earnings of the first and the second periods are* *given by:*

$$\alpha = \frac{(1+g)(1+n)}{1+(1+g)(1+n)}, \tag{15}$$

$$1-\alpha = \frac{1}{1+(1+g)(1+n)}. \tag{16}$$

Proof. First, differentiate $v(\alpha,\theta)$ partially with respect to α and factor $\partial u/\partial c_1$ out to get,

$$\frac{\partial v}{\partial \alpha} = \frac{\partial u}{\partial c_1}\left[\frac{\partial c_1}{\partial \alpha} + \frac{\partial u/\partial \ell_1}{\partial u/\partial c_1}\frac{\partial \ell_1}{\partial \alpha} + \frac{\partial u/\partial c_2}{\partial u/\partial c_1}\frac{\partial c_2}{\partial \alpha} + \frac{\partial u/\partial \ell_2}{\partial u/\partial c_1}\frac{\partial \ell_2}{\partial \alpha} + \frac{\partial u/\partial c_3}{\partial u/\partial c_1}\frac{\partial c_3}{\partial \alpha}\right]. \tag{17}$$

Then substitute from (6a)–(6d) in (17) and set $\partial v/\partial \alpha$ equal to zero. We have:

$$\frac{\partial c_1}{\partial \alpha} + w(1-\hat{\theta}_1)\frac{\partial \ell_1}{\partial \alpha} + \frac{1}{1+r}\,\frac{\partial c_2}{\partial \alpha} + \frac{w(1+g)(1-\hat{\theta}_2)}{1+r}\,\frac{\partial \ell_2}{\partial \alpha} + \frac{1}{(1+r)^2}\,\frac{\partial c_3}{\partial \alpha} = 0. \tag{18}$$

Second, differentiate (12) with respect to α and simplify. This yields:

$$\begin{aligned} \frac{\partial c_1}{\partial \alpha} &+ w\left\{1-\left[1-\frac{(1+g)^2(1+n)^2}{(1+r)^2}\right]\theta\right\}\frac{\partial \ell_1}{\partial \alpha} + \frac{1}{1+r}\,\frac{\partial c_2}{\partial \alpha} \\ &+ \frac{w(1+g)}{1+r}\left\{1-\left[1-\frac{(1+g)(1+n)}{1+r}\right]\theta\right\}\frac{\partial \ell_2}{\partial \alpha} + \frac{1}{(1+r)^2}\,\frac{\partial c_3}{\partial \alpha} = 0. \end{aligned} \tag{19}$$

Third, substitute for $\hat{\theta}_1$ and $\hat{\theta}_2$ from (5a) and (5b) in (18), subtract the resulting equation from (19) and simplify to get

$$\left[\alpha\mu - \theta(1+g)^2(1+n)^2\right]\frac{\partial \ell_1}{\partial \alpha} + (1+g)\left[(1-\alpha)\mu - \theta(1+g)(1+n)\right]\frac{\partial \ell_2}{\partial \alpha} = 0. \tag{20}$$

Finally, "solve" (8) for μ, substitute its value into (20), and simplify. This yields

$$\left[\alpha - \frac{(1+g)(1+n)}{1+(1+g)(1+n)}\right]\left[(1-\ell_2)\frac{\partial \ell_1}{\partial \alpha} + (1-\ell_1)\frac{\partial \ell_2}{\partial(1-\alpha)}\right] = 0. \tag{21}$$

It then follows from (21) that as long as $\partial \ell_1/\partial \alpha$ and $\partial \ell_2/\partial(1-\alpha)$ are of the same sign, it is the first bracketed expression in (21) that must be zero. The optimal weights are thus unique and given by (15)–(16).[13] ■

[13] Observe that this is an assumption regarding the *direction* of a change in leisure consumption to its price in the two periods and not their magnitudes. Increasing α lowers

The interesting feature of Proposition 2 is that the optimal weights are independent of preferences and interest rates. They are determined by population and productivity growth rates only and decline by age as long as population-plus-productivity growth rate is positive.

$$(1-\alpha)-\alpha=\frac{1-(1+g)(1+n)}{1+(1+g)(1+n)}<0.$$

Proposition 2 helps provide an intuitive explanation of this result. First, recall that (12) represents the constraint that must be satisfied in per capita terms by the young in order for the government's pay-as-you-go constraint to be satisfied. However, as we have seen earlier, each young person faces the budget constraint given by equation (4). These two equations describe two different constraints. The *individual's* budget constraint (4) is derived on the basis of a Nash-type behavioral assumption on the part of the individual. That is, he considers his retirement benefits, b, to be given by benefit formula (2) *independently* of his own actions (being only one among many individuals). Specifically, he considers the replacement factor μ to be fixed and independent of his own actions. The per capita young constraint (12), on the other hand, is derived on the basis of the fact that the government must satisfy its budget constraint *in the aggregate*, and μ is determined to ensure that this is the case. Of course, *in equilibrium*, *both* constraints must be satisfied. The important point is that if $\alpha=(1+g)(1+n)/[1+(1+g)(1+n)]$, then *the individual's budget constraint (4) will coincide with the per capita constraint (12).* Under this circumstance, the equilibrium is arrived at by maximization of the individual's utility subject to *one* constraint only, and not *two* constraints as in the general case. It is this coincidence of the individual's budget constraint with the per capita young feasibility constraint

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the effective net of tax wage in the first period and increasing $1-\alpha$ lowers the effective net of tax wage in the second period. Unless preferences are "very strange," the consumers' responses are bound to be similar in nature.

Indeed, we will prove later in Proposition 4 that if preferences are additively separable, $\partial\ell_1/\partial\alpha$ and $\partial\ell_2/\partial(1-\alpha)$ are both negative.

that is behind the optimality of $\alpha = (1+g)(1+n)/[1+(1+g)(1+n)]$.[14]

5 Optimal effective taxes

To gain a better intuitive understanding of our result, we next characterize the optimal effective tax rates implied by optimal weights (and a given value of θ.) This is done in Proposition 3.

Proposition 3 *The implied optimal replacement factor and the effective tax rates are given by,*

$$\begin{aligned} \mu^* &= (1+g)(1+n)[1+(1+g)(1+n)]\theta, &(22)\\ \left(\hat{\theta_1}\right)^* &= \left[1 - \frac{(1+g)^2(1+n)^2}{(1+r)^2}\right]\theta, &(23)\\ \left(\hat{\theta_2}\right)^* &= \left[1 - \frac{(1+g)(1+n)}{1+r}\right]\theta. &(24)\end{aligned}$$

Proof. Substitute the optimal weights in equation (8) and simplify. This yields

$$\left[\frac{\mu}{1+(1+g)(1+n)} - \theta(1+g)(1+n)\right][(1+n)(1-\ell_1)+(1-\ell_2)] = 0. \quad (25)$$

Equation (22) follows directly from the fact that $1-\ell_1$ and $1-\ell_2$ are positive. Substituting the resulting value of μ, as well as the optimal value of α, in equations (5a) and (5b) then proves (23)–(24). ■

From the expressions derived for $\left(\hat{\theta_1}\right)^*$ and $\left(\hat{\theta_2}\right)^*$ in (23)–(24), one sees immediately that they are both net subsidies if $(1+g)(1+n) > 1+r$, net taxes if $(1+g)(1+n) < 1+r$, and zero if $(1+g)(1+n) = 1+r$. This of course

[14] This argument is similar to the argument that a *compensated tax* scheme, where the tax proceeds are rebated to taxpayers in a way that they treat their compensations as *lump-sum*, lowers the taxpayers' welfare. A taxpayer's optimum under such a compensated tax scheme must not only satisfy his budget constraint but also the (per capita) feasibility constraint of the economy. The solution to this problem necessarily results in a utility level which is less than what can be achieved in the absence of the tax-cum-rebates (because of the substitution effect of the tax). Note also that without the tax the individual's budget constraint coincides with the economy's per capita feasibility constraint. [The compensated tax argument is due to Diamond (1970) where he also gives an insightful diagrammatic exposition of it (p. 215)].

has to do with the fact that whenever Samuelson's productivity-augmented biological rate of return exceeds the rate of interest, a pay-as-you-so social security system is a "good deal" (enhancing steady-state welfare even if ignoring the welfare of the generations living during the transition path). And whenever this rate falls short of the rate of interest, a pay-as-you-go system lowers the steady-state welfare and is a "bad deal". A good deal allows one to face a subsidy and a bad deal a tax (in every period).

Now consider the relationship between $\left(\hat{\theta}_1\right)^*$ and $\left(\hat{\theta}_2\right)^*$. It follows from (23)–(24) that,

$$\frac{\left(\hat{\theta}_1\right)^*}{\left(\hat{\theta}_2\right)^*} = 1 + \frac{(1+g)(1+n)}{1+r}.$$

This relationship is best understood if we differentiate between situations where the effective tax rates are negative (i.e. individuals receive net subsidies) and when they are positive so that individuals face a net tax.[15] When a subsidy, i.e. $(1+g)(1+n)/\,(1+r) > 1$, one wants the subsidy rate to be higher for earlier years by a factor more than twice that of later years. On the other hand, when a tax, i.e. $(1+g)(1+n)/\,(1+r) < 1$, one wants the tax rate to be higher for earlier years by a factor less than twice that of later years. Why? Either way, the idea is that optimality requires a *uniform* rate of return intertemporally (by choosing α). Investing in social security as opposed in real savings entails a rate of return of $(1+g)(1+n)/\,(1+r)$ for one period. The same uniform rate of return intertemporally requires a return of $[(1+g)(1+n)/\,(1+r)]^2$ after two periods. When a subsidy, this implies a subsidy rate of $(1+g)(1+n)/\,(1+r) - 1$ for one period and $[(1+g)(1+n)/\,(1+r)]^2 - 1$ for two periods. And when a tax, uniformity of the rate of return requires a tax rate of $1-(1+g)(1+n)/\,(1+r)$ for one period and $1 - [(1+g)(1+n)/\,(1+r)]^2$ for two periods.

[15]Recall that in this model, we take the value of θ as given and do not address the question of the size or optimality of having an unfunded social security system. So we can have positive effective tax rates.

What setting α optimally achieves, is to make the individual's "perceived" effective tax rates equal to the "actual" effective tax rates. The government levies a statuary tax to finance the system and signals the individuals how it calculates their future benefits [i.e. according to benefit formula (2)]. The individuals in turn calculate a net tax rate based on the statuary rate and the benefit formula. The tax is distortionary; but the government can minimize the distortion by giving the individuals the correct signals about its budget constraint. Sending the wrong signals will only create a second distortion with no offsetting benefits. The economy cannot gain by the government misinforming the individuals about its finances. It should just tell everyone how it has to balance its budget.

The force of the argument can most easily be seen when $(1 + g)(1 + n)/\,(1 + r) = 1$. In this case, as can be seen from (12), the *actual* effective tax rates facing taxpayers are zero. It is optimal for the individual to realize this and setting α optimally would achieve this by implying $\left(\hat{\theta}_1\right)^* = \left(\hat{\theta}_2\right)^* = 0$ regardless of the value of θ—an outcome that would not be possible if α is not chosen optimally. Under this circumstance, the statutory tax will have no effect and we have a first-best outcome.[16] On the other hand, if the individual perceives his tax rate to be non-zero which will be the case if α is not set optimally (i.e. if the benefit formula "misleads" him), then his utility will be reduced *notwithstanding* the fact that his actual net tax rates are zero. By setting α optimally, the government signals the individuals that their net tax rates are in fact zero.

This should also explain why α moves *positively* with population and productivity growth rates. With a growing and more productive population, the government will in fact collect more taxes from the younger generations to be handed in to the old. The benefit formula should reveal this infor-

[16] In a compensated tax scheme too *if* the individual knows that his compensation from the government is always set equal to his own tax payments rather than being lump-sum, he will internalize this information and the tax will have no effect. However, this can never be done.

mation. Our finding thus provides a justification for the current practice of wage indexing. It also puts forth an argument for "population indexing".

We must also point out that in our derivations we have assumed labor to be homogeneous and thus have made no distinction between wage rates for individuals of different ages. One can easily demonstrate that dropping this assumption will not change any of our results. In particular, the optimal value of α will remain unchanged.

6 Labor supply

In this section, we turn to a discussion of the labor supply response to the changes in earning weights in the benefit formula. In particular, we show that, assuming additive preferences and given a pay-as-you-go social security system, labor supply of the young increases while labor supply of the middle-aged decreases as more weight is given to the earlier years' earnings in calculating social security benefits.

The impact of changing α, the weight associated to first year earnings in the calculation of social security benefits, on the variables of our model can be derived by totally differentiating the system of equations consisting of (4), (6a)–(6d) and (8) with respect to α. The expressions that result are complex and impossible to sign. Nevertheless we can determine the direction of the labor supply response to the changes in α, if we assume additionally that preferences are additively separable in all goods. In this case, we have the following proposition which is proved in the Appendix.[17]

Proposition 4 *Assume that preferences are strictly convex and additively separable. Then a marginal increase in the weight assigned to the earnings of the young in the benefit formula of a pay-as-you-go social security program (and away from the weight assigned to the earnings of the middle-aged),*

[17] The separability assumption is employed for the purpose of this proposition only. It will be dropped in the rest of the paper.

evaluated at the net tax rates of zero, has the following consequences:

$$\frac{\partial \ell_1}{\partial \alpha} < 0, \tag{26}$$

$$\frac{\partial \ell_2}{\partial \alpha} > 0. \tag{27}$$

Proposition 4 indicates that as more weight is placed on the earnings of the earlier years, in calculating the social security benefits, the labor supply of the young would increase while the labor supply of the middle-aged would decline. The result may be explained intuitively. An increase in α, *ceteris paribus*, reduces $\hat{\theta}_1$ (the effective tax rate in the first period) and increases $\hat{\theta}_2$ (the effective tax rate in the second period). Given that the tax is "compensated" (in the sense that one receives retirement benefits for one's tax contributions), there will essentially be only a "substitution effect" here.[18] Consequently, labor supply increases in the first period and decreases in the second period unambiguously.

7 Concluding remarks

It is now almost universally accepted that the purpose of social security programs is to ensure that people will not experience a marked drop in their standard of living when they retire and that no person will find himself destitute in his old age. That these objectives may or may not be met in the absence of government intervention will likely always remain controversial—at least in the academic and policy circles. Uncertainty and asymmetric information surrounding one's longevity provide ample reasons for failure in insurance markets including social insurance. However, one should not underestimate the role that paternalism and myopic behavioral assumptions play in over-emphasizing such market failures.[19] The existence, extent, or absence of altruism, from children to parents and vice versa, as well as the

[18] The terms "compensated" and "substitution effect" are used rather loosely here. They are not the traditionally defined concepts associated with a change in tax rates.

[19] Feldstein (1985) discusses the optimal level of social security benefits under complete and partial myopia.

nature and reason for altruistic behavior also play a crucial role.[20] Regardless, social security reform cannot succeed without convincing the public that the reform will not harm the basic tenet of the program. Nor can a reform succeed if it does not adequately address the entitlement sensibilities of the public. It is natural that the program's history and the recipients' or would-be recipients' contributions to it would lead to formation of such feelings.

These requirements for a successful reform still leave a lot of room for disagreement and debate. Everyone would agree that a "basic level" of benefits should be guaranteed to all retirees; but to what extent? There is also widespread agreement on a "supplementary level" of benefits that would be related to one's contribution; but how strong should this link be? A tight link between benefits and contributions, a so called "Bismarckian system," or weak or non-existent link, a "Beveridgean system," have different implications for labor supply, retirement decision, and savings. They will also have different implications for intra-generational transfers. The incentives embedded in a system are extremely important considerations.

Who should provide the basic and supplementary pensions? Should it be the government or will it suffice if the government only mandates the purchase of such types of insurance? In the latter case, should the insurance be provided through one's employer or directly purchased from the private sector? What type of risk should be covered? Will longevity alone do or should we include disability and survivor's benefits as well? How should one set the level of benefits in the first year of retirement and how should lifetime benefits be indexed? What should be the retirement age for "full benefits"? What should be the minimum age for retirement? How should benefits be adjusted with the retirement age? Should we follow an actuarially fair scheme? Should we punish people for leaving the workforce early and/or

[20] See Cremer et al.'s (2016) contribution in this issue for a discussion of two-sided altruism in the context of the provision of long-term care insurance.

reward them for staying longer?

In the context of savings, the important question is what happens to the contributions. Will they be invested in real capital as in a pre-funded system, or will they be used to finance the retirees' pensions as in a pay-as-you-go system? This is not just the question of the so-called "Aaron inequality", namely that the real rate of interest exceeds the sum of growth rates of population and wage productivity. Aaron inequality implies that the rate of return on pre-funded systems is higher than on pay-as-you-go systems. This is of course a good thing. Additionally, however, translating savings into real investment increases capital accumulation and raises labor productivity and real wages.

Another aspect of a pay-as-you-go system is its inter-generational transfers in favor of the older generations at the time of instituting the program. The other side of this equation is the reverse redistribution that will occur as a result of changing an already existing pay-as-you-go system into a fully-funded system. Such a reform implies that the generations on the transitional path will have to pay not only for their own retirement benefits but also for the existing unfunded liabilities of the system. Whether or not the switch to a fully-funded system can be organized in such a way that these individuals are not hurt lies at the heart of the debate on the privatization of social security. For this to be possible, the expected extra rate of return on equities, and the benefits of higher capital accumulation and higher growth in labor productivity, must more than offset the double payments during the transition path.

Another question is that of defined contribution versus defined benefit systems. Currently, there is a trend to move away from the latter into the former type of scheme. A defined contribution system commits to a certain contribution schedule and adjusts benefits. A defined benefit system commits to a particular benefit schedule and adjusts contributions to keep financial balance. With defined contribution, one faces one's own investment risk. With defined benefit, the following generation faces the average risk. This is clearly important in the short run; but it has long term consequences as well. One might think that the two systems will have the same long term rates of return because both will have to be sustainable in the long run. However, the incentive effects of who bears the risk are crucially important in determining the long term rates of returns.

A burning issue, when it comes to the question of reform, is the demographic changes that are taking place (particularly in Europe and Japan). Increasing longevity and decreasing birth rates point to a future where old-age dependency ratio—ratio of people who are 65 years or older to those between 15 and 64—will drastically increase.[21] This also brings in the question of endogeneity of fertility and population growth rate which is treated as exogenous in this paper.[22]

This paper has ignored these pertinent issues; Feldstein (2002) provides an excellent and detailed discussion. Instead, it has concentrated on a simple reform that is both feasible (even politically) and costless. It has examined the labor supply effects of the linkage between the benefits of a pay-as-you-go social security program and the payroll taxes that finance them. It has also investigated the nature of the optimal linkage. We have demonstrated that as more weight is placed on earnings when young (in the benefit calculation), labor supply by the young increases while labor supply by the middle-aged decreases. Regarding optimal linkage, our main conclusion is that the weights that must be placed on earnings of different periods move positively with population and productivity growth rates.

The framework of this paper has been a three-period overlapping generations model in which gross factor returns were assumed fixed. Like any other research project, our results are based on the model employed. Fu-

[21] Projections from year 2000 to year 2050 indicate an increase from 18.5% to 40.9% for Canada, 26.9% to 42.1% for Sweden, 25.5% to 66.5% for Japan, 23.8% to 49.1% for Germany, 23.8% to 41.9% for UK, and 26.6% to 61.3% for Italy. The projected rate for the US is smaller and stands varies from 19.1% to 33.9%. See Penner (2007).

[22] See Gahvari (2009) for a survey.

ture research should concentrate on examining two natural extensions of our present study. On a theoretical level, it would be interesting to investigate how our results may change if the factor returns are not constant. The factor returns may indeed change as the level of capital intensity in the economy changes. And our paper has shown that, as the weights associated with the tax contributions of different periods change, the intertemporal leisure-consumption patterns change, and thus capital intensity changes.

The second avenue of research is empirical. For policy purposes, it is essential to have an idea of the magnitude of the suggested changes on labor supply, capital intensity, factor returns and welfare. It is only then that one may know with some degree of confidence how, if at all, the weights of different periods should be adjusted from their present uniform level. This in turn requires building a large scale multi-period simulation model. We leave both of these lines of research to future studies.

Appendix

Rewrite equation (8) as

$$\mu\left[\alpha(1-\ell_1)+(1-\alpha)(1+g)(1-\ell_2)\right]=\theta(1{+}g)^2\,(1+n)\left[(1+n)\,(1-\ell_1)+(1-\ell_2)\right].$$

Differentiating this equation partially with respect to α and "solving" it for $\partial\mu/\partial\alpha$, yields

$$\begin{aligned}\frac{\partial\mu}{\partial\alpha} &= \frac{\mu\left[(1+g)(1-\ell_2)-(1-\ell_1)\right]-\left[\theta(1+g)^2(1+n)^2-\alpha\mu\right](\partial\ell_1/\partial\alpha)}{\alpha(1-\ell_1)+(1-\alpha)(1+g)(1-\ell_2)} \\ &- \frac{\left[\theta(1+g)(1+n)-(1-\alpha)\mu\right](1+g)\,(\partial\ell_2/\partial\alpha)}{\alpha(1-\ell_1)+(1-\alpha)(1+g)(1-\ell_2)}. \end{aligned} \tag{A.1}$$

Next, totally differentiate equations (4) and (6a)–(6d) partially with respect to α while utilizing (5a)–(5b) and allowing μ to adjust according to (A.1), so that equation (8) is always satisfied. After a bit of algebraic manipulations, we have

$$\mathbf{Z}\begin{pmatrix}\frac{\partial c_1}{\partial\alpha}\\ \frac{\partial\ell_1}{\partial\alpha}\\ \frac{\partial c_2}{\partial\alpha}\\ \frac{\partial\ell_2}{\partial\alpha}\\ \frac{\partial c_3}{\partial\alpha}\end{pmatrix}=\begin{pmatrix}\frac{-\mu w(1+g)u_1(1-\ell_2)}{(1+r)^2[\alpha(1-\ell_1)+(1-\alpha)(1+g)(1-\ell_2)]}\\ 0\\ \frac{\mu w(1+g)u_1(1-\ell_1)}{(1+r)^2[\alpha(1-\ell_1)+(1-\alpha)(1+g)(1-\ell_2)]}\\ 0\\ 0\end{pmatrix}, \tag{A.2}$$

where $u_1\equiv\partial u/\partial c_1$, and $\mathbf{Z}$ is a 5×5 matrix whose elements are given by

$$\begin{aligned}
z_{11} &= w(1-\hat{\theta}_1)u_{11}-u_{21}\\
z_{12} &= w(1-\hat{\theta}_1)u_{12}-u_{22}-\frac{\alpha w u_1}{(1+r)^2}\cdot\frac{\theta(1+g)^2(1+n)^2-\alpha\mu}{\alpha(1-\ell_1)+(1-\alpha)(1+g)(1-\ell_2)}\\
z_{13} &= w(1-\hat{\theta}_1)u_{13}-u_{23}\\
z_{14} &= w(1-\hat{\theta}_1)u_{14}-u_{24}-\frac{\alpha w(1+g)u_1}{(1+r)^2}\cdot\frac{\theta(1+g)(1+n)-(1-\alpha)\mu}{\alpha(1-\ell_1)+(1-\alpha)(1+g)(1-\ell_2)}\\
z_{15} &= w(1-\hat{\theta}_1)u_{15}-u_{25}\\
z_{21} &= \frac{u_{11}}{1+r}-u_{31}\\
z_{22} &= \frac{u_{12}}{1+r}-u_{32}
\end{aligned}$$

$$
\begin{aligned}
z_{23} &= \frac{u_{13}}{1+r} - u_{33} \\
z_{24} &= \frac{u_{14}}{1+r} - u_{34} \\
z_{25} &= \frac{u_{15}}{1+r} - u_{35} \\
z_{31} &= \frac{w(1+g)(1-\hat{\theta}_2)}{1+r} u_{11} - u_{41} \\
z_{32} &= \frac{w(1+g)(1-\hat{\theta}_2)}{1+r} u_{12} - u_{42} - \frac{(1-\alpha)w(1+g)u_1}{(1+r)^2} \cdot \frac{\theta(1+g)^2(1+n)^2 - \alpha\mu}{\alpha(1-\ell_1)+(1-\alpha)(1+g)(1-\ell_2)} \\
z_{33} &= \frac{w(1+g)(1-\hat{\theta}_2)}{1+r} u_{13} - u_{43} \\
z_{34} &= \frac{w(1+g)(1-\hat{\theta}_2)}{1+r} u_{14} - u_{44} - \frac{(1-\alpha)w(1+g)^2u_1}{(1+r)^2} \cdot \frac{\theta(1+g)(1+n) - (1-\alpha)\mu}{\alpha(1-\ell_1)+(1-\alpha)(1+g)(1-\ell_2)} \\
z_{35} &= \frac{w(1+g)(1-\hat{\theta}_2)}{1+r} u_{15} - u_{45} \\
z_{41} &= \frac{u_{11}}{(1+r)^2} - u_{51} \\
z_{42} &= \frac{u_{12}}{(1+r)^2} - u_{52} \\
z_{43} &= \frac{u_{13}}{(1+r)^2} - u_{53} \\
z_{44} &= \frac{u_{14}}{(1+r)^2} - u_{54} \\
z_{45} &= \frac{u_{15}}{(1+r)^2} - u_{55} \\
z_{51} &= 1 \\
z_{52} &= w\left[1-\theta+\frac{\theta(1+g)^2(1+n)^2}{(1+r)^2}\right] \\
z_{53} &= \frac{1}{1+r} \\
z_{54} &= \frac{w}{1+r}\left[1-\theta+\frac{\theta(1+g)(1+n)}{1+r}\right] \\
z_{55} &= \frac{1}{(1+r)^2}.
\end{aligned}
$$

Note that in above $u_{ij} \equiv \partial^2 u/\partial i \partial j$, where i and j run from 1 to 5 and denote c_1, ℓ_1, c_2, ℓ_2 and c_3 respectively.

Premultiplying (A.2) by $\mathbf{Z}^{-1}$, yields

$$\begin{pmatrix} \frac{\partial c_1}{\partial \alpha} \\ \frac{\partial \ell_1}{\partial \alpha} \\ \frac{\partial c_2}{\partial \alpha} \\ \frac{\partial \ell_2}{\partial \alpha} \\ \frac{\partial c_3}{\partial \alpha} \end{pmatrix} = \frac{1}{|\mathbf{Z}|} \begin{pmatrix} Z_{11} & Z_{21} & Z_{31} & Z_{41} & Z_{51} \\ Z_{12} & Z_{22} & Z_{32} & Z_{42} & Z_{52} \\ Z_{13} & Z_{23} & Z_{33} & Z_{43} & Z_{53} \\ Z_{14} & Z_{24} & Z_{34} & Z_{44} & Z_{54} \\ Z_{15} & Z_{25} & Z_{35} & Z_{45} & Z_{55} \end{pmatrix} \begin{pmatrix} \frac{-\mu w(1+g)u_1(1-\ell_2)}{(1+r)^2[\alpha(1-\ell_1)+(1-\alpha)(1+g)(1-\ell_2)]} \\ 0 \\ \frac{\mu w(1+g)u_1(1-\ell_1)}{(1+r)^2[\alpha(1-\ell_1)+(1-\alpha)(1+g)(1-\ell_2)]} \\ 0 \\ 0 \end{pmatrix}, \tag{A.3}$$

where $|\mathbf{Z}|$ is the determinant of $\mathbf{Z}$ and Z_{ij} is the cofactor of z_{ij}. From (A.3) we will then have

$$\frac{\partial \ell_1}{\partial \alpha} = \frac{-\mu w(1+g)u_1}{(1+r)^2|\mathbf{Z}|} \left(\frac{(1-\ell_2)Z_{12} - (1-\ell_1)Z_{32}}{[\alpha(1-\ell_1)+(1-\alpha)(1+g)(1-\ell_2)]} \right), \tag{A.4a}$$

$$\frac{\partial \ell_2}{\partial \alpha} = \frac{-\mu w(1+g)u_1}{(1+r)^2|\mathbf{Z}|} \left(\frac{(1-\ell_2)Z_{14} - (1-\ell_1)Z_{34}}{[\alpha(1-\ell_1)+(1-\alpha)(1+g)(1-\ell_2)]} \right). \tag{A.4b}$$

Now assume that preferences are additively separable so that $u_{ij} = 0$ for $i \neq j$. In that case, evaluating (A.4a)–(A.4b) at $\hat{\theta_1} = \hat{\theta_2} = 0$ results in the following two expressions

$$\frac{\partial \ell_1}{\partial \alpha} = \left\{ \frac{w^2 u_{11}u_{33}u_{55}}{1+r}(1-\ell_1) + \left[\frac{u_{11}u_{44}u_{55}}{(1+r)^2} + \frac{w^2 u_{11}u_{33}u_{55}}{(1+r)^2} + \frac{u_{11}u_{33}u_{44}}{(1+r)^3} + u_{33}u_{44}u_{55} \right] (1-\ell_2) \right\} / D \tag{A.5a}$$

$$\frac{\partial \ell_2}{\partial \alpha} = -\left\{ \left[w^2 u_{11}u_{33}u_{55} + u_{22}u_{33}u_{55} + \frac{u_{11}u_{22}u_{55}}{(1+r)^2} + \frac{u_{11}u_{22}u_{33}}{(1+r)^4} \right] (1-\ell_1) + \frac{w^2 u_{11}u_{33}u_{55}}{1+r}\,(1-\ell_2) \right\} / D, \tag{A.5b}$$

where

$$D \equiv (1+r)^2 \left[\alpha(1-\ell_1) + (1-\alpha)(1+g)(1-\ell_2)\right] \left\{ w^2 u_{11}u_{33}u_{44}u_{55} + \frac{u_{11}u_{22}u_{44}u_{55}}{(1+r)^2} + u_{22}u_{33} \left[u_{44}u_{55} + \frac{w^2 u_{11}u_{55}}{(1+r)^2} + \frac{u_{11}u_{44}}{(1+r)^3} \right] \right\} \Big/ \mu w(1+g)u_1. \tag{A.6}$$

The signs of $\partial\ell_1/\partial\alpha$ and $\partial\ell_2/\partial\alpha$ can then easily be established by making use of the fact that convexity and additivity of preferences imply diminishing marginal utility for all goods so that $u_{ii} < 0$ for $i = 1, 2, \cdots, 5$.

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