Energy taxes under a quadratic almost ideal demand system

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Abstract

This paper uses 1996–1999 Consumer Expenditure Survey data to estimate a Quadratic Almost Ideal Demand System (QUAIDS) consisting of leisure, energy, and non-energy consumption goods for the US. It finds that leisure and energy are (gross) substitutes so that energy must be subsidized relative to non-energy goods in an optimized tax system. The paper also considers the welfare implications of revenue-neutral energy taxes whose proceeds are rebated uniformly to consumers keeping income tax rates unchanged. It finds that, despite its environmental benefits, a case for an energy tax can be made only if the society cares a great deal about inequality. Otherwise, energy taxes will be detrimental to the society’s welfare.

JEL classification: H21, H23

Keywords: Energy taxes, inequality aversion, leisure substitute, QUAIDS
1 Introduction

It is a well-known proposition in optimal tax theory that, everything else equal, at a second-best equilibrium leisure complements are taxed and leisure substitutes are subsidized (relative to one another). Corlett and Hague (1953) demonstrated this some sixty years ago within a Ramsey tax framework; but the proposition remains valid in second-best Mirrlesian settings as well.\(^1\) Intuitively, this works to mitigate the downward distortion on labor supply caused by taxing all consumption goods uniformly (or equivalently income taxation). Another way of looking at this is that taxing leisure complements generates additional income tax revenues for the government by boosting labor supply which then helps to lower the welfare loss associated with taxing the good in question.

This proposition notwithstanding, the applied literature on optimal taxation and tax reform has paid scant attention to estimating cross-price labor supply elasticities. Existing empirical studies typically assume that preferences are weakly separable in labor and goods. This assumption imposes severe restrictions on the values these elasticities can take.\(^2\) Nor is it supported by the papers that test for it.\(^3\) Under this circumstance, the estimates from a system which imposes separability are biased and inconsistent.

Two recent papers by West and Williams (2004, 2007) break from this tradition, directly estimate the cross-price elasticity with leisure, and use it to calculate the optimal tax rate on gasoline as well as studying the incidence of a gasoline tax.\(^4\) These papers use the 1996–1998 Consumer Expenditure Surveys, along with state-level price

\(^1\)See, Christiansen (1984) and more recently Kaplow (2010).

\(^2\)If one further assumes that the goods-subutility is homothetic, labor supply will have the same cross-price elasticity with all goods; see Sandmo (1974). This implies that all goods should be taxed at the same rate in a Ramsey setting. Without homotheticity, the uniformity result follows as long as incomes can be taxed nonlinearly; see Atkinson and Stiglitz (1976). See, e.g., LaFrance (1993) on the prevalence of the weak-separability assumption in applied welfare analysis.

\(^3\)See, e.g., Barnett (1979), Blundell and Walker (1982), Murphy and Thom (1986), and Browning and Meghir (1991).

\(^4\)Two earlier studies that estimate the cross-price elasticity with leisure are Madden (1995) and Diewert and Lawrence (1996); they are discussed in West and Williams (2007).
information from the American Chambers of Commerce Researchers’ Association, for their estimation. Given their focus, they divide goods into gasoline and “other”. Most importantly, they allow preferences to depend on leisure consumption as well in a non-separable way. The system they estimate is Deaton and Muellbauer’s (1980) Almost Ideal Demand System (AIDS). This provides a satisfactory estimation framework in that it is based on a well-defined indirect utility function that imposes neither separability nor homotheticity. They find that gasoline is a relative complement to leisure. This suggests that gasoline should be taxed relative to other goods (despite its generally being considered as regressive). This is of course on top of the tax component required for correcting its environmental damage.

Yet the AI demand system implies, rather implausibly, that all goods have Engel curves that vary linearly with the log of expenditures. Empirical Engel curves, on the other hand, often indicate relationships that are nonlinear. Simple quadratic polynomial regressions of budget shares on the log of expenditures in our data also point to nonlinearities. To allow for nonlinearity, we use Banks et al.’s (1997) Quadratic Almost Ideal Demand System (QUAIDS) for our estimation procedure. This is a generalization of Deaton and Muellbauer’s AIDS system with an underlying indirect utility function. As with West and Williams, we consider a 2-good demand system plus leisure. However, rather than focusing solely on gasoline, we concentrate on the tax treatment of energy goods in general. These are goods whose consumption entail some environmental damage. The composite energy good consists of gasoline consumption and household energy consumption—electricity, natural gas or home heating fuels and oils. The difference between total consumption and energy consumption constitutes a composite “clean good”. Leisure consumption is based on a time endowment of 16 hours per day per spouse in

\footnote{See Table 2.}

\footnote{QUAIDS is also preferable to other commonly used specifications. Log-linear models do not satisfy theory exactly, the linear expenditure system is overly restrictive, and flexible functional forms lead to representations of utility functions only approximately.}
Household consumption data is from the 1996-1999 Consumer Expenditure Survey. The Interview Survey component reports expenditures on various categories of goods, which allows us to create the composite energy good. In addition, the CEX reports household employment information, which allows us to calculate the household net wage rate. The marginal income tax rates are calculated using the NBER TaxSim program. The price for the clean and energy goods comes from the Bureau of Labor Statistics.

Our sample is separated based on marital status: single households and married households. We focus on the labor supply of the adults in the household; for married households this would be the respondent and their spouse. Since some adults choose to work zero hours, this leads to a sample selection problem which is controlled for by using the Heckman selection procedure to correct net wage rates. We then iteratively estimate the QUAIDS model using a three-stage least squares procedure. The results indicate that Engel curves are nonlinear for the clean good and leisure for single households, while only leisure (male) is nonlinear for married households.

The cross elasticity estimates find labor supply and the energy good to be complements. The uncompensated elasticity of labor supply with respect to the energy good price is \(-0.0090\) for single households, and \(-0.0013\) for males and \(-0.0220\) for females in married households. According to the Corlett-Hague rule then the energy good should be subsidized rather than taxed. Earlier, West and Williams (2004, 2007) had found positive values for the uncompensated elasticity of labor supply with respect to the price of gasoline. When re-estimating our model restricting the energy good to gasoline only, we also find positive values for the cross-price elasticities.

Putting our demand estimates to work, we evaluate the welfare implications of a change in the energy tax via a budget neutral tax reform. The exercise treats income tax rates as fixed, rebating the additional tax proceeds uniformly to all households. In line with our finding that energy and labor supply are complements, we find that energy
taxes may in fact be harmful to the economy despite their environmental benefits. This is the case even though the tax reform we are considering is redistributive towards the poor (who consume less energy than the rich and thus pay less in taxes but receive an equal share of the proceeds). Only if the society cares a great deal about inequality, energy tax reforms of this sort are welfare improving.

2 The model

The economy is populated with households who have identical tastes but different income levels. Households are either “single,” consisting of one potential working adult, or “married,” consisting of two potential working adults. The potential working adult in the single household is labeled the “primary” worker regardless of his gender and employment status. The potential working male in the married household is labeled the primary worker and the potential working female the “secondary” worker.\(^7\) Every adult in the household has one unit of time which he divides between working in the market or “leisure”.\(^8\)

Denote the labor supply by \(L\), leisure consumption by \(l\), time endowment by \(T\), and the net-of-tax wage by \(w\). Distinguish between variables pertaining to the primary and secondary workers through subscripts \(p\) and \(s\). To economize on notation, let \(L = L_p, l = l_p, T = T_p, w = w_p\) for single households and \(L = (L_p, L_s), l = (l_p, l_s), T = (T_p, T_s), w = (w_p, w_s)\) for married households. In a similar fashion, denote households’ non-labor income by \(m = m_p\) for single households and \(m = m_p + m_s\) for married households. Observe that \(m\) includes the “virtual income” required for linearizing the household’s budget constraint. Also denote consumption goods by \(x = (x_1, x_2, \ldots, x_n)\) and their corresponding consumer prices by \(p = (p_1, p_2, \ldots, p_n)\). The household’s lin-

\(^7\)This labeling is for convenience only and has no other connotation.
\(^8\)Not all the time one spends outside the work place can realistically be termed “leisure”. One has to sleep and perform some minimal everyday tasks. We assume these other functions take some fixed amount of time and deduct that from the 24 hours one has in a day to arrive at his “time endowment”. See Section 3 for more detail.
earized budget constraint is given by

\[ px = wL + m, \]

which one can rewrite as

\[ px + wl = wT + m \equiv I. \tag{1} \]

Observe that \( I = w_p T_p + m_p \) for single households and \( I = w_p T_p + m_p + w_s T_s + m_s \) for married households. We shall refer to \( I \) as “potential income”.

Households have preferences over \((x, I)\) and \(E\), the total level of emissions generated by consuming “energy goods”—a subset of consumption goods. We assume that preferences are separable in \((x, I)\) and \(E\) so that the non-emission component of preferences can be represented by the indirect utility function \( v = v(w, p, I) \).\(^9\) We further assume that this component subscribes to the Quadratic Almost Ideal Demand System (QUAIDS) introduced by Banks et al. (1997). The advantage of this formulation is that it allows Engel curves to vary with \( \ln I \) linearly for some goods and nonlinearly for others—a property often displayed by empirical Engel curves.\(^10\)

Thus

\[ \ln v = \left\{ \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-1} + \lambda(w, p) \right\}^{-1}, \tag{2} \]

\(^9\)To avoid cluttered notation, references to households are suppressed. However, it is clear that \( v, w, I, \) etc. differ across households \( h \).

\(^10\)If \( \lambda_i = 0 \), for all \( i = 1, 2, \ldots, n + 2 \), the indirect utility function (2) will be reduced to Deaton and Muellbauer’s (1980) Almost Ideal Demand System. In this case, Engel curves will be linear in \( \ln I \).
where, with married households, 

\[
\ln a(w, p) \equiv \alpha_0 + \alpha_p \ln w_p + \alpha_s \ln w_s + \sum_{i=1}^{n} \alpha_i \ln p_i + \ln w_p \sum_{i=1}^{n} \gamma_{ip} \ln p_i + \ln w_s \sum_{i=1}^{n} \gamma_{is} \ln p_i \\
+ \frac{1}{2} \left[ \gamma_{pp} (\ln w_p)^2 + 2 \gamma_{ps} (\ln w_p) (\ln w_s) + \gamma_{ss} (\ln w_s)^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln p_i \ln p_j \right],
\]

(3)

\[
b(w, p) \equiv (w_p)^{\beta_p} (w_s)^{\beta_s} \prod_{i=1}^{n} p_i^{\beta_i},
\]

(4)

\[
\lambda(w, p) \equiv \lambda_p \ln w_p + \lambda_s \ln w_s + \sum_{i=1}^{n} \lambda_i \ln p_i,
\]

(5)

with \(\alpha_0, \alpha_p, \alpha_s, \alpha_i, \beta_p, \beta_s, \beta_i, \lambda_p, \lambda_s, \lambda_i, \gamma_{ip}, \gamma_{is}, \gamma_{pp}, \gamma_{ps}, \gamma_{ss}\), and \(\gamma_{ij} (i, j = 1, 2, \ldots, n)\) being constants. With single households, equations (3)–(5) continue to apply as well with the further stipulation that \(\alpha_s = \beta_s = \lambda_s = \gamma_{is} = \gamma_{ps} = \gamma_{ss} = 0\). Let \(n + 1\) stand for subscript \(p\) and \(n + 2\) for \(s\). Imposing restrictions \(\sum_{i=1}^{n+2} \gamma_{ij} = \sum_{j=1}^{n+2} \gamma_{ij} = 0\), \(\sum_{i=1}^{n+2} \beta_i = \sum_{i=1}^{n+2} \lambda_i = 0\) and \(\sum_{i=1}^{n+2} \alpha_i = 1\) on the parameters of (3)–(5) ensures the demand system’s homogeneity of degree zero in income and prices, and its adding-up property. The symmetry restriction, of the Slutsky matrix, requires \(\gamma_{ij} = \gamma_{ji}\), for all \(i \neq j = 1, 2, \ldots, n + 2\). This is also imposed on the estimated parameters.

With the QUAIDS specification, as well as AIDS, it is simpler to estimate the goods’ expenditure shares rather than their quantity demanded. We have, from Roy’s identity, for consumption good \(i\) and leisure of primary and secondary workers \(k = p, s\),

\[
\omega_i \equiv \frac{p_i x_i}{I} = \frac{p_i}{I} \left( \frac{-\partial v/\partial p_i}{\partial v/\partial I} \right) = -\frac{p_i}{I} \frac{\partial \ln v}{\partial p_i},
\]

\[
\omega_k \equiv \frac{w_k l_k}{I} = \frac{w_k}{I} \left[ \frac{-\partial v/\partial w_k}{\partial v/\partial I} \right] = -\frac{w_k}{I} \frac{\partial \ln v}{\partial w_k},
\]

(6)

(7)

where \(\omega_i\) denotes the expenditure share for good \(i = 1, 2, \ldots, n\) and \(\omega_k, k = p, s\), denotes the share of the primary and secondary workers’ leisure in a household’s budget. Partially differentiate \(\ln v\) with respect to \(p_i, I, w_k\), and simplify through equations (3)–(5);
then substitute the resulting partial derivatives in equations (6)–(7) and simplify. One arrives at, for $i = 1, 2, \ldots, n$ and $k \neq e = p, s$,

$$
\omega_i = \alpha_i + \gamma_{ip} \ln w_p + \gamma_{is} \ln w_s + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln \frac{I}{a(w, p)} + \frac{\lambda_i}{b(w, p)} \left[ \ln \frac{I}{a(w, p)} \right]^2,
$$

(8)

$$
\omega_k = \alpha_k + \gamma_{kk} \ln w_k + \gamma_{ke} \ln w_e + \sum_{j=1}^{n} \gamma_{kj} \ln p_j + \beta_k \ln \frac{I}{a(w, p)} + \frac{\lambda_k}{b(w, p)} \left[ \ln \frac{I}{a(w, p)} \right]^2.
$$

(9)

### 3 Data and the Engel curves

The data comes from the Interview Survey component of the Consumer Expenditure Survey (CEX) covering the period 1996–1999. This is a quarterly data set that tracks households for four consecutive quarters. Although each household may appear in as many as four quarters, this is not a long enough time frame to create a panel. Therefore the sixteen quarters are pooled to form a cross-sectional data set.\textsuperscript{11} The unit of analysis in the survey is the household. To work with a homogeneous population in terms of the labor-leisure decision, we restrict the data to households where each potential working adult is between the ages 18–65. Nor do we include, for the same reason, households whose occupation codes appear as armed forces, self-employed, farming, forestry, or fishing. This yields a sample size of 28,367 households. Of these, 14,241 are single households and 14,126 are married households.

The CEX reports both total household expenditures and detailed expenditures on certain categories of goods such as food, apparel, entertainment, housing and transportation. It also reports detailed information on the actual items that comprise each category. We use these to create two broad categories of “energy” and “clean” goods. The first comprises all household expenditures on electricity, natural gas, home heating

\textsuperscript{11} Chapter 16 of the Bureau of Labor Statistics Handbook of Methods provides a brief description of the Consumer Expenditure Survey.
fuels and oils, and gasoline. The other is found as the difference between total household expenditures and energy good expenditures. We use week as our unit of time and thus convert the CEX figures, which are quarterly, to a weekly basis by assuming thirteen weeks in a quarter.\(^{12}\) The expenditure on leisure is the product of leisure consumption and the net wage. To compute leisure hours, we assign a time endowment of 16 hours per day and 5 days per week for a total of 80 hours to the potential working adults in the household, regardless of employment status.\(^{13}\) Subtracting the working hours, which the CEX also reports, from the time endowment yields leisure hours per week.

Turning to the calculation of the net wage, we first calculate a gross hourly wage for each spouse based on annual salary information, hours worked per week, and weeks worked per year; all reported by the CEX. To translate this into a net wage, one requires the household’s marginal income tax rate. We use the NBER TaxSim program to calculate the household’s effective federal, state, and FICA marginal income tax rates.\(^{14}\) This effective marginal tax rate is used to calculate an hourly net wage for each spouse.\(^{15}\)

Data on prices comes from the Bureau of Labor Statistics (BLS). The BLS has an “all items less energy,” price index, which we use for the clean good price, and an

\(^{12}\)There are 1,027 single households and 81 married households who report zero energy expenditures. This cannot be correct. Given that these households are, with few exceptions, renters, the most likely explanation for this zero reporting is that their rent must have included utilities. Not knowing their actual energy expenditures, we drop these households from our data.

\(^{13}\)An individual’s time can be separated into four different components: taxable work or labor, leisure, sleep, and non-taxable work. This last category consists of activities such as commuting to work, household chores, and other tasks such as grocery shopping etc. (Paid maid work is not a household chore.) It is unlikely that one has much flexibility with adjusting the time one spends on such activities or on sleeping. The 16 hours a day time allotment thus assumes that one spends 8 hours a day on sleeping and non-taxable work. This being somewhat uncertain, we use several different time endowments (12 hours to 18). There is no significant change in the results due to time endowment variations.

\(^{14}\)www.nber.org/taxsim

\(^{15}\)Denote the marginal income tax rates household \(h\) faces by \(\theta^h_p\) for the primary worker and by \(\theta^h_s\) for the secondary worker. Denoting their corresponding gross-of-tax wage rates by \(w^h_p\) and \(w^h_s\), we have \(w^h_p \equiv w^h_p (1 - \theta^h_p)\) and \(w^h_s = w^h_s (1 - \theta^h_s)\). In conformity with our notation, \(\Theta^h = (\theta^h_p, \theta^h_s)\), \(\text{wg}^h = (w^h_p, w^h_s)\) for single households and \(\Theta^h = (\theta^h_p, \theta^h_s, \theta^h_s)\), \(\text{wg}^h = (w^h_p, w^h_s, w^h_s)\) for married households.
“energy” price index, which we use for the energy good price. The indices are divided by 100 so they represent a dollar price. Both price indices are national indices reported on a monthly basis. We calculate their three month averages to correspond to the household’s three-month reporting period in the CEX.

Table 1 reports sample statistics for both subsamples. Single households spend almost half of potential income on consumption of the clean good, 46.13%, while married households spend slightly over one-third, 35.00%. Leisure consumption is also a sizable expenditure. Leisure expenditures consist of 50.12% of potential income for single households and 62.26% for married households. For both subsamples, expenditure on the energy good never exceeds 4% (3.75% for single households and 2.73% for married households).

The effective marginal tax rate reported in the tables combines the federal, state and FICA tax rates. The average marginal tax rate for single households is 28.82% and for married households is 36.62% for males and 29.26% for females. The hourly net wage is $7.40 for single households and $9.53 and $7.08 for the male spouse and female spouse in married households.

### 3.1 Engel curves

As a first step to examining whether or not a linear specification for Engel curves is appropriate, we estimate simple quadratic polynomial regressions. Each of the three goods is first regressed on ln of potential income (linear specification), and then regressed

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16 Appendix 3, Chapter 17 of the BLS Handbook of Methods lists the components of various aggregate price indices. The “energy” price index is comprised of gasoline, electricity, natural gas, and home heating fuels and oils.

17 Unlike the ACCRA Coli price indices that vary both by state and time, the BLS price indices vary only by time. Despite this limitation, we have opted for BLS prices because the Coli price index for energy does not include the price of gasoline.

18 Our theoretical model for married households distinguishes between a “primary” worker and a “secondary” worker, where the primary worker is the higher earner. The empirical labor supply literature traditionally distinguishes between male labor supply and female labor supply. Consequently, we calculate labor supply elasticities for the male spouse and the female spouse rather than identifying the higher wage earner and labeling he/she the “primary” worker.
Table 1: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Single Households</th>
<th>Married Households</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Mean (M)</td>
<td>Std. Dev</td>
</tr>
<tr>
<td>Clean Good Exp ($)</td>
<td>343.55</td>
<td>(216.03)</td>
<td>433.28</td>
<td>(238.56)</td>
</tr>
<tr>
<td>Energy Good Exp ($)</td>
<td>25.74</td>
<td>(17.86)</td>
<td>31.69</td>
<td>(13.40)</td>
</tr>
<tr>
<td>Leisure Exp ($)</td>
<td>339.59</td>
<td>(143.13)</td>
<td>367.20</td>
<td>(165.43)</td>
</tr>
<tr>
<td>Total Expenditures ($)</td>
<td>708.88</td>
<td>(243.63)</td>
<td>1201.49</td>
<td>(356.16)</td>
</tr>
<tr>
<td>Clean Good Share (%)</td>
<td>46.13</td>
<td>(17.61)</td>
<td>35.00</td>
<td>(12.97)</td>
</tr>
<tr>
<td>Energy Good Share (%)</td>
<td>3.75</td>
<td>(2.42)</td>
<td>2.73</td>
<td>(1.09)</td>
</tr>
<tr>
<td>Leisure Share (%)</td>
<td>50.12</td>
<td>(18.00)</td>
<td>30.80</td>
<td>(10.57)</td>
</tr>
<tr>
<td>Clean Good Price ($)</td>
<td>1.69</td>
<td>(0.04)</td>
<td>1.69</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Energy Good Price ($)</td>
<td>1.07</td>
<td>(0.04)</td>
<td>1.07</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Hourly Gross Wage ($)</td>
<td>11.59</td>
<td>(4.93)</td>
<td>17.38</td>
<td>(14.53)</td>
</tr>
<tr>
<td>Marginal Tax Rate (%)</td>
<td>28.82</td>
<td>(22.56)</td>
<td>36.62</td>
<td>(15.06)</td>
</tr>
<tr>
<td>Hourly Net Wage ($)</td>
<td>7.40</td>
<td>(1.89)</td>
<td>9.53</td>
<td>(2.40)</td>
</tr>
<tr>
<td>Hours Worked (Wkly)</td>
<td>32.78</td>
<td>(18.93)</td>
<td>40.89</td>
<td>(16.06)</td>
</tr>
<tr>
<td>Hours Leisure (Wkly)</td>
<td>47.22</td>
<td>(18.93)</td>
<td>39.11</td>
<td>(16.06)</td>
</tr>
<tr>
<td>Ln(Clean Price) ($)</td>
<td>0.52</td>
<td>(0.03)</td>
<td>0.52</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Ln(Energy Price) ($)</td>
<td>0.07</td>
<td>(0.04)</td>
<td>0.07</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Ln(Net Wage) ($)</td>
<td>1.97</td>
<td>(0.25)</td>
<td>2.22</td>
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<td>Age</td>
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<td>41.14</td>
<td>(10.83)</td>
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<td>No HS Diploma (%)</td>
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<td>–</td>
<td>11.31</td>
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<td>25.55</td>
<td>–</td>
<td>27.70</td>
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<tr>
<td>Some College (%)</td>
<td>33.61</td>
<td>–</td>
<td>27.96</td>
<td>–</td>
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<tr>
<td>Bachelor's Degree (%)</td>
<td>20.20</td>
<td>–</td>
<td>21.39</td>
<td>–</td>
</tr>
<tr>
<td>Graduate Degree (%)</td>
<td>8.29</td>
<td>–</td>
<td>11.64</td>
<td>–</td>
</tr>
<tr>
<td>Male (%)</td>
<td>40.48</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>White (%)</td>
<td>76.79</td>
<td>–</td>
<td>87.43</td>
<td>–</td>
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<tr>
<td>Black (%)</td>
<td>18.67</td>
<td>–</td>
<td>7.53</td>
<td>–</td>
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<tr>
<td>Asian (%)</td>
<td>3.86</td>
<td>–</td>
<td>4.20</td>
<td>–</td>
</tr>
<tr>
<td>Other (%)</td>
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<td>–</td>
<td>0.84</td>
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<td>No Children (%)</td>
<td>74.49</td>
<td>–</td>
<td>39.81</td>
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<tr>
<td>One Child (%)</td>
<td>11.65</td>
<td>–</td>
<td>21.05</td>
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<tr>
<td>Two Children (%)</td>
<td>8.50</td>
<td>–</td>
<td>25.19</td>
<td>–</td>
</tr>
<tr>
<td>Three or More (%)</td>
<td>5.36</td>
<td>–</td>
<td>13.95</td>
<td>–</td>
</tr>
<tr>
<td>Own Home (%)</td>
<td>35.00</td>
<td>–</td>
<td>76.24</td>
<td>–</td>
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<tr>
<td>No Cars (%)</td>
<td>37.59</td>
<td>–</td>
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</tr>
<tr>
<td>One Car (%)</td>
<td>53.61</td>
<td>–</td>
<td>47.08</td>
<td>–</td>
</tr>
<tr>
<td>Two Cars (%)</td>
<td>7.44</td>
<td>–</td>
<td>29.20</td>
<td>–</td>
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<tr>
<td>Three or More (%)</td>
<td>1.36</td>
<td>–</td>
<td>6.94</td>
<td>–</td>
</tr>
<tr>
<td># of Observations</td>
<td>14,241</td>
<td>–</td>
<td>14,126</td>
<td>–</td>
</tr>
</tbody>
</table>

*Data is from the 1996 – 1999 CEX survey. The energy good is gasoline, electricity, natural gas, and home heating fuels and oils. The clean good is remaining household expenditures.*
### Table 2: Quadratic Polynomial Regression Results

<table>
<thead>
<tr>
<th></th>
<th>Clean Good</th>
<th>Energy Good</th>
<th>Leisure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear Quadratic</td>
<td>Linear Quadratic</td>
<td>Linear Quadratic</td>
</tr>
<tr>
<td>Single Households</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \ln I )</td>
<td>0.1889( ^\dagger )</td>
<td>-1.4851( ^\dagger )</td>
<td>0.0243</td>
</tr>
<tr>
<td></td>
<td>(0.0040)</td>
<td>(0.1091)</td>
<td>(0.0161)</td>
</tr>
<tr>
<td>( \ln I^2 )</td>
<td>-0.1291( ^\dagger )</td>
<td>-0.0027( ^\dagger )</td>
<td>-0.1264( ^\dagger )</td>
</tr>
<tr>
<td></td>
<td>(0.0084)</td>
<td>(0.0012)</td>
<td>(0.0087)</td>
</tr>
<tr>
<td>Married Households</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \ln I )</td>
<td>0.1163( ^\dagger )</td>
<td>-1.0233( ^\dagger )</td>
<td>0.0331</td>
</tr>
<tr>
<td></td>
<td>(0.0035)</td>
<td>(0.1245)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>( \ln I^2 )</td>
<td>0.0811( ^\dagger )</td>
<td>-0.0031( ^\dagger )</td>
<td>-0.0491( ^\dagger )</td>
</tr>
<tr>
<td></td>
<td>(0.0089)</td>
<td>(0.0007)</td>
<td>(0.0075)</td>
</tr>
</tbody>
</table>

\( ^\dagger \) significance at 5\% level.

Regression results test a linear log income specification versus a quadratic log income specification.

These results are reported in Table 2. The top frame gives the results for single households and the bottom frame gives the results for married households. With the linear specification, the potential income term is significant for all three goods for both single and married households. The results for the quadratic specification, on the other hand, are mixed. Potential income and its square are both significant for the clean good and leisure in the case of single households, while significant for all goods in the case of married households. These estimates provide prima facie evidence for using the QUAIDS specification, where the Engel curves are allowed to vary linearly with \( \ln \) of potential income for some goods and nonlinearly for others.\(^\text{19}\)

### 4 Estimation

Expenditure shares \( \omega_i \) and \( \omega_k \) constitute our estimating equations, where \( i = 1 \) refers to energy good, \( i = 2 \) refers to clean good, \( \omega_p \) refers to leisure for primary workers, and \( \omega_s \) refers to leisure for secondary workers. They are given by (8)–(9). A particular difficulty

\(^{19}\)These Engel curve results provide an initial picture of the income-demand relationship. The exact nature of the QUAIDS specification, and which goods are restricted to a linear income specification (i.e. \( \lambda_i = 0 \)), is determined by the QUAIDS estimation results.
with estimating these equations is that they are nonlinear in parameters. However, they are conditionally linear if the value of the price indices $a(w, p)$ and $b(w, p)$ are known. We thus follow the Iterated Linear Least Squares Estimator (ILLE) procedure of Blundell and Robin (1999) to estimate our model.\footnote{The iteration procedure begins by estimating the AIDS specification while approximating $\ln a(w, p)$ with Stone’s Index:}

$$\ln a(w, p) \cong \sum_{i=1}^{2} \omega_i \ln p_i + \omega_p \ln w_p + \omega_s \ln w_s,$$

where $\omega_i$, $\omega_p$, and $\omega_s$ are the sample average budget share values. We use the resulting parameter estimates to recalculate $\ln a(w, p)$, using equation 3, and calculate $b(w, p)$ for the next round in the iteration process, which from this stage on is based on the QUAIDS specification. In the subsequent iterations, we use the QUAIDS parameters to update the price indices until the parameters converge.\footnote{West and Williams (2007) also use the three-stage least squares procedure to estimate the AIDS model. However their parameter estimates are based on Stone’s Index as a proxy for the $\ln a(w, p)$ price index, rather than iteratively estimating the parameters.}

The three-stage least squares procedure is needed because it combines a two-stage least squares procedure with a seemingly unrelated regression model. The two-stage least squares component allows for the use of instruments in controlling for endogeneity. The net wage is endogenous because the marginal tax rate is based on household income. We thus instrument for it using a sample average net wage based on occupation-, state- and gender-specific sample cells. In addition, error terms are potentially correlated across equations because the right-hand side variables are identical. The seemingly unrelated regression model controls for the endogenous error term by taking into account the correlated error structure and also allowing for the imposition of the cross-equation restrictions.\footnote{The iteration procedure begins by estimating the AIDS specification while approximating $\ln a(w, p)$ with Stone’s Index:}

The demographic variables include age, age squared, education dummy variables, ethnicity dummy variables, home ownership dummy variable, number of children dummy variables, and number of cars dummy variables as well as state and month fixed effects. A dummy variable for gender is included in the single household estimation, while demographic characteristics for both the male and female spouses are included in the
married household estimation. Dummy variables for home ownership and the number of cars control for energy consumption differences based on owning versus renting and whether one drives. State fixed effects are included to control for differences in energy consumption across states.\(^{22}\) Month fixed effects are included to control for seasonal variation in energy consumption.\(^{23}\) Cross-sectional wage variation, within each state, is used to estimate the cross-price elasticity of labor supply; and variation in prices over time is used to estimate the cross-price elasticity of the energy good.

Estimation of labor supply creates additional econometric issues that do not apply to estimating demand for market goods. The primary issue is the sample selection bias due to some adults choosing not to work. One does not observe a wage for those who do not work, but their decision to work is positively correlated with the wage they could potentially earn. Traditionally, the labor supply literature has treated the selection issue as random for males, given their high labor participation rate, while the selection issue is considered an integral part of female labor supply estimation.\(^{24}\) We resolve this issue by estimating Heckman-corrected net wage rates for all adults. This procedure also allows us to impute a wage for adults who are not working.

We estimate separate Heckman selection models for single and married household subsamples and estimate separate selection models for males and females within each subsample. The wage equation has ln net wage as the dependent variable and is a function of age, education, race and region. For workers, the wage equation also includes occupation code. The selection equation includes the same demographic variables as well as number of children, log of clean and energy good prices, and state-specific unemployment rates, which serve as the exclusion restriction. For married households, spousal

\(^{22}\)Weather differs by region. Regions also differ in car usage. Public transportation is prevalent in large cities such as New York City, Washington D.C., and Chicago. However it is much less so in other large cities such as Los Angeles, Atlanta, and Houston.

\(^{23}\)Heating and air-conditioning increase energy consumption in the winter and summer months, while families may drive more during the summer for family vacations.

salary and demographic variables are included as well, with spousal salary information serving as an additional exclusion restriction.\textsuperscript{25}

To estimate the QUAIDS model, we drop the clean good equation and only estimate the remaining two equations (three for married households). The three-stage least squares procedure allows us to impose the symmetry and homogeneity restrictions during the estimation procedure. We then use the adding up (as well as symmetry and homogeneity) restrictions to compute the parameters for the clean good equation. To calculate standard errors, we use a bootstrap procedure consisting of 1,500 replications. The bootstrap procedure clusters observations by households since a household may appear in as many as four quarters. Initially, we ran all the estimations with no restrictions on the $\ln$ income squared coefficient, $\lambda$, in any of the equations. The data did not support a statistically significant non-zero $\lambda$ value for the energy equation in the case of single households and for the clean good, energy good, and female labor supply in the case of married households. Subsequently, we re-estimated all the equations, but with $\lambda = 0$ for these goods. Tables 3 and 4 report these final estimates for single and married households.\textsuperscript{26}

Education level has a significant impact on energy consumption for single households. Relative to high school, households with higher levels of education spend a smaller share of their income on energy consumption. This is unsurprising given the strong correlation between income and education levels combined with the energy good being a necessity. The same is true for married households; higher education levels and the share of income going to energy consumption are negatively correlated. However, for the male spouse, only the bachelor’s and the graduate degree dummy variables are significant. For females, it is the no high school diploma dummy variable that is significant.

The share of income devoted to energy consumption is affected by home ownership,

\textsuperscript{25}This is essentially the same procedure as in West and Williams (2004, 2007).

\textsuperscript{26}The results for the initial regressions where $\lambda \neq 0$ for all goods are not reported; they are available from the authors upon request.
Table 3: Parameter Estimates (Single Households)

<table>
<thead>
<tr>
<th></th>
<th>Clean Good</th>
<th>Energy Good</th>
<th>Leisure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>Std. Error</td>
<td>Estimate</td>
</tr>
<tr>
<td>Constant</td>
<td>0.6706</td>
<td>(0.1169)</td>
<td>0.0645</td>
</tr>
<tr>
<td>ln (Clean Price)</td>
<td>-0.0663</td>
<td>(0.0769)</td>
<td>-0.0199</td>
</tr>
<tr>
<td>ln (Energy Price)</td>
<td>-0.0199‡</td>
<td>(0.0061)</td>
<td>0.0139</td>
</tr>
<tr>
<td>ln (Net Wage)</td>
<td>0.0862</td>
<td>(0.0804)</td>
<td>0.0060</td>
</tr>
<tr>
<td>ln (Real Income)</td>
<td>-0.4396‡</td>
<td>(0.1106)</td>
<td>-0.0188</td>
</tr>
<tr>
<td>ln (Real Income²)</td>
<td>0.2858‡</td>
<td>(0.0609)</td>
<td>-</td>
</tr>
<tr>
<td>Age</td>
<td>0.0016</td>
<td>(0.0064)</td>
<td>0.0009‡</td>
</tr>
<tr>
<td>Age Sq</td>
<td>-0.0001‡</td>
<td>(0.0001)</td>
<td>-0.00001‡</td>
</tr>
<tr>
<td>Male</td>
<td>-0.0273</td>
<td>(0.0208)</td>
<td>0.0021</td>
</tr>
<tr>
<td>No HS Diploma</td>
<td>-0.0198‡</td>
<td>(0.0213)</td>
<td>-0.0030‡</td>
</tr>
<tr>
<td>Some College</td>
<td>0.0009</td>
<td>(0.0129)</td>
<td>-0.0032‡</td>
</tr>
<tr>
<td>Bachelor’s Deg</td>
<td>-0.0106‡</td>
<td>(0.0348)</td>
<td>-0.0055‡</td>
</tr>
<tr>
<td>Graduate Deg</td>
<td>-0.0168‡</td>
<td>(0.0474)</td>
<td>-0.0093‡</td>
</tr>
<tr>
<td>Black</td>
<td>-0.0270‡</td>
<td>(0.0092)</td>
<td>-0.0004</td>
</tr>
<tr>
<td>Asian</td>
<td>-0.0217‡</td>
<td>(0.0196)</td>
<td>-0.0043‡</td>
</tr>
<tr>
<td>Other</td>
<td>-0.0812‡</td>
<td>(0.0309)</td>
<td>-0.0003</td>
</tr>
<tr>
<td>Own Home</td>
<td>0.0331‡</td>
<td>(0.0057)</td>
<td>0.0139‡</td>
</tr>
<tr>
<td>No Car</td>
<td>-0.0369‡</td>
<td>(0.0046)</td>
<td>-0.0048‡</td>
</tr>
<tr>
<td>Two Cars</td>
<td>0.0153‡</td>
<td>(0.0073)</td>
<td>0.0057‡</td>
</tr>
<tr>
<td>Three or More</td>
<td>0.0535‡</td>
<td>(0.0165)</td>
<td>0.0060‡</td>
</tr>
<tr>
<td>No Children</td>
<td>0.0510‡</td>
<td>(0.0071)</td>
<td>0.0046‡</td>
</tr>
<tr>
<td>Two Children</td>
<td>-0.0360‡</td>
<td>(0.0086)</td>
<td>-0.0054‡</td>
</tr>
<tr>
<td>Three or More</td>
<td>-0.0970‡</td>
<td>(0.0103)</td>
<td>-0.0056‡</td>
</tr>
</tbody>
</table>

System of 3 demand equations, the clean good equation is dropped. Parameters for the clean good are calculated based on cross-equation restrictions. $\lambda \neq 0$ for clean good and leisure. Regression includes state and month fixed effects. Standard errors are calculated using a bootstrapping procedure (1,500 replications). ‡ significance at 5% level.

number of cars, and number of children. Households who own their own home devote a larger share of income to energy consumption. This is true for both single and married households and most likely reflects the fact that homes are larger than apartments and thus require more energy use. All three dummy variables for car ownership are significant for single households, while only the three or more cars dummy variable is significant for married households. The dummy variables have the appropriate sign when significant, that is owning more cars will increase the share spent on energy goods. Both single and married households with two or more children devote a smaller share of income to energy consumption.
Table 4: Parameter Estimates (Married Households)

<table>
<thead>
<tr>
<th></th>
<th>Clean Good</th>
<th></th>
<th>Energy Good</th>
<th></th>
<th>Male Leisure</th>
<th></th>
<th>Female Leisure</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>Std. Error</td>
<td>Estimate</td>
<td>Std. Error</td>
<td>Estimate</td>
<td>Std. Error</td>
<td>Estimate</td>
<td>Std. Error</td>
</tr>
<tr>
<td>Constant</td>
<td>0.1791‡</td>
<td>(0.0352)</td>
<td>0.0276‡</td>
<td>(0.0031)</td>
<td>0.3316‡</td>
<td>(0.0343)</td>
<td>0.3617‡</td>
<td>(0.0311)</td>
</tr>
<tr>
<td>ln (Clean Price)</td>
<td>-0.0118</td>
<td>(0.0280)</td>
<td>-0.0131‡</td>
<td>(0.0023)</td>
<td>-0.0140</td>
<td>(0.0226)</td>
<td>0.0390‡</td>
<td>(0.0182)</td>
</tr>
<tr>
<td>ln (Energy Price)</td>
<td>-0.0131‡</td>
<td>(0.0023)</td>
<td>0.0109‡</td>
<td>(0.0019)</td>
<td>0.0002</td>
<td>(0.0014)</td>
<td>0.0020</td>
<td>(0.0012)</td>
</tr>
<tr>
<td>ln (Net Wage) (M)</td>
<td>-0.0140</td>
<td>(0.0226)</td>
<td>0.0002</td>
<td>(0.0014)</td>
<td>0.0707‡</td>
<td>(0.0255)</td>
<td>-0.0569‡</td>
<td>(0.0109)</td>
</tr>
<tr>
<td>ln (Net Wage) (F)</td>
<td>0.0390‡</td>
<td>(0.0182)</td>
<td>0.0020</td>
<td>(0.0012)</td>
<td>-0.0569‡</td>
<td>(0.0109)</td>
<td>0.0160</td>
<td>(0.0198)</td>
</tr>
<tr>
<td>ln (Real Income)</td>
<td>-0.0382‡</td>
<td>(0.0300)</td>
<td>-0.0154‡</td>
<td>(0.0006)</td>
<td>0.1110‡</td>
<td>(0.0303)</td>
<td>-0.0574‡</td>
<td>(0.0066)</td>
</tr>
<tr>
<td>ln (Real Income²)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Age (M)</td>
<td>0.0037</td>
<td>(0.0025)</td>
<td>0.0001</td>
<td>(0.0002)</td>
<td>-0.0044</td>
<td>(0.0027)</td>
<td>0.0006</td>
<td>(0.0017)</td>
</tr>
<tr>
<td>Age Sq (M)</td>
<td>-0.0001‡</td>
<td>(0.00063)</td>
<td>0.0000</td>
<td>(0.00000)</td>
<td>0.0001‡</td>
<td>(0.00003)</td>
<td>-0.0003‡</td>
<td>(0.00002)</td>
</tr>
<tr>
<td>Age (F)</td>
<td>0.0039</td>
<td>(0.0021)</td>
<td>0.0006‡</td>
<td>(0.0002)</td>
<td>-0.0028</td>
<td>(0.0017)</td>
<td>-0.0037</td>
<td>(0.0022)</td>
</tr>
<tr>
<td>Age Sq (F)</td>
<td>-0.0001‡</td>
<td>(0.00000)</td>
<td>-0.0001‡</td>
<td>(0.00000)</td>
<td>0.0001</td>
<td>(0.00002)</td>
<td>0.0001‡</td>
<td>(0.00003)</td>
</tr>
<tr>
<td>No HS Diploma (M)</td>
<td>-0.0155‡</td>
<td>(0.0134)</td>
<td>-0.0009</td>
<td>(0.0000)</td>
<td>-0.0001</td>
<td>(0.0084)</td>
<td>0.0165‡</td>
<td>(0.0064)</td>
</tr>
<tr>
<td>Some College (M)</td>
<td>0.0018</td>
<td>(0.0075)</td>
<td>-0.0001</td>
<td>(0.00004)</td>
<td>0.0021</td>
<td>(0.0050)</td>
<td>-0.0038</td>
<td>(0.0038)</td>
</tr>
<tr>
<td>Bachelor’s Deg (M)</td>
<td>-0.0042‡</td>
<td>(0.0129)</td>
<td>-0.0018‡</td>
<td>(0.0005)</td>
<td>0.0043</td>
<td>(0.0091)</td>
<td>0.0016</td>
<td>(0.0049)</td>
</tr>
<tr>
<td>Graduate Deg (M)</td>
<td>-0.0112‡</td>
<td>(0.0174)</td>
<td>-0.0018‡</td>
<td>(0.0007)</td>
<td>0.0032</td>
<td>(0.0119)</td>
<td>0.0098</td>
<td>(0.0065)</td>
</tr>
<tr>
<td>No HS Diploma (F)</td>
<td>-0.0153‡</td>
<td>(0.0083)</td>
<td>-0.0018‡</td>
<td>(0.0006)</td>
<td>0.0276‡</td>
<td>(0.0068)</td>
<td>-0.0104</td>
<td>(0.0101)</td>
</tr>
<tr>
<td>Some College (F)</td>
<td>0.0009‡</td>
<td>(0.0056)</td>
<td>-0.0002</td>
<td>(0.0004)</td>
<td>-0.056</td>
<td>(0.0040)</td>
<td>0.0049</td>
<td>(0.0057)</td>
</tr>
<tr>
<td>Bachelor’s Deg (F)</td>
<td>-0.0051‡</td>
<td>(0.0087)</td>
<td>-0.0003</td>
<td>(0.0006)</td>
<td>-0.0175‡</td>
<td>(0.0055)</td>
<td>0.0229‡</td>
<td>(0.0090)</td>
</tr>
<tr>
<td>Graduate Deg (F)</td>
<td>-0.0190‡</td>
<td>(0.0117)</td>
<td>-0.0012</td>
<td>(0.0008)</td>
<td>0.0004</td>
<td>(0.0076)</td>
<td>0.0198</td>
<td>(0.0122)</td>
</tr>
<tr>
<td>Black (M)</td>
<td>0.0316‡</td>
<td>(0.0141)</td>
<td>0.0001</td>
<td>(0.0011)</td>
<td>-0.0040</td>
<td>(0.0139)</td>
<td>-0.0277‡</td>
<td>(0.0129)</td>
</tr>
<tr>
<td>Asian (M)</td>
<td>-0.0074‡</td>
<td>(0.0129)</td>
<td>-0.0005</td>
<td>(0.0009)</td>
<td>-0.0031</td>
<td>(0.0137)</td>
<td>0.0111</td>
<td>(0.0108)</td>
</tr>
<tr>
<td>Other (M)</td>
<td>-0.0295‡</td>
<td>(0.0277)</td>
<td>-0.0028</td>
<td>(0.0015)</td>
<td>0.0528‡</td>
<td>(0.0242)</td>
<td>-0.0205</td>
<td>(0.0209)</td>
</tr>
<tr>
<td>Black (F)</td>
<td>-0.0314‡</td>
<td>(0.0147)</td>
<td>0.0001</td>
<td>(0.0012)</td>
<td>0.0140</td>
<td>(0.0140)</td>
<td>0.0174</td>
<td>(0.0143)</td>
</tr>
<tr>
<td>Asian (F)</td>
<td>-0.0071‡</td>
<td>(0.0134)</td>
<td>-0.0024‡</td>
<td>(0.0009)</td>
<td>0.0177</td>
<td>(0.0119)</td>
<td>-0.0082</td>
<td>(0.0120)</td>
</tr>
<tr>
<td>Other (F)</td>
<td>-0.0387‡</td>
<td>(0.0306)</td>
<td>0.0012</td>
<td>(0.0015)</td>
<td>-0.0220</td>
<td>(0.0191)</td>
<td>0.0558</td>
<td>(0.0328)</td>
</tr>
<tr>
<td>Own Home</td>
<td>0.0293‡</td>
<td>(0.0039)</td>
<td>0.0047‡</td>
<td>(0.0003)</td>
<td>-0.0167‡</td>
<td>(0.0036)</td>
<td>-0.0173‡</td>
<td>(0.0034)</td>
</tr>
<tr>
<td>No Car</td>
<td>0.0020</td>
<td>(0.0045)</td>
<td>-0.0005</td>
<td>(0.0004)</td>
<td>-0.0087‡</td>
<td>(0.0038)</td>
<td>0.0072</td>
<td>(0.0037)</td>
</tr>
<tr>
<td>Two Car</td>
<td>0.0105‡</td>
<td>(0.0034)</td>
<td>-0.0003</td>
<td>(0.0003)</td>
<td>-0.0021</td>
<td>(0.0031)</td>
<td>-0.0080‡</td>
<td>(0.0030)</td>
</tr>
<tr>
<td>Three or More</td>
<td>0.0347‡</td>
<td>(0.0061)</td>
<td>0.0017‡</td>
<td>(0.0005)</td>
<td>-0.0119‡</td>
<td>(0.0050)</td>
<td>-0.0245‡</td>
<td>(0.0051)</td>
</tr>
<tr>
<td>No Child</td>
<td>0.0455‡</td>
<td>(0.0044)</td>
<td>0.0032‡</td>
<td>(0.0004)</td>
<td>-0.0966‡</td>
<td>(0.0036)</td>
<td>-0.0392‡</td>
<td>(0.0037)</td>
</tr>
<tr>
<td>Two Child</td>
<td>-0.0315‡</td>
<td>(0.0048)</td>
<td>-0.0024‡</td>
<td>(0.0004)</td>
<td>0.0088‡</td>
<td>(0.0038)</td>
<td>0.0250</td>
<td>(0.0039)</td>
</tr>
<tr>
<td>Three or More</td>
<td>-0.0623‡</td>
<td>(0.0055)</td>
<td>-0.0032‡</td>
<td>(0.0004)</td>
<td>-0.0528‡</td>
<td>(0.0047)</td>
<td>-0.0528‡</td>
<td>(0.0049)</td>
</tr>
</tbody>
</table>

*System of 4 demand equations, the clean good equation is dropped. Parameters for the clean good are calculated based on cross-equation restrictions. λ ≠ 0 for leisure (male) only. Regression includes state and month fixed effects. Standard errors are calculated using a bootstrap procedure (1,500 replications). ‡ significance at 5% level.
Finally, as expected, energy shares are also affected by incomes and prices. Log of real income has a significant negative effect on the share of income devoted to energy consumption which is consistent with the energy good being a necessity. The effects of prices, and wage rates, are more complicated to discern from the parameter estimates. The point is that prices and wages affect budget shares not only through log $p_j$ and log $w_k$ terms but also through the income terms via the $a(w, p)$ and $b(w, p)$ indices.

5 Elasticities

Denote the income elasticity of demand for good $i=1, 2$ and for leisure $k=p, s$ by $\eta_i$ and $\eta_k$; the own- and cross-price elasticities of demand for good $i=1, 2$ with respect to good $j=1, 2$ by $\varepsilon_{ij}$; the cross-price elasticity of demand for good $i=1, 2$ with respect to leisure $k=p, s$ by $\varepsilon_{ik}$; the cross-price elasticity of demand for leisure $k=p, s$ with respect to good $j=1, 2$ by $\varepsilon_{kj}$; and the own- and cross-price elasticities of demand for leisure $k=p, s$ with respect to leisure $e=p, s$ by $\varepsilon_{ke}$. To relate these elasticity terms to the estimating equations, one can rewrite them in terms of the budget shares. We have, for $i$ and $j=1, 2$, and for $k$ and $e=p, s$,

$$\eta_i \equiv \frac{\partial x_i}{\partial I} \frac{1}{x_i} = \frac{1}{\omega_i} \frac{\partial \omega_i}{\partial \ln I} + 1,$$

$$\eta_k \equiv \frac{\partial l_k}{\partial I} \frac{1}{l_k} = \frac{1}{\omega_k} \frac{\partial \omega_k}{\partial \ln I} + 1,$$

$$\varepsilon_{ij} \equiv \frac{\partial x_i}{\partial p_j} \frac{1}{p_j} = \frac{1}{\omega_i} \frac{\partial \omega_i}{\partial \ln p_j} - \delta_{ij},$$

$$\varepsilon_{kj} \equiv \frac{\partial l_k}{\partial p_j} \frac{1}{p_j} = \frac{1}{\omega_k} \frac{\partial \omega_k}{\partial \ln p_j},$$

$$\varepsilon_{ie} \equiv \frac{\partial x_i}{\partial w_e} \frac{1}{w_e} = \frac{1}{\omega_i} \frac{\partial \omega_i}{\partial \ln w_e} \left| 1 + \frac{1}{\omega_i} \frac{\partial \omega_i}{\partial \ln I} \right| \left( 1 + \frac{1}{\omega_i} \frac{\partial \omega_i}{\partial \ln I} \right),$$

$$\varepsilon_{ke} \equiv \frac{\partial l_k}{\partial w_e} \frac{1}{w_e} = \frac{1}{\omega_k} \frac{\partial \omega_k}{\partial \ln w_e} \left| 1 + \frac{1}{\omega_k} \frac{\partial \omega_k}{\partial \ln I} \right| \left( 1 + \frac{1}{\omega_k} \frac{\partial \omega_k}{\partial \ln I} \right) - \delta_{ke},$$

where $\delta_{ij}$ and $\delta_{ke}$ denote the Kronecker delta; see the Appendix. Then partially differentiate $\omega_i$ and $\omega_k$, as given by equations (8)–(9), with respect to $\ln I, \ln p_j, \ln w_e$ and
substitute in (10)–(15). This yields for all \( i \) and \( j = 1, 2, \) and \( k \) and \( e = p, s \), as shown in the Appendix,

\[
\begin{align*}
\eta_i & = 1 + \frac{1}{\omega_i} \left[ \beta_i + \frac{2\lambda_i}{b(w, p)} \ln \frac{I}{a(w, p)} \right], \\
\eta_k & = 1 + \frac{1}{\omega_k} \left[ \beta_k + \frac{2\lambda_k}{b(w, p)} \ln \frac{I}{a(w, p)} \right], \\
\varepsilon_{ij} & = -\delta_{ij} + \frac{\gamma_{ij}}{\omega_i} - \frac{1}{\omega_i} \left[ \alpha_j + \gamma_{jp} \ln w_p + \gamma_{js} \ln w_s + \sum_{i=1}^{2} \gamma_{ij} \ln p_i \right] \\
& \times \left[ \beta_i + \frac{2\lambda_i}{b(w, p)} \ln \frac{I}{a(w, p)} \right] - \lambda_i \beta_j \frac{\ln \frac{I}{a(w, p)}}{\omega_i b(w, p)} \left[ \ln \frac{I}{a(w, p)} \right]^2, \\
\varepsilon_{kj} & = \frac{\gamma_{kj}}{\omega_k} - \frac{1}{\omega_k} \left[ \alpha_j + \gamma_{jp} \ln w_p + \gamma_{js} \ln w_s + \sum_{i=1}^{n} \gamma_{ij} \ln p_i \right] \\
& \times \left[ \beta_k + \frac{2\lambda_k}{b(w, p)} \ln \frac{I}{a(w, p)} \right] - \lambda_k \beta_j \frac{\ln \frac{I}{a(w, p)}}{\omega_k b(w, p)} \left[ \ln \frac{I}{a(w, p)} \right]^2, \\
\varepsilon_{ie} & = \frac{1}{\omega_i} \gamma_{ie} - \frac{1}{\omega_i} \left[ \beta_i + \frac{2\lambda_i}{b(w, p)} \ln \frac{I}{a(w, p)} \right] \times \\
& \left[ \alpha_e + \gamma_{ee} \ln w_e + \gamma_{ek} \ln w_k + \sum_{i=1}^{n} \gamma_{ie} \ln p_i - \frac{w_e T_e}{I} \right], \\
\varepsilon_{ke} & = \frac{1}{\omega_k} \gamma_{ke} - \frac{1}{\omega_k} \left[ \beta_k + \frac{2\lambda_k}{b(w, p)} \ln \frac{I}{a(w, p)} \right] \times \\
& \left[ \alpha_e + \gamma_{ee} \ln w_e + \gamma_{ek} \ln w_k + \sum_{i=1}^{n} \gamma_{ie} \ln p_i - \frac{w_e T_e}{I} \right].
\end{align*}
\]

Observe that if \( \lambda_i = 0 \), both the income and the cross-price elasticity of demand for good \( i \) is independent of potential income.

Using the parameter estimates given in Tables 3–4, we calculate and report the income and price elasticities of demand for single and married household subsamples...
in Tables 5–6. As one would expect the compensated own-price elasticities are negative and the compensated wage rate elasticities are positive. The uncompensated and compensated own-price elasticity of demand for energy is \(-0.5958\) and \(-0.5771\) for the “representative” single household. These are somewhat less elastic than West and Williams’ (2007) estimates of \(-0.771\) and \(-0.750\) for gasoline. Our estimates for the “representative” married household are \(-0.5869\) and \(-0.5750\) as compared to \(-0.283\) and \(-0.209\) for West and Williams (2007).

The uncompensated and compensated labor supply elasticities are \(-0.0205\) and \(0.3990\) for single households; they are \(0.1436\) and \(0.4356\) for males, and \(0.9777\) and \(1.2678\) for females in the married households. Surveys of the labor supply literature, and the corresponding elasticity estimates, are found in Pencavel (1986), Killingsworth and Heckman (1986), Blundell and MaCurdy (1999), and Keane (2011). The majority of studies find uncompensated labor supply elasticities close to zero for males, but much larger values for females. Thus, amongst our estimates, the married households’ uncompensated male labor supply elasticity appears to be on the high side. However, statistically, this estimate is not significantly different from zero. The female elasticity estimate, while high, is still within the range of other studies.

The most interesting estimate from our perspective is the cross-price elasticity between labor supply and energy. The uncompensated elasticity of labor supply with respect to the price of energy is \(-0.0090\) for single households, and \(-0.0013\) for males and \(-0.0220\) for females in married households. That this elasticity is negative, and for all subsamples, indicates that energy is a complement to labor supply (a substitute to leisure). As such, it suggests that one wants to subsidize energy relative to other goods (ignoring its environmental benefits). This is in addition to equity considerations that

\[\text{By a “representative” household we mean a hypothetical household endowed with the sample average potential income facing the sample mean for the energy price, clean good price, and net wage rates.}\]

\[\text{West and Williams (2007) have estimated these to be 0.040 and 0.353 for single households; 0.062 and 0.187 for males, and 0.242 and 0.337 for females in the married household sample.}\]
may also call for subsidizing energy due to its being a “necessity”. By contrast, considering gasoline alone, West and Williams (2007) have estimated the uncompensated elasticity of labor supply with respect to the price of gasoline to be positive and equal to 0.003 for single households and 0.013 for males and 0.002 for females in married households. Their result suggests that gasoline is a complement to leisure and should be taxed relative to other goods. Re-estimating our model and using gasoline as the sole energy good, we find mixed results. The uncompensated elasticity of labor supply with respect to the price of gasoline is –0.0199 for single households, again suggesting substitutability with leisure. On the other hand, for the married household subsample we find this elasticity to be 0.0029 for males (suggesting complementarity with leisure) but –0.0043 for females (suggesting substitutability with leisure).

6 Optimal energy tax?

Deriving a value for the optimal energy tax poses a number of difficulties. One problem is that the optimal tax on energy, and on any other good for that matter, depends on the other tax instruments used by the government. A second, and related, problem is that the value of the tax also depends on whether or not the government optimizes over the other tax instruments. Third, the optimal tax depends on the way one aggregates efficiency and equity concerns through the use of a particular social welfare function.

In our estimation procedure, we have recognized the fact that the government relies

\[ ^{29} \text{In support of their finding, West and Williams (2007) offer the observation that higher gas prices must lead to a reduction in leisure driving. This argument is appealing. The income effect of a rise in the price of gasoline lowers the consumption of all normal goods including leisure. As far as the substitution effect is concerned, the price of leisure driving is not just the wages foregone but also the cost of gasoline used for driving. The increase in the price of gasoline thus increases the price of leisure driving reducing leisure consumption. However, if this is the story then one should also have a compensated complementarity relationship between gasoline and leisure. But West and Williams find the compensated elasticity of labor supply with respect to gasoline price to be } -0.009 \text{ so that gasoline is a net substitute to leisure and not a net complement.} \]

\[ ^{30} \text{It is well known that in the absence of differential lump-sum taxation, one cannot separate efficiency and equity concerns unless preferences are of a special form not satisfied by QUAIDS preferences (nor by AIDS). This separation requires preferences that are of Gorman-polar form. This means that the} \]
Table 5: Elasticity Estimates (Single Households)

<table>
<thead>
<tr>
<th></th>
<th>Clean Price</th>
<th>Energy Price</th>
<th>Net Wage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Income Elasticities</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clean Good</td>
<td>1.3049</td>
<td>(1.2585, 1.3513)</td>
<td></td>
</tr>
<tr>
<td>Energy Good</td>
<td>0.4989</td>
<td>(0.4387, 0.5591)</td>
<td></td>
</tr>
<tr>
<td>Labor Supply</td>
<td>-1.0902</td>
<td>(-1.1756, -1.0048)</td>
<td></td>
</tr>
<tr>
<td><strong>Uncompensated Elasticities</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clean Good</td>
<td>-0.8317</td>
<td>(-1.1207, -0.5426)</td>
<td>0.6559</td>
</tr>
<tr>
<td></td>
<td>-0.0396</td>
<td>(-0.0655, -0.0138)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.3756, 0.9361)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy Good</td>
<td>-0.1270</td>
<td>(-0.3902, 0.1362)</td>
<td>0.6404</td>
</tr>
<tr>
<td></td>
<td>-0.5958</td>
<td>(-0.8218, -0.3698)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.4383, 0.8425)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor Supply</td>
<td>0.2094</td>
<td>(-0.1718, 0.5906)</td>
<td>-0.0205</td>
</tr>
<tr>
<td></td>
<td>-0.0090</td>
<td>(-0.0368, 0.0189)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.4273, 0.3864)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Compensated Elasticities</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clean Good</td>
<td>-0.2298</td>
<td>(-0.5049, 0.0454)</td>
<td>0.1538</td>
</tr>
<tr>
<td></td>
<td>0.0093</td>
<td>(-0.0156, 0.0342)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1304, 0.4380)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy Good</td>
<td>0.1031</td>
<td>(-0.1636, 0.3698)</td>
<td>0.4485</td>
</tr>
<tr>
<td></td>
<td>-0.5771</td>
<td>(-0.8035, -0.3506)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.2398, 0.6571)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor Supply</td>
<td>-0.2934</td>
<td>(-0.6696, 0.0828)</td>
<td>0.3900</td>
</tr>
<tr>
<td></td>
<td>-0.0499</td>
<td>(-0.0773, -0.0224)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0102, 0.7878)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*95% confidence intervals are calculated using a bootstrapping procedure (1,500 replications). QUAIDS specification, $\lambda \neq 0$, for clean good and leisure.*
Table 6: Elasticity Estimates (Married Households)

<table>
<thead>
<tr>
<th></th>
<th>Clean Price</th>
<th>Energy Price</th>
<th>Net Wage (Male)</th>
<th>Net Wage (Female)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Income Elasticities</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clean Good</td>
<td>0.8910</td>
<td>(0.7230, 1.0589)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy Good</td>
<td>0.4350</td>
<td>(0.3925, 0.4776)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor Supply (M)</td>
<td>-0.8310</td>
<td>(0.8821, 0.7798)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor Supply (F)</td>
<td>-1.6203</td>
<td>(1.7172, 1.5234)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Uncompensated Elasticities</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clean Good</td>
<td>-1.0103</td>
<td>(1.1474, 0.8731)</td>
<td>0.5657</td>
<td>0.5737</td>
</tr>
<tr>
<td>Energy Good</td>
<td>-0.3586</td>
<td>(0.5184, 0.1988)</td>
<td>0.4947</td>
<td>0.4969</td>
</tr>
<tr>
<td>Labor Supply (M)</td>
<td>0.0231</td>
<td>(0.1169, 0.1631)</td>
<td>0.1436</td>
<td>-0.2536</td>
</tr>
<tr>
<td>Labor Supply (F)</td>
<td>-0.3237</td>
<td>(0.5675, 0.0799)</td>
<td>-0.0220</td>
<td>0.9777</td>
</tr>
<tr>
<td><strong>Compensated Elasticities</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clean Good</td>
<td>-0.6984</td>
<td>(0.8596, 0.5372)</td>
<td>0.2527</td>
<td>0.4141</td>
</tr>
<tr>
<td>Energy Good</td>
<td>-0.2063</td>
<td>(0.3676, 0.0451)</td>
<td>0.3418</td>
<td>0.4190</td>
</tr>
<tr>
<td>Labor Supply (M)</td>
<td>-0.2678</td>
<td>(0.4091, 0.1264)</td>
<td>0.4356</td>
<td>-0.1047</td>
</tr>
<tr>
<td>Labor Supply (F)</td>
<td>-0.8908</td>
<td>(1.1382, 0.6434)</td>
<td>-0.2345</td>
<td>1.2678</td>
</tr>
</tbody>
</table>

*95% confidence intervals are calculated using a bootstrapping procedure (1,500 replications). QUAIDS specification, λ ≠ 0, for male leisure only.*
on a general income tax. However, optimizing over a general income tax is a complicated procedure and beyond the scope of this paper. Given that we have to rely on non-optimal income tax rates, one cannot properly speak of an optimal energy tax. In what follows we thus follow a “tax reform” approach. That is, we fix the income tax rates as they currently are and evaluate the welfare implications of energy taxes. We make the exercise budget neutral by assuming that all extra tax proceeds are rebated through uniform rebates (which can be accommodated through the income tax schedule). Specifically, let

\[
R(t_1, t_2, \theta^h, a^h) = \sum_{i=1}^{2} t_i x_i \left( w^h, p_i, I^h \right) + \theta^h w g^h L \left( w^h, p, I^h \right) - a^h,
\]

denote total taxes paid by household \( h \), where \( t_1 \) and \( t_2 \) denote the current taxes on energy and other goods, and \( a^h \) is the “virtual income” component of \( m^h \). When the tax on energy increases from \( t_1 \) to \( t'_1 \), \( a^h \) is increased by an amount equal to \( a \) for all households such that

\[
R(t'_1, t_2, \theta^h, a^h + a) = R(t_1, t_2, \theta^h, a^h).
\]

Regarding the social welfare function, we use the iso-elastic specification introduced by Atkinson (1970). This is particularly useful as it allows for a wide range of attitudes

indirect utility can be written as

\[
v(w^h, p, I^h) = a(w^h, p) + b(p) I^h.
\]

31 We have

\[
R(t'_1, t_2, \theta^h, a^h + a) = \sum_{i=1}^{2} t'_i x_i \left( w^h, p'_i, I^h + a \right) + \theta^h w g^h L \left( w^h, p', I^h + a \right) - \left( a^h + a \right),
\]

where \( p' \) is the new vector of consumer prices.
towards inequality in the society. It is defined by

\[
W = \frac{1}{1 - \eta} \sum_{h=1}^{H} \pi^h \left( u^h \right)^{1-\eta} \quad \eta \neq 1 \quad \text{and} \quad 0 \leq \eta < \infty, \tag{22}
\]

\[
= \sum_{h=1}^{H} \pi^h \ln u^h, \quad \eta = 1,
\]

where \( \eta \geq 0 \) denotes the inequality aversion index and \( u^h \) is the utility of household \( h \). As is well-known, this social welfare function reduces to the utilitarian function for \( \eta = 0 \) and to the Rawlsian function when \( \eta \to \infty \).

In an attempt to differentiate between environmental and non-environmental effects of the energy tax, we proceed as follows.

6.1 Energy tax and the environment

In calculating the effect of the energy tax on the environment, we have to first determine the carbon dioxide emissions associated with one unit of energy as we have defined it. A unit of energy is a weighted average of four components: gasoline, electricity, natural gas, and home heating fuels and oils. The weights are the share of each component in energy expenditures over the sample period.\(^{32}\) We calculate the carbon dioxide emission of each component by first expressing it in terms of its customary measurement unit (gallon for gasoline, kilowatt hour for electricity, therm for natural gas, and gallon for home heating fuels and oils). We next use the “Greenhouse Gas Equivalences Calculator” from the EPA to calculate the carbon dioxide emission of each component.\(^{33}\) Adding them up we find \( 8.206 \times 10^{-3} \) metric tons of \( CO_2 \) emissions per unit of the energy good. Finally, to translate this figure into dollars, we rely on Glaeser and Kahn (2008) who estimate the social cost per metric ton of emissions to be \$43. This number, in combination with the figure of \( 8.206 \times 10^{-3} \) metric tons of \( CO_2 \) emissions per unit of the energy good,

\(^{32}\)We calculate these weights to be 0.470494 for gasoline, 0.373129 for electricity, 0.130854 for natural gas, and 0.025523 for home heating fuels and oils.

\(^{33}\)www.epa.gov/cleanenergy/energy-resources/calculator.html.
We are now in a position to determine the environmental benefit of a revenue neutral energy-tax-cum-rebate reform. For this purpose, based on our demand system estimate, we calculate how much a particular tax reduces the consumption of energy (on a weekly basis). Our calculations indicate a 7.2% reduction in energy consumption as a result of levying a $0.25 tax on energy. Multiplying this reduction by the $0.35 social cost per unit yields the associated environmental benefit (assuming a constant marginal social damage). 

Table 7 reports the environmental benefits corresponding to each tax on a household basis.

Table 7: Weekly Environmental Benefits per Household

<table>
<thead>
<tr>
<th>Tax</th>
<th>$0.25</th>
<th>$0.50</th>
<th>$0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduction in consumption of energy</td>
<td>1.2009</td>
<td>2.2948</td>
<td>3.2563</td>
</tr>
<tr>
<td>Environmental benefit ($)</td>
<td>0.4237</td>
<td>0.8097</td>
<td>1.1490</td>
</tr>
</tbody>
</table>

yields a social cost of $0.35 per unit of the energy good.34

34To be more precise, $0.352858. Observe that each unit of energy could be purchased at an average price of $1.07294 during the sample period. The social cost of $0.352858 thus amounts to 33% of the price of energy over the sample period.

35Given an average price of $1.07294 per unit of energy over the sample period, a $0.25 tax on energy amounts to a 23.30% increase in price. We thus have a 7.2% reduction in energy consumption as a result of a 23.30% increase in price. Put differently, we have a 0.309% reduction for a 1% increase in price. Interestingly, this is very close to the figure Bento et al. (2009) find for the reduction in consumption of gasoline as a result of a $0.25 tax per gallon. They report a reduction of 5.09%. Now the Energy Information Administration lists the average price of gasoline in 2001 to be $1.38. Thus a $0.25 tax per gallon amounts to an 18.1% increase in price. A 5.09% reduction in consumption due to an 18.1% increase in price is equivalent to a 0.281% reduction for a 1% increase in price.

36Considering that each unit of energy could be purchased at an average price of $1.07294 during the sample period, these taxes amount to 23.3%, 46.6% and 69.9% of the price of energy over the sample period.

37It is possible that these benefits are on the “low side”. Energy consumption entails other social costs besides those associated with CO2 emissions. This is particularly true for gasoline consumption which has a 47% share in total expenditures on energy. Considering congestion and accidents, Parry and Small (2005) estimate the social cost associated with a gallon of gasoline to be $0.75. On the other hand, the $43 figure per metric ton of CO2 emissions translate into a social cost of $0.38 per gallon of gasoline ($8.92 \times 10^{-3} \times 43 = 0.38356$). If one were to include these other costs, the environmental benefits of energy taxes will be higher than our calculations in Table 7.
6.2 Non-environmental welfare

As a rule, one cannot separate the welfare effects of taxes into separate environmental and non-environmental portions. To get around this difficulty, we simply assume that the non-environmental effects are what one gets in a hypothetical economy akin to ours but without an externality term. This is an economy where preferences are represented by \( v(w^h, p, I^h) \) rather than \( v(w^h, p, I^h) - \varphi(E) \). The energy-tax-cum-rebate policy then affects only \( v(w^h, p, I^h) \).

To arrive at a dollar value for the non-environmental utility changes for a society populated with heterogeneous agents, we use the concept of the “social equivalent variation”, \( EV^s \). We define this analogously to the Hicksian concept of equivalent variation for a \( h \)-household who faces a higher price for energy while being compensated by a uniform rebate, \( a \). Specifically, one defines \( EV^s \) from the following relationship:

\[
\sum_{h=1}^{H} \pi^h \left[ v\left(w^h, p, I^h + EV^s\right) \right]^{1-\eta} = \sum_{h=1}^{H} \pi^h \left[ v\left(w^h, p', I^h + a\right) \right]^{1-\eta}.
\] (23)

It measures the amount of money one can give each household under the current tax regime such that the non-environmental component of social welfare will be equivalent to its value under the proposed reform (of levying an energy tax and rebating its proceeds uniformly to all households). Observe that \( EV^s \) is positive if the reform is welfare improving and negative if it is welfare reducing (not including the environmental benefits).

Table 8 reports \( EV^s \) for the three energy taxes for different \( \eta \) values. Given that the tax/rebate program is redistributive towards the less well-off households, \( EV^s \) increases with \( \eta \).\(^{38}\) Consequently, the most interesting feature of the reported numbers is that they are negative even when \( \eta \) is large. This tells us that, ignoring environmental benefits, energy taxes coupled with uniform rebates are harmful to the economy. These

\(^{38}\)The more well-off household consume more energy than the less well-off households and thus pay more in energy taxes. At the same time, all households receive the same amount of rebate.
taxes exacerbate rather than ameliorate the distortionary-cum-redistributive effects of the existing income taxes. Put differently, their second dividend, in the parlance of the “double dividend” controversy, is negative. Of course, this feature may very well be different if the energy tax is accompanied with a reduction in the marginal income tax rates which is not studied here.

6.3 Net welfare effect

To gauge the net welfare effects of energy taxes, one cannot simply add up the two components we have calculated under environmental and non-environmental welfare effects. For this purpose, we again use the concept of the social equivalent variation. This time though, $EV^s$ measures the amount of money one can give each household under the current tax regime such that the social welfare will be equivalent to that under the proposed reform (of levying an energy tax and rebating its proceeds uniformly to all households). It is defined by

$$
\sum_{h=1}^{H} \pi^h \left[ v \left( w^h, p, I^h + EV^s \right) - \varphi \left( E \left( t_1, t_2, \theta^h, a^h \right) \right) \right]^{1-\eta} = \\
\sum_{h=1}^{H} \pi^h \left[ v \left( w^h, p', I^h + a \right) - \varphi \left( E \left( t_1', t_2, \theta^h, a^h + a \right) \right) \right]^{1-\eta}. \tag{24}
$$

To simplify matters, we assume that $\varphi(E)$ is linear in $E$ so that the marginal social damage of emissions is constant. This allows us to write $\varphi \left( E \left( t_1, t_2, \theta^h, a^h \right) \right) =
Table 9: Weekly Net Benefits

<table>
<thead>
<tr>
<th>η / tax</th>
<th>$0.25</th>
<th>$0.50</th>
<th>$0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>-1.77</td>
<td>-3.57</td>
<td>-5.39</td>
</tr>
<tr>
<td>0.50</td>
<td>-1.46</td>
<td>-2.96</td>
<td>-4.51</td>
</tr>
<tr>
<td>1.00</td>
<td>-1.13</td>
<td>-2.33</td>
<td>-3.60</td>
</tr>
<tr>
<td>2.00</td>
<td>-0.50</td>
<td>-1.14</td>
<td>-1.92</td>
</tr>
<tr>
<td>5.00</td>
<td>0.48</td>
<td>0.74</td>
<td>0.77</td>
</tr>
<tr>
<td>10.0</td>
<td>1.19</td>
<td>2.15</td>
<td>2.84</td>
</tr>
</tbody>
</table>

ϕ · E (t₁, t₂, θₕ, aₕ). To go any further we need to estimate ϕ. We proceed as follows. Let a denote the dollar cost to the society as a whole when a unit of energy is consumed ($0.35 in our calculations). The corresponding cost to one household is a/H where H is the number of households. This is in dollars; to translate it into “societal utils,” we have to multiply this figure by the marginal utility of income to the society. We use the average of all households’ marginal utility of income, which we can find from the data, for this figure. Specifically, this is equal to \( \frac{\sum_{h=1}^{H} \alpha^h}{H} \) where \( \alpha^h \equiv \frac{\partial v (w^h, p, I^h)}{\partial I^h} \) denotes the marginal utility of income for household h. Multiplying a/H by \( \frac{\sum_{h=1}^{H} \alpha^h}{H} \) provides us with an estimate of ϕ. Armed with this estimate, and our estimates of the demand system and the non-environmental utility function, we are able to calculate the EVs values.

Table 9 reports EVs for the three energy taxes for different η values. The striking feature of these numbers is that they are all negative for “small values” of η but turn positive at higher values of η. This is not surprising in light of our earlier calculations that showed the non-environmental effects of the energy taxes to be welfare reducing but decreasing in η. The environmental benefits are not large enough to offset the high non-environmental costs when η is low, but large enough to offset the moderate non-environmental costs when η is high.
7 Concluding remarks

This paper has used 1996–1999 Consumer Expenditure Survey data to estimate a Quadratic Almost Ideal Demand System (QUAIDS) consisting of leisure, energy, and non-energy consumption goods for the US. It has found that leisure and energy are (gross) substitutes so that energy must be subsidized relative to non-energy goods in an optimized tax system (not taking into account its environmental benefits). However, neither this finding nor our demand system estimates allow us to provide a unique answer to the question of what is the optimal energy tax. The trouble lies with the question which is not quite well specified. The “optimal” energy tax depends on the other tax instruments available to the government and the (non-optimized) values that these other instruments take. In particular, the most important tax instrument in the economy, namely the income tax, is far from being optimized. Consequently, what we take as granted as far as the income tax structure is concerned matters a lot in the determination of the optimal energy tax.

Given these difficulties, the paper has followed a “tax reform” approach. That is, it has fixed the income tax rates as they currently are and has evaluated the welfare implications of three levels of energy taxes. To make the exercise budget neutral, it has assumed that all extra tax proceeds are rebated through uniform rebates (which can be accommodated through the income tax schedule). Given this setup, it has found that, despite its environmental benefits, a case for an energy tax can be made only if the society cares a great deal about inequality. Otherwise, energy taxes will be detrimental to the society’s welfare. Of course this answer may very well change if the energy tax is accompanied by changing the income tax rates.
Appendix

Derivation of (8)–(9): Step 1. Partially differentiate

\[
\ln v = \left\{ \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-1} + \lambda(w, p) \right\}^{-1},
\]

with respect to \(p_i, w_k\), and \(I\):

\[
\frac{\partial \ln v}{\partial p_i} = \Lambda \left\{ \frac{\partial}{\partial p_i} \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-1} + \frac{\partial}{\partial p_i} \lambda(w, p) \right\},
\]

\[
\frac{\partial \ln v}{\partial w_k} \bigg|_I = \Lambda \left\{ \frac{\partial}{\partial w_k} \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-1} + \frac{\partial}{\partial w_k} \lambda(w, p) \right\},
\]

\[
\frac{\partial \ln v}{\partial I} = \Lambda \left\{ \frac{\partial}{\partial I} \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-1} + \frac{\partial}{\partial I} \lambda(w, p) \right\},
\]

where

\[
\Lambda \equiv - \left\{ \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-1} + \lambda(w, p) \right\}^{-2}.
\]

Simplifying yields,

\[
\frac{\partial \ln v}{\partial p_i} = \Lambda \frac{\partial}{\partial p_i} \lambda(w, p) - \Lambda \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-2} \times \left[ \frac{-b(w, p)(\partial \ln a(w, p)/\partial p_i) - (\ln I - \ln a(w, p))(\partial b(w, p)/\partial p_i)}{(b(w, p))^2} \right],
\]

(A1)

\[
\frac{\partial \ln v}{\partial w_k} \bigg|_I = \Lambda \frac{\partial}{\partial w_k} \lambda(w, p) - \Lambda \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-2} \times \left[ \frac{b(w, p)(-\partial \ln a(w, p)/\partial w_k) - (\ln I - \ln a(w, p))(\partial b(w, p)/\partial w_k)}{(b(w, p))^2} \right],
\]

(A2)

\[
\frac{\partial \ln v}{\partial I} = \Lambda \frac{\partial}{\partial I} \lambda(w, p) - \Lambda \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-2} \times \left[ \frac{b(w, p)(1/I - \partial \ln a(w, p)/\partial I) - (\ln I - \ln a(w, p))(\partial b(w, p)/\partial I)}{(b(w, p))^2} \right].
\]

(A3)

Step 2. Partially differentiate \(b(w, p)\) and \(\lambda(w, p)\) with respect to \(p_i, w_k\), and \(I\) to
get

\[
\begin{align*}
\frac{\partial b(w, p)}{\partial p_i} &= \frac{\beta_i}{p_i} b(w, p), \\
\frac{\partial b(w, p)}{\partial w_k} &= \frac{\beta_k}{w_k} b(w, p), \\
\frac{\partial b(w, p)}{\partial I} &= 0, \\
\frac{\partial \lambda(w, p)}{\partial p_i} &= \frac{\lambda_i}{p_i}, \\
\frac{\partial \lambda(w, p)}{\partial w_k} &= \frac{\lambda_k}{w_k}, \\
\frac{\partial \lambda(w, p)}{\partial I} &= 0.
\end{align*}
\]

Also note that \(\ln a(w, p)\) is independent of \(I\) so that

\[
\frac{\partial \ln a(w, p)}{\partial I} = 0.
\]

Substituting these expressions in (A1)–(A3),

\[
\begin{align*}
\frac{\partial \ln v}{\partial p_i} &= \Lambda \frac{\lambda_i}{p_i} - \Lambda \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-2} \\
&\quad \times \left[ -b(w, p) \frac{\partial \ln a(w, p)}{\partial p_i} - (\ln I - \ln a(w, p)) \frac{\partial b(w, p)}{p_i} \right], \tag{A4}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \ln v}{\partial w_k} |_{I} &= \Lambda \frac{\lambda_k}{w_k} - \Lambda \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-2} \\
&\quad \times \left[ b(w, p) \left( -\frac{\partial \ln a(w, p)}{\partial w_k} - (\ln I - \ln a(w, p)) \frac{\partial b(w, p)}{w_k} \right) \right], \tag{A5}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \ln v}{\partial I} &= -\Lambda \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-2} \frac{b(w, p) (1/I)}{(b(w, p))^2}. \tag{A6}
\end{align*}
\]

Dividing (A4) and (A5) by (A6) then results in

\[
\begin{align*}
\frac{\partial \ln v}{\partial p_i} &= \Lambda \frac{\lambda_i}{p_i} - \Lambda \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-2} \\
&\quad \times \left[ -b(w, p) \frac{\partial \ln a(w, p)}{\partial p_i} - (\ln I - \ln a(w, p)) \frac{\partial b(w, p)}{p_i} \right], \\
\frac{\partial \ln v}{\partial w_k} |_{I} &= \Lambda \frac{\lambda_k}{w_k} - \Lambda \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-2} \\
&\quad \times \left[ b(w, p) \left( -\frac{\partial \ln a(w, p)}{\partial w_k} - (\ln I - \ln a(w, p)) \frac{\partial b(w, p)}{w_k} \right) \right], \\
\frac{\partial \ln v}{\partial I} &= -\Lambda \left[ \frac{\ln I - \ln a(w, p)}{b(w, p)} \right]^{-2} \frac{b(w, p) (1/I)}{(b(w, p))^2}.
\end{align*}
\]

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Simplifying yields,

\[
\frac{\partial \ln v}{\partial p_i} = - \left[ \frac{\partial \ln a(w, p)}{\partial p_i} + \beta_i \ln \frac{I}{a(w, p)} \right] I - \frac{\lambda_i}{p_i} b(w, p) \left[ \ln \frac{I}{a(w, p)} \right]^2,
\]

\[
\frac{\partial \ln v}{\partial w_k} = - \left[ \frac{\partial \ln a(w, p)}{\partial w_k} + \frac{\beta_k}{w_k} \ln \frac{I}{a(w, p)} \right] I - \frac{\lambda_k}{w_k} b(w, p) \left[ \ln \frac{I}{a(w, p)} \right]^2.
\]

Step 3. Substitute the above expressions in (6)–(7) to get

\[
\omega_i = - p_i \frac{\partial \ln v}{\partial p_i} I \frac{\partial \ln v}{\partial I}
\]

\[
= p_i \frac{\partial \ln a(w, p)}{\partial p_i} + \left[ \frac{\beta_i}{p_i} \ln \frac{I}{a(w, p)} + \frac{\lambda_i}{p_i} \ln \frac{I}{a(w, p)} \right] I - \beta_i \ln \frac{I}{a(w, p)} \left[ \ln \frac{I}{a(w, p)} \right]^2,
\]

\[
\omega_k = \frac{w_k}{I} \frac{\partial \ln v}{\partial w_k}
\]

\[
= w_k \frac{\partial \ln a(w, p)}{\partial w_k} + \beta_k \ln \frac{I}{a(w, p)} + \lambda_k \ln \frac{I}{a(w, p)} \left[ \ln \frac{I}{a(w, p)} \right]^2.
\]

Step 4. Partially differentiate \( \ln a(w, p) \) with respect to \( p_i \) and \( w_k \):

\[
\frac{\partial \ln a(w, p)}{\partial p_i} = \frac{1}{p_i} \left[ \alpha_i + \gamma_{ip} \ln w_p + \gamma_{is} \ln w_s + \sum_{j=1}^{n} \gamma_{ji} \ln p_j \right],
\]

\[
\frac{\partial \ln a(w, p)}{\partial w_k} = \frac{1}{w_k} \left[ \alpha_k + \sum_{i=1}^{n} \gamma_{ik} \ln p_i + \gamma_{kk} \ln w_k + \gamma_{ke} \ln w_e \right].
\]

Then substitute these expressions in the expressions for \( \omega_i \) and \( \omega_k \) derived in step 3. This yields equations (8)–(9).

**Derivation of (10)–(15):** From the definition of budget shares, \( \omega_i \equiv p_i x_i / I \) and \( \omega_k \equiv w_k l_k / I \). Rearranging, we have, \( x_i = I \omega_i / p_i \) and \( l_k = I \omega_k / w_k \). Partially differentiating
these two relationships with respect to $I, p_j$, and $w_e$ yields,

\[
\frac{\partial x_i}{\partial I} = \frac{\omega_i}{p_i} + \frac{1}{p_i} \frac{\partial \omega_i}{\partial I} = \frac{1}{p_i} \left[ \omega_i + \frac{\partial \omega_i}{\partial \ln I} \right], 
\]

(A7)

\[
\frac{\partial l_k}{\partial I} = \frac{\omega_k}{w_k} + \frac{1}{w_k} \frac{\partial \omega_k}{\partial I} = \frac{1}{w_k} \left[ \omega_k + \frac{\partial \omega_k}{\partial \ln I} \right] 
\]

(A8)

\[
\frac{\partial x_i}{\partial p_j} = I \left[ \frac{1}{p_i} \frac{\partial \omega_i}{\partial p_j} - \frac{\omega_i}{(p_i)^2} \frac{\partial p_i}{\partial p_j} \right] = \frac{1}{p_i} \left[ \frac{1}{p_j} \frac{\partial \omega_i}{\partial \ln p_j} - \frac{\omega_i}{p_i} \delta_{ij} \right] 
\]

(A9)

\[
\frac{\partial l_k}{\partial p_j} = \frac{1}{p_k} \frac{\partial \omega_k}{\partial p_j} = \frac{1}{p_j w_k} \frac{\partial \omega_k}{\partial \ln p_j} 
\]

(A10)

\[
\frac{\partial x_i}{\partial w_e} = \frac{\partial x_i}{\partial w_e} |_I + T_e \frac{\partial x_i}{\partial I} = \frac{1}{p_i} \frac{\partial \omega_i}{\partial w_e} |_I + T_e \frac{\partial x_i}{\partial I} 
\]

\[
= \frac{1}{p_i} \left[ \frac{I}{w_e} \frac{\partial \omega_i}{\partial \ln w_e} |_I + T_e \left( \omega_i + \frac{\partial \omega_i}{\partial \ln I} \right) \right] 
\]

(A11)

\[
\frac{\partial l_k}{\partial w_e} = \frac{\partial l_k}{\partial w_e} |_I + T_e \frac{\partial l_k}{\partial I} 
\]

\[
= I \left[ (\partial \omega_k / \partial w_e) w_k - (\partial w_k / \partial w_e) \omega_k \right] + T_e \frac{1}{w_k} \left[ \omega_k + \frac{\partial \omega_k}{\partial \ln I} \right] 
\]

\[
= \frac{I}{w_k} \left( \frac{1}{w_e} \frac{\partial \omega_k}{\partial \ln w_e} |_I - \omega_k \delta_{ke} \right) + T_e \frac{1}{w_k} \left( \omega_k + \frac{\partial \omega_k}{\partial \ln I} \right). 
\]

(A12)

Substituting these derivatives in the various definition of elasticity terms given in (10)–(15) yields the equalities presented in these expressions.

**Derivation of (16)–(21):** Step 1. Partially differentiate equations (8)–(9) with respect
to $I, p_j, w_e$, and simplify:

$$\frac{\partial \omega_i}{\partial \ln I} = \beta_i + \frac{2\lambda_i}{b(w, p)} \ln \frac{I}{a(w, p)},$$  \hspace{1cm} (A13)

$$\frac{\partial \omega_k}{\partial \ln I} = \beta_k + \frac{2\lambda_k}{b(w, p)} \ln \frac{I}{a(w, p)},$$  \hspace{1cm} (A14)

$$\frac{\partial \omega_i}{\partial \ln p_j} = \gamma_{ij} - \left[ \beta_i + \frac{2\lambda_i}{b(w, p)} \ln \frac{I}{a(w, p)} \right] \frac{\partial \ln a(w, p)}{\partial \ln p_j}$$

$$\frac{\partial \omega_k}{\partial \ln p_j} = \gamma_{kj} - \left[ \beta_k + \frac{2\lambda_k}{b(w, p)} \ln \frac{I}{a(w, p)} \right] \frac{\partial \ln a(w, p)}{\partial \ln p_j},$$  \hspace{1cm} (A15)

$$\frac{\partial \omega_i}{\partial \ln w_e} \Big|_l = \gamma_{ie} - \left[ \beta_i + \frac{2\lambda_i}{b(w, p)} \ln \frac{I}{a(w, p)} \right] \frac{\partial \ln a(w, p)}{\partial \ln w_e}$$

$$\frac{\partial \omega_k}{\partial \ln w_e} \Big|_l = \gamma_{ke} - \left[ \beta_k + \frac{2\lambda_k}{b(w, p)} \ln \frac{I}{a(w, p)} \right] \frac{\partial \ln a(w, p)}{\partial \ln w_e},$$  \hspace{1cm} (A16)

$$\frac{\partial \omega_k}{\partial \ln w_e} \Big|_l = \gamma_{ke} - \left[ \beta_k + \frac{2\lambda_k}{b(w, p)} \ln \frac{I}{a(w, p)} \right] \frac{\partial \ln a(w, p)}{\partial \ln w_e}$$

Step 2. Partially differentiate equations (3)–(4) with respect to $p_j$ and $w_e$:

$$\frac{\partial \ln a(w, p)}{\partial \ln p_j} = \alpha_j + \gamma_{jp} \ln w_p + \gamma_{js} \ln w_s + \sum_{i=1}^{n} \gamma_{ij} \ln p_i,$$

$$\frac{\partial \ln a(w, p)}{\partial \ln w_e} = \alpha_e + \gamma_{ee} \ln w_e + \gamma_{ek} \ln w_k + \sum_{i=1}^{n} \gamma_{ie} \ln p_i,$$

$$\frac{\partial b(w, p)}{\partial \ln p_j} = p_j \frac{\partial b(w, p)}{\partial p_j} = p_j \frac{\beta_j}{p_j} b(w, p),$$

$$\frac{\partial b(w, p)}{\partial \ln w_e} = w_e \frac{\partial b(w, p)}{\partial w_e} = w_e \frac{\beta_e}{w_e} b(w, p).$$
Substitute the above equations in (A13)–(A18) to get,

\[
\frac{\partial \omega_i}{\partial \ln I} = \beta_i + \frac{2\lambda_i}{b(w, p)} \ln \frac{I}{a(w, p)}, \quad (A19)
\]

\[
\frac{\partial \omega_k}{\partial \ln I} = \beta_k + \frac{2\lambda_k}{b(w, p)} \ln \frac{I}{a(w, p)}, \quad (A20)
\]

\[
\frac{\partial \omega_i}{\partial \ln p_j} = \gamma_{ij} - \left[ \beta_i + \frac{2\lambda_i}{b(w, p)} \ln \frac{I}{a(w, p)} \right] \times \left[ \alpha_j + \gamma_{jp} \ln w_p + \gamma_{js} \ln w_s + \sum_{i=1}^{n} \gamma_{ij} \ln p_i \right] - \frac{\lambda_i \beta_j}{b(w, p)} \left[ \ln \frac{I}{a(w, p)} \right]^2 \quad (A21)
\]

\[
\frac{\partial \omega_k}{\partial \ln p_j} = \gamma_{kj} - \left[ \beta_j + \frac{2\lambda_k}{b(w, p)} \ln \frac{I}{a(w, p)} \right] \times \left[ \alpha_j + \gamma_{jp} \ln w_p + \gamma_{js} \ln w_s + \sum_{i=1}^{n} \gamma_{ij} \ln p_i \right] - \frac{\lambda_k \beta_j}{b(w, p)} \left[ \ln \frac{I}{a(w, p)} \right]^2 \quad (A22)
\]

\[
\frac{\partial \omega_i}{\partial \ln w_e} \bigg|_{I} = \gamma_{ie} - \left[ \beta_i + \frac{2\lambda_i}{b(w, p)} \ln \frac{I}{a(w, p)} \right] \times \left[ \alpha_e + \gamma_{ee} \ln w_e + \gamma_{ek} \ln w_k + \sum_{i=1}^{n} \gamma_{ie} \ln p_i \right] - \frac{\lambda_i \beta_e}{b(w, p)} \left[ \ln \frac{I}{a(w, p)} \right]^2 \quad (A23)
\]

\[
\frac{\partial \omega_k}{\partial \ln w_e} \bigg|_{I} = \gamma_{ke} - \left[ \beta_k + \frac{2\lambda_i}{b(w, p)} \ln \frac{I}{a(w, p)} \right] \times \left[ \alpha_e + \gamma_{ee} \ln w_e + \gamma_{ek} \ln w_k + \sum_{i=1}^{n} \gamma_{ie} \ln p_i \right] - \frac{\lambda_k \beta_e}{b(w, p)} \left[ \ln \frac{I}{a(w, p)} \right]^2 \quad (A24)
\]

Substituting (A19)–(A24) in (10)–(15) and simplifying leads to (16)–(21).
References


