Business Cycle Theory II

(Nominal/Demand Shocks)

Introduction

Theoretical research on business cycles has branched out considerably over the last 20 years. In addition to the *Real Business Cycle* branch, there is now a second major branch known as *New Keynesian* models. Together, these branches go under the heading of *Dynamics Stochastic General Equilibrium* (DSGE) Models.

As suggested by its name, the New Keynesian models adhere to the view held dear by old Keynesian economists that frictions, especially in the form of price stickiness, are essential to understanding the economy. As we will show, these models are very similar to the Keynesian IS-LM model you probably studied in your intermediate class. The new models typically include an IS curve and a Phillips curve, both centerpieces of the old Keynesian model. Some of the New Keynesian models also include an LM curve, but this depends on how monetary policy is modeled.

Where new and old Keynesians differ is that New Keynesian models are founded on microeconomics, starting at the level of preferences and technology. In this respect, the New Keynesian paradigm is more similar to the RBC one. There is in fact a good deal of agreement between those economists who consider themselves as practitioners of the RBC paradigm and those who consider themselves practitioners of the New Keynesian paradigm. Practitioners of RBC theory are not adverse to the notion that frictions are important for understanding the business cycle. Indeed, an approach known as *Business Cycle Accounting* pioneered by V.V. Chari, Patrick J. Kehoe, and Ellen McGrattan provides strong evidence that two wedges, a TFP wedge and a labor wedge, which arise because of frictions, are the source of most of the business cycle fluctuations.

There are however some major differences. The first is more of a philosophical one that involves the right amount of model abstraction. For the most part, proponents of RBC prefer simpler structures, with a small number of model parameters. Proponents of the RBC approach are not embarrassed that their models do not match all dimensions of the macro data. In contrast, proponents of the New Keynesian approach prefer models that fit many more dimensions of the aggregate data. This requires the addition of large number of features and shocks be added to the model, and with them, a large set of parameters that need to be estimated.

An important issue, particularly with respect to the quantitative analysis used in these chapters, is whether the New Keynesian models are being calibrated? In most cases, the assignment of parameters is done so that the model matches or fits the data. On the face of it, this would not seem inconsistent with the calibration procedure. However, as a pure calibration exercise a number of criticisms have been made. Robert E. Lucas once said "Beware of Economists bearing free parameters", and so there is a lot to be aware of in the case of these New Keynesian models. By setting the parameters to match observations, particularly the ones the theory is interested in accounting for, these models are not being tested as in step 5 of our calibration procedure. Many practitioners of the RBC school have criticized New Keynesians for not using micro level data to restrict the values of these added parameters. Paul Romer from New York University goes further in his article the Trouble with Macroeconomics to criticize the standard approaches used by practitioners of these models to get around the identification problem associated with the large number of parameters. He is not kind in his assessment, arguing that for the most parts parameter values are tied down by feeding in a fact with an unknown truth value (FWUTV). Although Dr. Romer is harsh in his criticism of the New Keynesian models, he is no fan of the RBC models either.

Despite these criticisms, there is really no other game in town for the purpose of evaluating monetary policy. RBC models, at least the earliest versions, had nothing to say about policy. Absent frictions, the competitive equilibrium is efficient, and there is no reason for government intervention. New Keynesian models have become the backbone of policy analysis at most of the central banks.

II. Background: The Real Effects of Monetary Policy?

Sargent and Wallace (1975) showed that introducing rational expectations into the Keyneisan IS/LM model had important implications for monetary policy. In particular, they showed that the monetary authority could not on average change the level of the economy's output or employment when people formed expectations rationally. If peoples' expectations are rational, then they cannot be making errors in their forecasts in a systematic manner. The entire premise underlying the mechanism by which monetary policy has real effects works through surprising firms and households. Without surprises, and assuming some firms or workers are not locked into set prices in the period, the changes in the money supply would have no real effect. This is the famous ***Neutrality of Money*** result, which goes back to Classical Economics.

The neutrality of money result has come to be associated with Real Business Cycle Theory. This is a bit unfortunate. It is little known that the working paper of Kydland and Prescott's 1982 *Econometrica* article included a monetary side. However, they found the effects of monetary shocks in the calibration exercise to be small and so Kydland and Prescott simplified the model in the final version of the paper by eliminating the monetary side.

Recall from our chapter on Business Cycle Facts, that M2 is procyclical with a lead of 3 to 6 months. This would suggest that money has real effects. However, some economists argue that the positive correlation reflects reverse causation. Reverse causation works as follows. People and firms expect a high TFP 3 or 6 months from now. With a higher expected TFP, capital will be more productive, so businesses will want to increase their investments and hence their demand for loans. Banks will meet these higher loans from their excess reserves. As you probably learned in your studies of Money and Banking, the money multiplier depends on the amount of excess reserves that the banking sector holds, with the multiplier being larger when less reserves are held. This expectation of a future productivity shock, therefore, causes the money supply to increase today.

The introduction of rational expectations did not however end the debate or view that monetary policy had real effects. What it did was sharpened the focus, and led to the need to identify surprise changes to the money supply. For this purpose***, Vector Auto Regression*** (VAR) analysis has become an integral part of monetary economics and the New Keynesian paradigm. This research tool was introduced to the profession by Chris Sims (1980).

VARS and Impulse Response Functions

A major tool used in New Keynesian analysis is Vector Auto Regressions and their implied impulse response functions. This tool is used both in documenting the facts to be accounted for as well as testing the theory.

Basically, a VAR is a system of N-linear equations where N is the number of variables. In a given linear equation, a particular variable is assumed to be a linear function of its past variables as well as a linear function of the past variables of the N-1 other variables. In each linear equation, there is also an error or shock term. To illustrate, consider the VAR studied by Christiano, Eichenbaum and Evans (1999). The three macro variables are inflation, π_t, real GNP, y_t, and the nominal interest rate, i_t. The VAR thus constitutes three equations specified as

$$\pi_t = a_{\pi 1}\pi_{t-1} + ... + a_{\pi P}\pi_{t-P} + a_{y1}y_{t-1} + ... + a_{yP}y_{t-P} + a_{i1}i_{t-1} + ... + a_{iP}i_{t-P} + \mu_{\pi_t}$$

$$y_t = b_{\pi 1}\pi_{t-1} + ... + b_{\pi P}\pi_{t-P} + b_{y1}y_{t-1} + ... + b_{yP}y_{t-P} + b_{i1}i_{t-1} + ... + b_{iP}i_{t-P} + \mu_{y_t}$$

$$i_t = c_{\pi 1}\pi_{t-1} + ... + c_{\pi P}\pi_{t-P} + c_{y1}y_{t-1} + ... + c_{yP}y_{t-P} + c_{i1}i_{t-1} + ... + c_{iP}i_{t-P} + \mu_{i_t}$$

In the above equations, the letter P denotes the number of lag variables in the system. All variables represent deviations from their trend or steady states. Letting μ_t denoting the vector of error terms, the variance-covariance matrix is E($\mu\mu'$), which we denote by V. In the VAR, the vector of error terms satisfy E(μ)=0 and E($\mu_t\mu_{t+k}$)=0 so that there is no correlation of the errors over time.

One could easily estimate the equations, one by one using *Ordinary Least Squares*. Done in this way, $\mu_{\pi t}$, μ_{yt}, and μ_{it} represent forecast errors- the difference between the value of each variable as predicted by the estimated equation and the actual value. These forecast errors cannot be interpreted as fundamental shocks to the economy. For example, for the purpose of determining the effect of monetary policy we would like to be able to determine the effect a surprise increase or decrease in the interest rate engineered by the Central Bank. Although this fundamental shock will influence μ_{it}, it will not be the sole determinant of the forecast error. In particular, the forecast error in each equation will be correlated with all the other forecast errors. Think about a change in the interest rate. If the central bank is following a rule as hypothesized by John Taylor where the interest rate is set as a function of the gap between output and its trend, then a fundamental shock to the output equation will affect the interest rate equation.

Herein lies the challenge of the VAR approach: to identify the fundamental shocks to each equation. Without this, one cannot say anything about how a shock in monetary policy affects the economy. The identification strategy in the VAR approach specifies each forecast term to be a function of the fundamental shocks to each of the variables. Let $\varepsilon_{\pi t}$, ε_{yt}, and ε_{it}, denote the fundamental shocks to the three variables. Then

$$\mu_{\pi t} = g_{\pi 1}\varepsilon_{\pi t} + g_{y1}\varepsilon_{yt} + g_{i1}\varepsilon_{it}$$

$$\mu_{yt} = g_{\pi 2}\varepsilon_{\pi t} + g_{y2}\varepsilon_{yt} + g_{i2}\varepsilon_{it}$$

$$\mu_{it} = g_{\pi 3}\varepsilon_{\pi t} + g_{y3}\varepsilon_{yt} + g_{i3}\varepsilon_{it}$$

In matrix form, the system of equations is written as $\mu_t = G\varepsilon_t$ with GG′=V. The assumption is that the variance-covariance matrix of the fundamental shocks is the identity matrix, i.e. E($\varepsilon\varepsilon'$)=I

The challenge in the VAR paradigm is to impose restrictions that allow one to determine the parameter values denoted by the g's. There are a number of strategies that have been used. All of them require that one have some idea of the way the economy works. We present only one of the many identification strategies used in this literature: the Recursive method used by Sims (1980, 1989). Here the idea is to order the equations so that the first equation is only affected by its own fundamental error. The second equation is affected by its own fundamental error as well as the fundamental error in the first equation. Finally, the error term in the third equation is assumed to be a function of all three fundamental shocks.

In the current model, Christiano, Eichenbaum and Evans (1999) order the equations with the first being the inflation error equation, with the logic that only current fundamental shocks to the inflation rate affect today's inflation rate, the second being the output error equation with the logic that it is affected by its own fundamental shock as well as by the inflation shock, and the third being the interest error equation, with the logic that it is affected by the inflation shock, the output shock and its own fundamental shock . With this recursive identification scheme, the error equations simply to

$$\mu_{\pi t} = g_{\pi 1}\varepsilon_{\pi t}$$

$$\mu_{yt} = g_{\pi 2}\varepsilon_{\pi t} + g_{y2}\varepsilon_{yt}$$

$$\mu_{it} = g_{\pi 3}\varepsilon_{\pi t} + g_{y3}\varepsilon_{yt} + g_{i3}\varepsilon_{it}$$

In this case, the matrix G is a lower triangle, namely,

$$G = \begin{bmatrix} g_{\pi 1} & 0 & 0 \\ g_{\pi 2} & g_{y2} & 0 \\ g_{\pi 3} & g_{y3} & g_{i3} \end{bmatrix} .$$

The matrix G is invertible and hence we can solve for the fundamental shocks as a function of the error terms. Since we have the variance co-variance matrix of the error terms from the original system, we can solve the matrix G since $GG'=V$. In this way, the VAR identifies the fundamental shocks.

After identifying the fundamental shocks, one can determine the impulse response functions of each variable associated with any of the three fundamental shocks. The impulse response functions trace out the values of the y, π, and i following a one unit change in one of the fundamental shocks. There are no other shocks introduced to the system other than this shock today. For instance, we could consider how the system is affected by a shock to interest rates. Given that we have all the parameters of the model (i.e., a,b, c coefficients and G matrix), we can trace out the time series for inflation, output and interest rates following a fundamental shock in the interest rate.

The following figure is taken from a paper that appeared in the Economic Review of the Federal Reserve Band of Dallas written by Nathan Balke and Kenneth M. Emery called Understanding the Price Puzzle. It shows the impulse responses of GDP, and the price level (PCE) to an unexpected increase in the Federal Funds rate that is implied by a simple SVAR analysis. The SVAR analysis is conducted separately for the entire 1960-934 period and then for just the price level for the 1960-1979 subperiod and the later 1983-93 subperiod. The response of GDP to output accords well with intuition; tighter monetary policy is expected to cause a contraction in the economy. The movement of the price level is very puzzling, at least in the earlier period, when it increases following a contraction in the money supply. This is indeed a puzzle, first identified by Chris Sims in (1992). Much work continues in this area with some researchers taking the puzzle as a fact and trying to account for it within a new Keynesian model and with other researchers pursuing the idea that the problem is associated with misspecification of the VAR.

Figure 2
Impulse Response of *Y* and *P*
To Federal Funds Rate Innovation, 1960–93

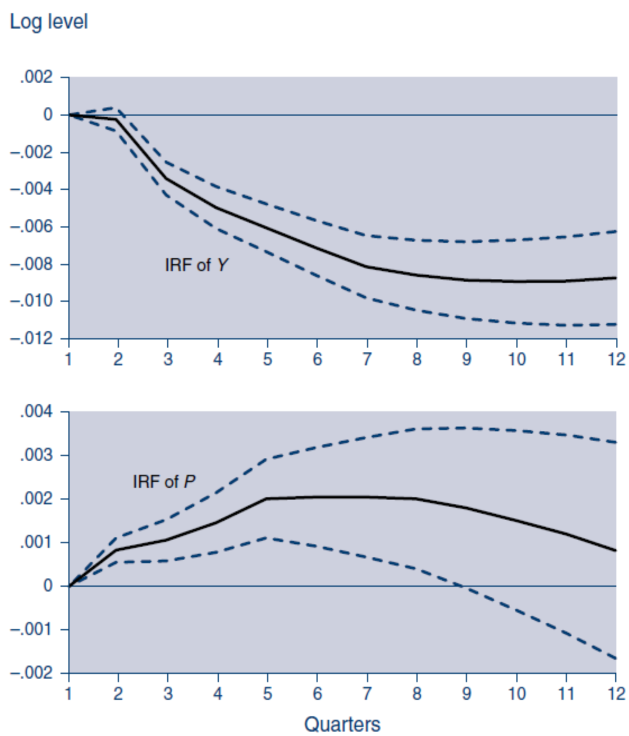
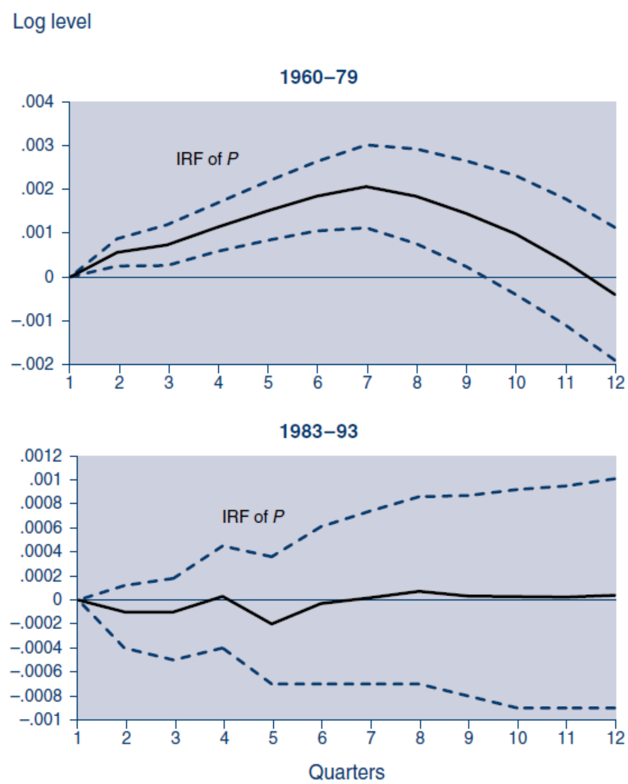


Figure 3
Impulse Response of *P*
To Federal Funds Rate Innovation



The impulse response functions are used both as a way of documenting the properties of business cycles and as a way of testing the DSGE model, (typically of the New Keynesian variety). As we shall explain more fully below, the equations of the New Keynesian model include fundamental shocks. With the parameters of the equation in hand, one can trace out the economy's path associated with a one-time shock to a key component of the system.

Criticisms of VARs

VAR's are rather controversial. They are not truly a-theoretical, nor truly theoretical. For example, in the identification scheme used above, they require an ordering that is motivated by theory. Although advances in VAR analysis now allow for a large set of variables, there is the issue of what variables should be included. With all these questions, a valid concern is whether the impulse responses are facts, or just artifacts of an imperfect analytical tool. For example, in the above figures, one observes that the price level responds in a counterintuitive way. Theory suggests that an increase in the interest rate should cause the price level to fall as the economy contracts. This counterintuitive finding is called the *Price Puzzle*, being first identified by Sims (1990).

Variance Decomposition

Although, impulse response functions allow one to trace out the effect of some fundamental shock on the endogenous variables of interest, they do not tell one how big one shocks contribution is to the overall forecast errors. Variance Decomposition, in contrast, does just this. For the variance of each forecast error, the contribution of each of the fundamental shocks is determined. The general finding is that the fundamental shock to monetary policy accounts for less than 25% of the forecasting error of real gdp. The biggest contributor to the overall variance of the forecast error of real GDP is the fundamental shock to real GDP, ε_{yt}.

Basic New Keynesian Model

The simplest NK model consists of three equations: an IS equation, a Phillips Curve Equation and a monetary policy rule equation.[1]

The IS equation

[1] The model described in these pages is based on Benigno (2009)

The IS equation is derived using 3 equilibrium conditions. They are the utility optimizing condition that the marginal rate of substitution between date t and t+1 consumption is equal to the gross real interest rate, (known as the Euler Equation), the Fisher Relation, and the goods market clearing condition.

Utility Optimization

The household in each period derives utility from consumption of a final good and leisure. The discounted stream of utility is

(PV-Utility) $$\sum_{t=0}^{\infty}\beta^{t}[U(C_t)-V(N_t)]$$

The functional form for $U(C_t)$

$$U(C_t)=\frac{C_t^{1-\sigma}-1}{1-\sigma}$$

And the functional form for $V(N_t)$ is

$$V(N_t)=\frac{N_t^{1+\eta}}{1+\eta}$$

The inverse of the parameter, σ, is the intertemporal elasticity of substitution with respect to consumption, and the inverse of the parameter, η, is the intertemporal elasticity of substitution with respect to labor. These parameters determine how willing the household is willing to substitute todays consumption for tomorrow's consumption. Importantly, notice that utility is defined over labor. This explains the negative sign before $V(N_t)$.

The consumption utility optimizing condition is

(MRSC) $$\frac{C_t^{-\sigma}}{\beta C_{t+1}^{-\sigma}}=1+r_t$$

In the context of business cycles, the idea is to think of period t as the short-run and period t+1 as the long-run, or the steady state. In view of this interpretation, we will drop the time subscripts and use $\overline{C}$ to denote the steady state value. Also, in the context of business cycles, there should be an Expectation at time t in the (MRSC) equation.

Taking the log of both sides of (MRSC) and exploiting the properties of logs, (MRSC) can be rewritten as

(EE-L) $\bar{c} - c \cong \frac{1}{\sigma} r + \frac{1}{\sigma} \beta$

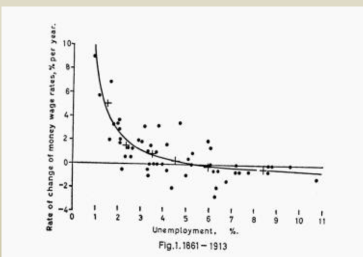
This equation uses the result that ln(1+x)≅x. This explains why we have used the approximately equal sign in (EE-L). Note that we use the convention of using a lower case letter to denote the natural logarithm of a variable.

The Fisher Relation is a no arbritrage condition that relates the real interest rate, the nominal interest rate and the inflation rate. You have probably come upon it in the statement that the real rate of interest rate is approximately equal to the nominal rate less the inflation rate. The Fisher Relation actually is

(Fisher R) $1 + r_t = (1 + i_t)\frac{P_t}{P_{t+1}}$

The left hand side the real gross return from saving one unit of output. Alternatively, if you took that one unit of output, given its nominal price P_t, one could obtain P_t units of currency. These dollars could be placed in an interest bearing savings account that pays a gross nominal interest rate of $(1+i_t)$. Thus after one period, you would have $(1+i_i)P_t$ dollars. To determine its real value, we would divide by the nominal price of goods at date t+1, P_{t+1}. The right hand side is the real return to saving in the form of nominal bonds. Again, in the context of business cycles, P_{t+1} should be expected.

The Phillips curve is named after the New Zealand born economist A.W. Phillips who in 1958 plotted the rate of change in nominal wage rates against the unemployment rate for the UK between 1861and 1957in an article published in Economica. The figure is reprinted below.



Taking logs of both sides, we arrive at

(FR-L) $r = i - (\bar{p} - p)$

Market Clearing

In the simplest of New Keynesian models, there is no capital and hence no investment, and no government. The Final Goods Market clearing condition

(GM) $Y = C$.

If we substitute (FR-L) and (G-M) into the (MRSC) equation, we obtain the IS curve. This is

(IS) $y = \bar{y} - \sigma^{-1}[i - (\bar{p} - p)] - \sigma^{-1} \ln \beta$

The Phillips Curve

Compared to the Phillips Curve, the IS Curve is easy to derive. The Phillips Curve is derived from the profit maximization of intermediate good monopolists, the utility maximizing condition for today's consumption and leisure, and the market clearing condition. The production side of the economy consists of a perfectly competitive final goods sector that uses intermediate goods to produce the final good according to the following CES elasticity of substitution production technology

(CES) $$Y = \left(\int_0^1 X(j)^{\frac{\varepsilon-1}{\varepsilon}} d_j \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Below this is the intermediate good sector which consists of monopolies that set prices to maximize profits subject to the demand for their product from the final goods sectors. Profit maximization by final good producers yields the demand for intermediate good j

(X-Demand) $$X(j) = \left(\frac{P_j}{P} \right)^{-\varepsilon} Y$$

Since the final good sector is perfectly competitive, firms must earn zero profits, namely,

(Zero-Profit) $$PY = \int_0^1 P(j)X(j)dj$$

where the right hand side is the cost of buying the intermediate goods. Using (X-Demand) and (Zero Profits), one can show that the Price of the final good is

(Price Index) $$P = \left(\int_0^1 P(j)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

To produce a given quantity of intermediated goods X requires H/A units of labor where A is TFP. More specifically,

(X-Production) $$X(j) = AH(j)$$

Profit maximization of an intermediate producer yields the following the result that the optimal price is a constant mark up over the marginal cost. Namely,

(Price equation) $$P(j) = \frac{W}{A} \frac{\varepsilon}{\varepsilon - 1}$$

where W is that nominal wage rate.

The idea of the steady state, or full-employment level of output, $\bar{Y}$, is the equilibrium quantity in the case that all intermediate producers can re-optimize their prices in each period.

Critical to the New Keynesian theory is the existence of frictions. There are two approaches. The first and the one adopted here is the assumption that not all intermediate firms are able to adjust their prices in any given period. This harkens back to the work of Calvo (1983) and is referred to as the sticky price assumption. An alternative that is attributed to the work of Mankiw and Reiss (2002) is the sticky information assumption. With sticky information, not all intermediate good producers obtain the current information. Those that do will set their prices upon the current information, but those that do not will set it based on old information.

In the sticky price formulation, each intermediate good producer is has a random chance of being able to change its price in the period. More specifically, with probability θ, a firm cannot change its price in the period, and with probability $(1-\theta)$ the firm can change its price in the period. The inability of all firms to reset prices is the key to having monetary shocks have real effects to the economy. If all firms could adjust their prices in the period, then we would just have the standard Classical result. The nominal interest rate and the inflation rate will adjust to keep the real interest rate constant. However, if some firms cannot adjust their prices, then their goods will be relatively cheaper compared to the firms that can raise their price. These stuck firms will see an increase in sales. They will no longer be maximizing profits but because of their monopoly status do still realize some profitsGiven that the shock is independently and identically distributed, the average time between price changes is $1/(1-\theta)$. A firm that is able to reset its price in the period must take this into account, that it may be many periods before they are able to change their price again. Let us assume that a firm does this by minimizing the log price, z_t, that minimizes the expected discounted sum of square "errors" between the set price and the perfectly flexible price. In particular, z_t, is chosen to minimize

(Loss Function) $$L(z_t) = \sum_{j=0}^{\infty} (\theta\beta)^j E_t (z_t - p^*_{t+j})^2$$

In (Loss Function), p^*_{t+j} is the optimal price chosen in period t+j if prices were flexible. The idea is that the firm, like the household, discounts the future at a rate β. Additionally, the probability that j periods from now the firm has not able to reset its price is θ^j.

In a more rigorous formulation, the firm would choose its log price, z_t, to maximize its expected profits. In a certain sense, the loss function given by the quadratic function is intended to approximate the expected profits.

To find the optimal reset price, we differentiate the (Loss Function) with respect z_t and set the

$$L'(z_t) = 2\sum_{j=0}^{\infty} (\theta\beta)^j E_t (z_t - p^*_{t+j}) = 0$$

Since the derivative is linear and separable in z_t, this reduces to

(LF FONC) $$z_t \sum_{j=0}^{\infty} (\theta\beta)^j {}_{t} = \sum_{j=0}^{\infty} (\theta\beta)^j E_t p^*_{t+j}$$

The left hand side summation is a geometric sum. For a geometric sum, $\sum_j^{\infty} x^j$ with x<1, the quantity is equal to $\frac{1}{1-x}$.

Hence, the left hand side of (LF FONC) reduces to

$$\frac{z_t}{1-\theta\beta} = \sum_{j=0}^{\infty} (\theta\beta)^j E_t p^*_{t+j} .$$

Cross multiplying by (1-θβ), we arrive at

(ORP1) $$z_t = (1-\theta\beta)\sum_{j=0}^{\infty}(\theta\beta)^j E_t p^*_{t+j}$$

There is a rather intuitive explanation for this expression. Specifically, the firm in setting its price today will do so that it is keeping its price to the average expected optimal price.

The next step in deriving the New Keynesian Phillips curve is to characterize the optimal price in each period, p^*_{t+j}. Here, we return to the (Pricing Equation). In logs, the (Pricing Equation) is

$$p^*_{t+j} = w_{t+j} - a_{t+j} + \ln\varepsilon - \ln(\varepsilon - 1)$$

Where w is the log of the nominal wage, W, and a is the log of the TFP, A. Their difference, w-a, is the log of the marginal cost of producing an intermediate good. For convenience, let $mc_{t+j}=w_{t+j}-a_{t+j}$ and let $\mu=\ln\varepsilon-\ln(\varepsilon-1)$. Then

$$p^*_{t+j} = mc_{t+j} + \mu \, .$$

Next, we substitute the above equation into (ORP1). This is

(ORP2) $$z_t = (1-\theta\beta)\sum_{j=0}^{\infty}(\theta\beta)^j E_t (mc_{t+j} + \mu)$$

The next step is to consider the law of motion for the aggregate price level. Returning to the (Price Index) equation, it follows that the price level is a weighted average of the intermediate good producers pirces, a fraction Given this stickiness, the law of motion for the price level is

(Price LM) $$P_t = [\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \, .$$

Where does this come from? Think that in each period fraction (1-θ) intermediate good producers change their price. That means today that (1-θ) firms set their price to P_t^*. One period ago, another (1-θ) were able to set their price to P_{t-1}^*, and of these fraction θ did not change their price. Proceeding in this fashion, we can write the current price level as a weighted average of the past prices set by the monopolists. This is

(Recursive P) $P_t = [(1-\theta)(P^*_{t-1})^{1-\varepsilon} + (1-\theta)\theta(P^*_{t-1})^{1-\varepsilon} + (1-\theta)\theta^2(P^*_{t-1})^{1-\varepsilon} + ...]^{\frac{1}{1-\varepsilon}}$

We arrive at the (Price LM) by noting that the infinite sum of the terms on the right hand side of the above equation starting with $(1-\theta)\theta(P^*_{t-1})^{1-\varepsilon}$ equals the aggregate price level at time t-1., i.e, P_{t-1}.

We next proceed by taking the First Order Taylor approximation of the (Recursive P) equation about its steady state. This yields

$$P_t = \overline{P} + \theta(P_{t-1} - \overline{P}_{t-1}) + (1-\theta)(P_t * - \overline{P}_t *)$$

Next we use the following trick. Define $\tilde{X}_t \equiv \ln X_t - \ln \overline{X}$. Here the idea is that $\tilde{X}_t$ is the deviation of the log of the variables from the steady state. Next, we can use the properties of logs to arrive at the following relation $\tilde{X}_t \equiv \ln X_t - \ln \overline{X} = \ln\left(\frac{X_t}{\overline{X}}\right) = \ln(1 + \frac{X_t - \vec{X}}{\overline{X}}) \cong \frac{X_t - \overline{X}}{\overline{X}}$. From here it is easy to see that

$$X_t \cong \overline{X}(1 + \tilde{X}_t)$$

The First Order Taylor Approximation then is

$$\overline{P}(1 + \tilde{P}_t) = \overline{P} + \theta\overline{P}_{t-1}(1 + \tilde{P}_{t-1}) + (1-\theta)\overline{P}_t *(1 + \tilde{P}_t *)$$

In a steady state $\overline{P} = \overline{P}_{t-1} = \overline{P}_t *$. Using this result, the above equation simplifies to

$\tilde{P}_t = \theta\tilde{P}_{t-1} + (1-\theta)\tilde{P}_t *$. And using the definition of $\tilde{X}_t \equiv \ln X_t - \ln \overline{X}$, we arrive at (aggregate price) $p_t = \theta p_{t-1} + (1-\theta)\,p_t\,*$.

As p_t^* is z_t, in our notation, the aggregate price equation can be rewritten as

$$p_t = \theta p_{t-1} + (1-\theta) z_t$$

From here we can solve for z_t, namely,

(ORP from LM) $z_t = \frac{1}{1-\theta}(p_t - \theta p_{t-1})$

The next step returns to (ORP2). Here we make use of a result from ordinary stochastic difference equations. Namely, if we have a first order stochastic difference equation of the form

$$v_t = a x_t + b E_t v_{t+1},$$

then its solution is

$v_t = a\sum_{j=0}^{\infty} b^k E_t x_{t+k}$. In our model,

$v_t = z_t$, $x_t = \mu + mc_t$, $a = 1-\theta\beta$, and $b = \theta\beta$. The optimal reset price (ORP2) is actually the solution to the First order stochastic difference equation given by

(ORP FOSDE) $z_t = \theta\beta E_t z_{t+1} + (1-\theta\beta)(u + mc_t)$

From here, we substitute for the reset price in (ORP FOSDE) using (ORP from LM). This is

$$\frac{1}{1-\theta}(p_t - \theta p_{t-1}) = \frac{\theta\beta}{1-\theta\beta} E_t(p_{t+1} - \theta p_t) + (1-\theta\beta)(u + mc_t)$$

The next step is to rewrite the above equation in terms of the inflation rate, π_t. Recall that $\pi_t = P_t / P_{t-1} - 1$ so that $\pi_t + 1 = P_t / P_{t-1}$. If we take the log of both sides and use the result that

$\ln(1+x) \approx x$, then $\pi_t = p_t - p_{t-1}$. Rearranging the above equation so that we have expressions for π_t and π_{t+1} on each side, we arrive at

(NKPC MC form) $$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}(\mu + mc_t - p_t)$$

This says that today's inflation is a function of the expected inflation rate and the excess of the marginal cost over the price. The reason why inflation is a positive function of the gap between the marginal cost and price level is that it implies a larger price reset for those firms that are able to reset prices.

The New Keynesian Phillips curve is usually expressed as a function of the output gap. To do this, we first show that we can redefine the final good to be a linear function of the total labor supply, namely,

$$Y_t = A_t N_t$$

Where $N_t = \int_0^1 N_t(j) d_t$ and $A_t = \left(\int_0^1 (A_t(j)^{\varepsilon-1} dj \right)^{\frac{1}{\varepsilon-1}}$.

First, we use the demand equation and the intermediate production function. This is

$A(j)N(j) = \left(\frac{P_j}{P} \right)^{-\varepsilon} Y$. Using the price equation, this is

(1) $$A(j)N(j) = \left(\frac{W}{A(j)} \frac{1}{P} \frac{\varepsilon}{\varepsilon-1} \right)^{-\varepsilon} Y$$

Now solve for A(j) in the above equation and substitute it into the production function for the final good. This yields

$Y = \left(\int_0^1 X(j)^{\frac{\varepsilon-1}{\varepsilon}} d_j \right)^{\frac{\varepsilon}{\varepsilon-1}} = \left(\int_0^1 A(j)^{\varepsilon-1} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{W}{P} \frac{\varepsilon}{\varepsilon-1} \right)^{-\varepsilon} Y$, which implies

(2) $\left(\int_0^1 A(j)^{\varepsilon-1}dj\right)^{\frac{\varepsilon}{\varepsilon-1}} = \left(\frac{W}{P}\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}$

Now rearrange (1) so that we solve for N(j). This is

$N(j) = A(j)^{\varepsilon-1}\left(\frac{W}{P}\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon} Y$ and intergrate over j. Then

(3) $\int_0^1 N(j)dj = \left(\int_0^1 A(j)^{\varepsilon-1}dj\right)\left(\frac{W}{P}\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon} Y$. The left hand side of this equation is just N by the labor market clearing condition. Now substitute (2) into (3) which yields

$N = \left(\int_0^1 A(j)^{\varepsilon-1}dj\right)^{\frac{1}{1-\varepsilon}} Y$. This allows us to write

$Y_t = A_t N_t$ with

$$A_t = \left(\int_0^1 (A_t(j)^{\varepsilon-1}dj\right)^{\frac{1}{\varepsilon-1}}.$$

Now we go to the utility maximizing condition for labor and consumption. This is

(MRSC-labor) $\frac{N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t}$

Using the goods market condition and the aggregate production function, we can rewrite (MRSC-Labor) as

$$\frac{(Y/A)^{\eta}}{Y^{-\sigma}} = \frac{Y^{\sigma+\eta}}{A^{\eta}} = \frac{W_t}{P_t}.$$

In logs, this is

(Actual output) $(\sigma+\eta)y_t - \eta a_t = w_t - p_t$

Now we return to the (Price Equation) under the assumption that all firms can adjust their prices in the period. Given the (Price index), it follows that under price flexibility, P(j)=P, provided that A(j) is the same across firms. Thus,

$$1 = \frac{W}{A}\frac{1}{P}\frac{\varepsilon}{\varepsilon - 1}.$$

Using the (MRSC-labor) condition, this can be rewritten as

$1 = \frac{Y^{\sigma}(Y/A)^{\eta}}{A}\frac{\varepsilon}{\varepsilon-1}$, which in logs is

(Natural Rate) $a = \frac{\sigma+\eta}{1+\eta} y^n + \frac{1}{1+\eta}\mu$

The superscript on the log of output indicates that this is the natural rate of output, i.e. the output for the economy when all firms are able to adjust their prices.

To complete the derivation of the Phillips Curve we substitute w_t-p_t and a_t in the (NKPC MC form) equation using the (Natural Rate) equation and (Actual Output) equation. This yields,

(NKPC) $\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)(\sigma+\eta)}{\theta}(y_t - y^n)$

Monetary Policy

The final equation in the New Keynesian model is a monetary policy rule that determines the nominal interest rate. Namely, (Monetary Rule) $$i_t = \rho + \phi_\pi \pi_t + \phi_y y_t + \upsilon_t$$

The term, υ_t, is a true shock to monetary policy. The monetary policy coefficients, ϕ_y and ϕ_π are both positive and are understood to be set by the monetary authority.

The monetary rule specified above is referred to as a Taylor rule, being named after the economist John Taylor who first proposed it 1992. The above formulation is a simple version of the rule Taylor proposed, which had the output gap, namely, y_t-y^n, instead of actual output, and the deviation of inflation from its target, i.e., π_t-π^*, instead of the actual inflation rate in the monetary policy rule.

Solving the model

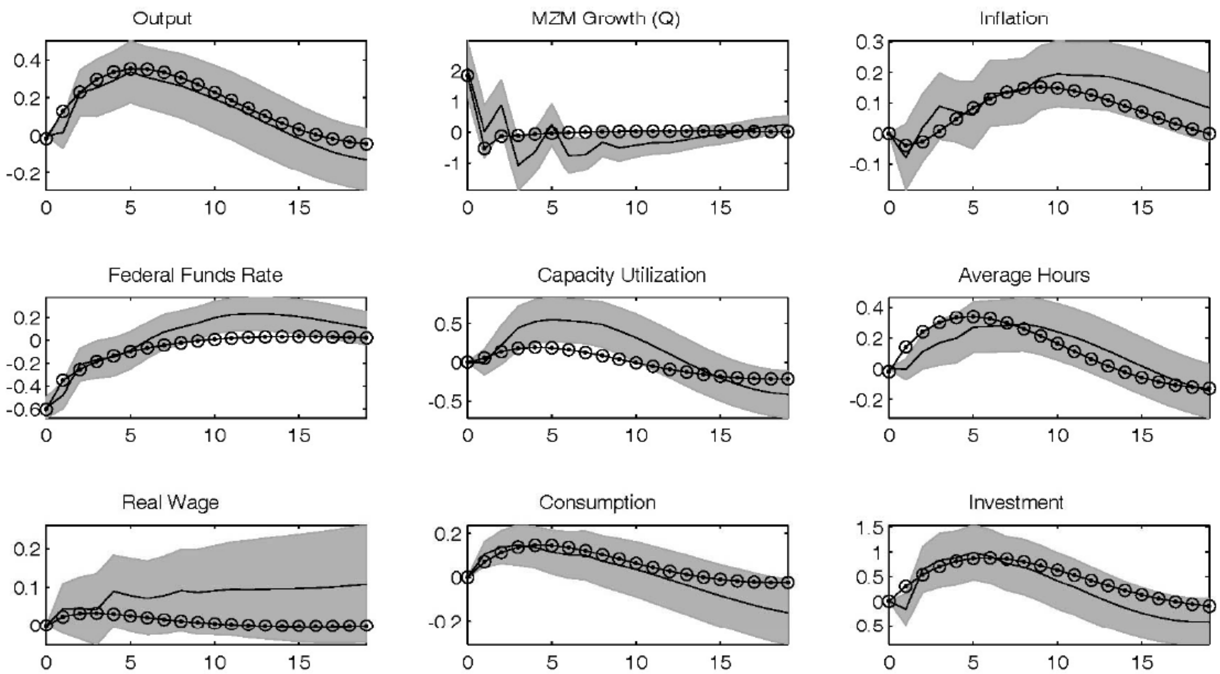
Solving the model is not trivial, even if the system is log linearized. For some very simplistic systems, it is possible to solve using pen and paper, but mostly it is done on the computer. Indeed, certain software packages are ideally suited for solving these log linearized systems. Indeed, software packages such as DYNARE have made it relatively cheap for economists to write a paper using a DSGE model.

Monetary Shocks: Developing Intuition

What is the effect of an unexpected decrease in the interest rate associated with the Taylor Rule? The first thing this does is decrease the nominal interest rate, i. By the (IS) curve this will work to increase demand provided that the interest rate rise is not perfectly offset by a rise in the price level. And here is the critical feature of the New Keynesian model; with Sticky Prices following Calvo (1983), this is not possible. Hence, the unexpected fall in the nominal interest rate translates into a decrease in the real interest rate, and hence an increase in Aggregate Demand.

From the (NKPC), the inflation rate will increase as output is above the steady state. This does not require an increase in the expected inflation rate although if this were to increase, the effect on the actual inflation rate will be even larger.

Below are the impulse responses for GDP, the real wage, the inflation rate associated with a negative shock to the federal funds rate and output based on a lecture by Lawrence Christiano called [Consensus New Keynesian DSGE Model]. Importantly, the model predicts that the shock to monetary policy causes real output and inflation to rise.



This is the general conclusion of the quantitative exercises, which in some ways are calibration exercises and in other ways are estimation exercises. Practitioners of this approach typically assign parameters based on other people's work, and then estimate the other parameters typically using sophisticated Econometric techniques. What is not obviously clear from these exercises is what these parameters are being fitted to. Some fit the model to the impulse responses that they are trying to explain, which makes these estimation exercises, or very bad calibration exercises.

Conclusion

The New Keynesian Model has become the dominant model choice of business cycle practicioners in the last ten years. There are obvious reasons for this. First, it really is the only game in town for the purpose of studying monetary economics. Second, despite the abandonment of the old Keynesian model in the last part of the twentieth century, many economists never flinched from their beliefs that Keynes was right.

Although New Keynesian models have offered economists a way to study monetary policy all the while of staying true to the principle of micro foundations, they are not subject to criticism. First, they are no better than their RBC counterparts in predicting or understanding the Great Recessions. With maybe one exception, they are silent about unemployment like their RBC counterparts. Additionally, they have had moderate success in accounting for the persistence in output changes or price changes that are implied by the VARS. For this they have added all sorts of extra parameters, many of which are not well grounded in microeconomics.

References

Benigno, Pierpaolo. (2009) New Keynesian Economics: an AS-AD View. NBER Working Paper 14824.

Calvo (1983)

Sargent, Thomas and Neil Wallace 1975. Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule. *Journal of Political Economy* 83, 241-254.

Taylor, John. 1979. Staggered Wage Setting in a Macro Model. *American Economic Review* 69(2): 108-113.

Chari, V, P. Kehoe and E. McGrattan.

Sims, Christopher (1980). "Macroeconomics and Reality". *Econometrica* **48** (1): 1–48.

Extensions

To rewrite the (GM) equation in terms of logs, one needs to make use of some other properties of logarithms.

Define $\tilde{X}_t \equiv \ln X_t - \ln \bar{X}$. Here the idea is that $\tilde{X}_t$ is the deviation of the log of the variables from the steady state. Next, we can use the properties of logs to arrive at the following relation $\tilde{X}_t \equiv \ln X_t - \ln \bar{X} = \ln\left(\frac{X_t}{\bar{X}}\right) = \ln(1 + \frac{X_t - \vec{X}}{\bar{X}}) \cong \frac{X_t - \bar{X}}{\bar{X}}$. From here it is easy to see that

$$X_t \cong \bar{X}(1 + \tilde{X}_t)$$

Now we shall apply this result to the goods market clearing condition. This yields

$$\bar{Y}(1 + \tilde{Y}_t) = \bar{C}(1 + \tilde{C}_t) + \bar{G}(1 + \tilde{G}_t)$$

Next apply the distributive law and make use of the steady state goods market clearing condition. In doing this, the above equation simplifies to

$$\bar{Y}\tilde{Y}_t = \bar{C}\tilde{C}_t + \bar{G}\tilde{G}_t$$

Let $s_c \equiv \bar{C}/\bar{Y}$ and $s_g \equiv \bar{G}/\bar{Y}$. Then we can rewrite the above equation as $\tilde{Y}_t = s_c\tilde{C}_t + s_g\tilde{G}_t$. And now applying the definition of our variables with the tildes, we arrive at

$$y - \bar{y} = s_c(c - \bar{c}) + s_g(g - \bar{g})$$

The last step in deriving the IS Equation is to substitute the above equation and Equation (FE-L) into the (EE-L). This yields

(IS) $$y = \bar{y} - \hat{s}_c[i - (\bar{p} - p)] + \hat{s}_g(g - \bar{g}) - \phi$$

Where $\phi = \frac{s_c}{\sigma} \ln \beta$, $\hat{s}_c \equiv \frac{s_c}{\sigma}$ $\hat{s}_g \equiv s_c s_g$. Note that in (IS), when the price level is below its steady state, output will be above its steady state. The (IS) equation here intuitively can be thought of as an aggregate demand curve.