Microfoundations and General Equilibrium

I. Introduction

Before beginning our study of specific macroeconomic questions, we review some micro economic theories and the general equilibrium approach to economic analysis. Recall, that microfoundations are the essence of modern macro models, and general equilibrium is the analysis of choice.

For this purpose, it is useful to review the idea of a model economy, and to set clear the difference between endogenous variables, exogenous variables, and parameters. A *model*, by definitions, is an abstraction of reality. All models, therefore, are false. In any model, some variables are endogenous whereas others are exogenous. Additionally, mathematical models will contain parameters. The definition of an endogenous variable, exogenous variable and parameter are as follows:

An *Endogenous Variable*- is a variable whose value is determined within the model itself. These are the variables the values for which we will be solving. In macroeconomic models, endogenous variables are output, consumption, wages, investment, work hours and interest rates.

An *Exogenous Variable* – is a variable whose value is assumed to be determined outside the model. From the standpoint of the model, its value is taken as given, and hence not explained. Many variables related to economic policy, such as tax rates, government expenditures, and growth rate of the money supply are treated as exogenous variables in macroeconomic models. In conducting policy analysis, the researcher will typically consider different values for these exogenous variables, and analyze how they change the values of the endogenous variables.

In addition, a model contains *parameters*. Parameters are like exogenous variables in that their values are taken as given. They are distinct, however, from exogenous variables in that they tend to represent things that are given by nature such as consumer preferences or production technologies. In the case of many exogenous variables, the idea is that they determined by some process, say a political one, or even an economic outcome. However, because the model is an abstraction of reality, it is appropriate to not model the determination of these variables. This is not the case with parameters.

Most macroeconomics models are made up of people, usually referred to as households. Additionally model contains firms, and sometimes a government. Households typically supply inputs to firms, and buy final goods and service. Firms rent or buy inputs supplied by households to produce goods and services. Households and firms determine the supply and demand for goods and inputs. Government policy also affects these supply and demands either directly by having the government demand or supply some good, or indirectly through altering the choices of households and firms, primarily through taxes. We will take up the government in a later chapter. For now, we will only deal with firms, consumers and the general equilibrium associated with their demands and supplies.

II. Theory of the Firm

We begin with the study of the firm, and specifically its profit maximization problem. A firm's objective is to maximize its profits (revenue less cost of inputs) subject to the technology constraint. The technology constraint corresponds to the production function, which gives the combination of inputs and the quantity of output.

Properties of Production Function

A production function describes the technology for combining inputs into output. For macroeconomics, we will almost always think of the output being some all encompassing good, rather than a specific good such as apples. Let Y denote the output and $X_1,X_2,,,,,X_n$ be the inputs. Then the production function is a function F such that

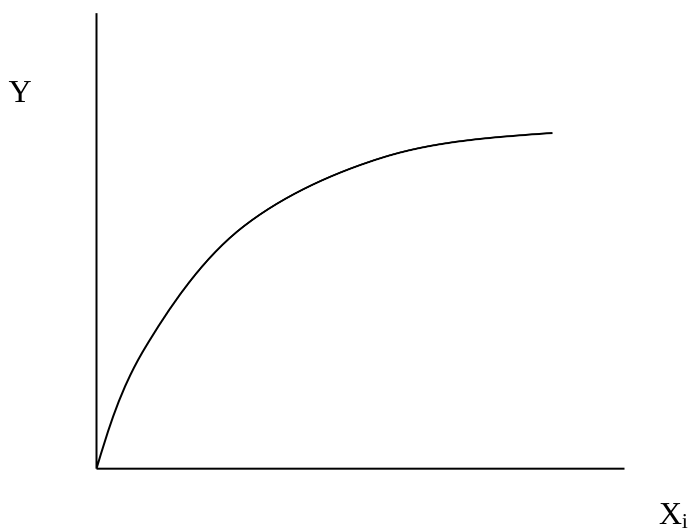
(PF) $$Y = F(X_1, X_2,,, X_n)$$

There are two key properties of the production function that are generally assumed. First, the production is increasing in each of input. This is to say that the derivative of F with respect to each individual input, $\partial F/\partial X_i$, is positive, (assuming that the function F is continuously differentiable in each of its variables). This derivative, which is the change in output associated with a change in a given output is referred to as the marginal product of factor input i.

The second key property is that the marginal product of any factor decreases as the quantity of the input increases. This is known as the *Law of Diminishing Returns*. Specifically, the *Law of Diminishing Returns* states that as one factor of production is increased, holding all other factors and technology fixed, the increases in output associated with increasing the factor (eventually) become smaller. In other word, the Law of Diminishing Returns implies a downward sloping marginal product curve. Mathematically, the law of Diminishing returns implies a second derivative which is negative, i.e., $\partial^2F/\partial X^2<0$.

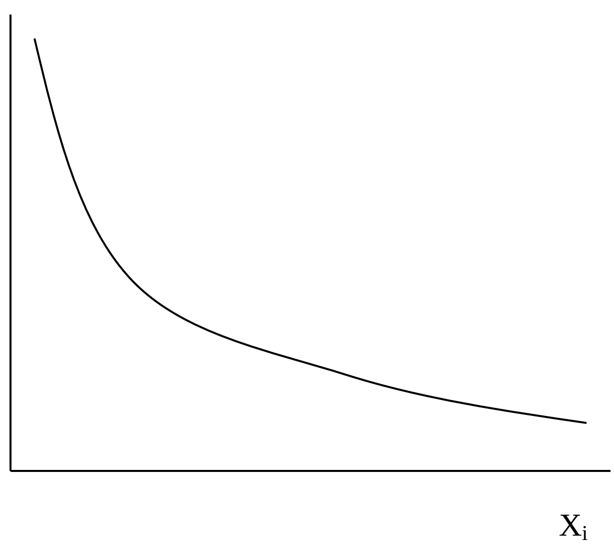
Graphically, a production function that has these two properties has the following graphical representation. It is important to note that in the graph, only one input is being increased; the quantities of all the other inputs are being held fix in this plot. The first property is apparent by the fact that the curve is upward sloping, namely output increases as the ith input increases. This corresponds to the slope or first derivative of the function being positive.

Figure 1: Production Function



The second property deals with how the slope of the production function, which corresponds to the *marginal (physical) product* of the input, changes over time. Recall that the slope of a curve at a point is the slope of the tangency line at that point. The marginal products, i.e., the slopes, are shown in Figure 2. Notice that the marginal product is measured in terms of the output. For this production function, the marginal product is a decreasing function that approaches zero in the limit. This is a special case, but one that applies to the Solow and Neoclassical growth models that we will use throughout these chapters. As we shall see, decreasing marginal product that goes to zero is the critical property of the model that drives some important results related to economic growth.

Figure 2. Marginal Product of Capital



If you look at Figure 1, you will notice that the law of diminishing returns implies a production function that is bowed outward to the origin. This latter property is what is known as called concavity. To say that a curve is concave means that the slope of the function declines as we increase the independent variable, in this case X_i. Concavity corresponds to a second derivative that is negative. Concavity has a very important economic meaning. In particular, concavity implies there are diminishing returns to the factor of production being held fixed.

Other Properties of Production Functions

Another property of production functions relates to the returns to scale. A production function can either be characterized by Constant, Increasing, or Decreasing Returns to Scale. To determine if a production functions has constant, increasing or decreasing returns to scale, we need to change **all** of the inputs by the same fraction and determine by how much output changes. If we double all the inputs and output exactly doubles, we say the production function is characterized by constant returns to scale; if we double all the inputs and output increases by more than a factor 2, we say the production function is characterized by increasing returns to scale; finally, if we double all the inputs and output increases by less than a factor 2, we say the production function is characterized by decreasing returns to scale.

Notice that the law of diminishing returns deals with the case of increasing only one input, holding all other fixed while constant returns deals with the case of increasing all inputs by the same proportion. Thus it is a separate, distinct property than returns to scale property of the production function.

Profit maximization

Throughout these chapters we will assume that production is undertaken by firms whose objective is to maximize profits. In most of these chapters we will further assume that markets are perfectly competitive and thus firms are price takers. [1]Profits are defined as revenues less costs. In our example of the production function, the firm's profits are

(Profits) $$P_Y F(X_1, X_2,,, X_n) - P_1 X_1 - P_2 X_2 - .... - P_n X_n$$

Where P_Y is the dollar/euro price of the output and P_i is the rental price of the ith factor of production. A maximizing point of the profit function is where the derivative of the function is zero. (This assumes an interior solution, where the input that maximizes profits is strictly greater than zero.) Namely, for each X_i we have the following profit maximizing condition

(MRP=MC) $$P_Y \frac{\partial F}{\partial X_i} - P_i = 0$$

Rearranging the above equation, we have that the marginal product equals the real rental price, or marginal cost

(MP=MC) $$\frac{\partial F}{\partial X_i} = \frac{P_i}{P_Y}$$

The real rental rate or price of the input, i.e. marginal cost, is the payment to the input in units of the output. Hence, nominal units are unimportant to the firm's decisions. This is

[1] In two chapters, we will consider the case where firms have some monopoly power. We will delay a description of the monopolist's problems until those chapters. Here, we describe the profit maximization problem of price taking firms.

something we shall see in the models that we study. They are real models in that nominal prices are irrelevant to the choices of firms and people. As such, the unit of account in our models will most often be its output.

III. Utility Maximization

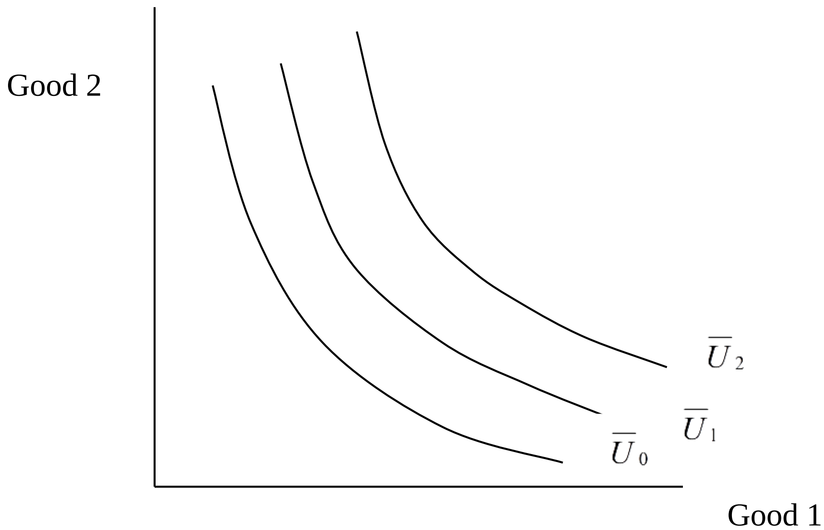
The other main economic actors in our macro models are consumers or households. We now review consumer theory from microeconomics. We assume that households derive happiness or utility from the consumption of various goods and services. Whereas micro economics typically start at the level of a preference ordering, we macroeconomists start at the level of a utility function.

More specifically, we assume that a household's utility is a function of the quantities of the goods and services it consumes. Let $U(c_1, c_2,,, c_n)$ be the household's utility associated with eating quantities $c_1, c_2,,, c_n$ of the n available goods and services in the economy. In many of our macro models, the utility function will be defined over a single consumption good and leisure.

Indifference Curves

For generating intuition, it is useful to study the case where the utility function is defined over only two goods, say apples and oranges. In this case, we can characterize the solution to the consumer's decisions graphically via the indifference curve diagram, shown below.

Figure 3: Indifference Curves



An indifference curve shows the combinations of two goods that yield the same utility to the consumer. The consumer, hence, is indifferent between any of the consumption bundles along a given curve.   As long as more consumption of each good is desirable, the indifference curve must be downward sloping. Additionally, as long as people like variety, the indifference curve, is convex, namely, bowed in toward the origin.   In effect, you would rather have 5 apples and 5 oranges than 10 oranges.

As an indifference curve represents all of the consumption bundles that yield the exact same utility level, there is a separate indifference curve corresponding to different utility levels; there is not a single indifference curve then, but a continuum of different ones. As the utility level is increased, the corresponding indifference curve lies further to the right and top of the diagram. Thus, in Figure 3, $\overline{U}_2 > \overline{U}_1 > \overline{U}_0$.

The Budget Constraint

The household's object is to purchase the consumption bundle that maximizes his or her utility. Of course, he must be able to afford that consumption bundle. The combinations of the goods that a household can afford to buy are given by its budget constraint. Let V be the dollar value of the household's wealth- the value of the things it owns, and let p_1 and p_2 be the dollar prices of the two goods. Then the household's budget constraint is given by

(BC) $$p_1 c_1 + p_2 c_2 \leq V .$$

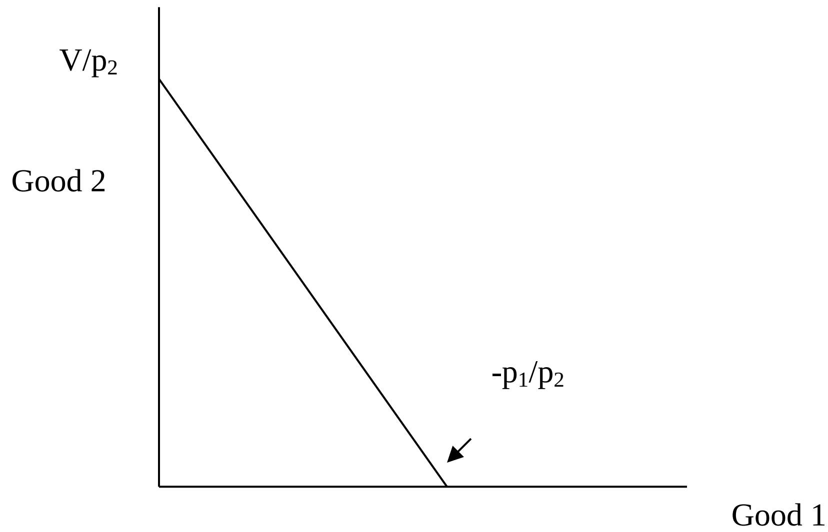
The value of things the household owns can take a variety of forms. It can own a stock or a bond. It may also own some resources, land, for example. It also owns time, 24 hours per day. Economists refer to the things that are owned by the household that it enters the model with as its *Endowments*. For all practical purposes, endowments are akin to model parameters.

For graphical purposes, it is useful to solve for c_2 in the above equation. This is

$$c_2 \leq \frac{V}{p_2} - \frac{p_1}{p_2} c_1 .$$

For the relation in which holds with equality, this is just a straight line with y-intercept V/p_2 and slope $-p_1/p_2$. This is graphed in Figure 4. The inequality means that the consumer can afford to buy any combination of the two goods that is below the budget constraint.

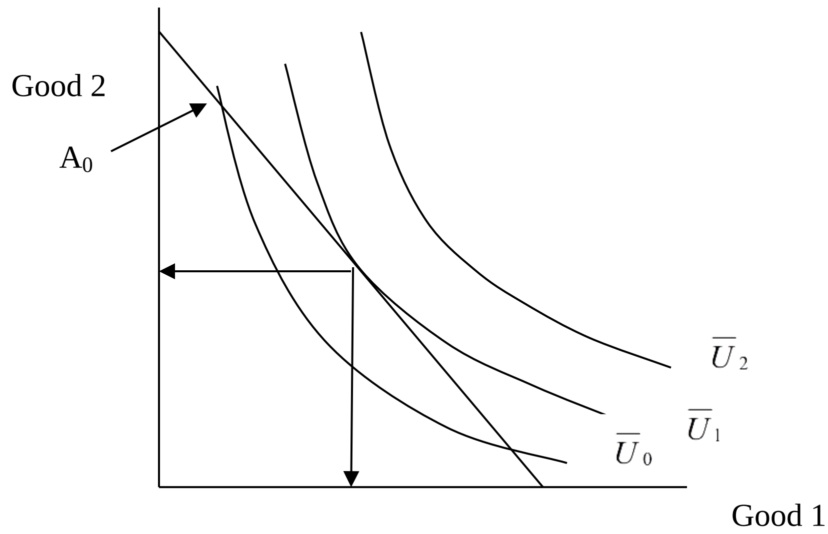
Figure 4: Budget Constraint



Utility Maximization

Maximizing utility means reaching the highest indifference that the household can reach given its budget constraint/line. The highest indifference curve that can be reached is the one that is just tangent to the budget line. In Figure 5, this indifference curve corresponds to $\overline{U}_1$. The household could certainly afford to buy the consumption bundle given by point A_0 which is on $\overline{U}_0$, but clearly he is not optimizing in this case.

Figure 5: Utility Maximization



We want to characterize mathematically this optimizing behavior. Recall, that the slope of a function or curve at a point is the slope of the tangent at that point. The tangent just happens to be the budget constraint so that the slope at the utility maximizing point is $-p_1/p_2$. Therefore, the slope of the indifference curve at the utility maximizing consumption bundle is equal to the number, $-p_1/p_2$. Although this is what the slope must equal if the household is maximizing, it does not tell us what the slope is in terms of utility and the goods. Recall, that by definition, the slope is the change in the dependent variable associated with a change in the independent variable. Here, the dependent variable is utility and there are two independent variables. The indifference curve $\overline{U}_1$ represents all the consumption bundles for which $\overline{U}_1 = U(c_1, c_2)$. Along $\overline{U}_1$, we can change c_1 and c_2 with no change in utility. The change in total utility along this indifference curve is

(Change in Utility) $\partial U / \partial c_1 \, \Delta c_1 + \partial U / \partial c_2 \, \Delta c_2 = 0.$

This just says that the change in total utility equals the change in c_1 times the change in utility from changing $c_1, \partial U / \partial c_1$, plus the change in c_2 times the change in utility from changing c_2, $\partial U / \partial c_2$. The right hand side of equation (Change in Utility) is zero because we are moving along the same indifference curve, so the level of utility does not change. Rearranging equation (Change in Utility) we find that the slope of the indifference curve, $\Delta c_2 / \Delta c_1$, is

(MRSC) $\quad \Delta c_2 / \Delta c_1 = -\partial U / \partial c_1 / \partial U / \partial c_2$.

This is to say that the change in c_2 resulting from a change in c_1 along the indifference curve is the ratio of the marginal utilities. This ratio of marginal utilities is what is called the *Marginal Rate of Substitution in Consumption*. It is the rate the consumer is willing to exchange the two goods so as to keep his utility constant.

Since we now have the slope of the indifference curve, and we know that it must equal the ratio of the prices, $-p_1/p_2$, it follows that the utility maximizing bundle, (c_1^*, c_2^*) satisfies

(MRSE=MRSC) $\quad p_1 / p_2 = \partial U / \partial c_1 / \partial U / \partial c_2$.

The left hand side is what is referred to as the *Marginal Rate of Substitution in Exchange* as it is the rate at which the household can exchange good 2 for good 1. Utility maximization requires that the marginal rate of substitution in exchange be equal to the marginal rate of substitution in consumption.

In what follows, it will be more useful to flip the numerators and denominators of equation (MRSE=MRSC). This is

(Utility Max) $p_2 / p_1 = \partial U / \partial c_2 / \partial U / \partial c_1$.

Note, that the only thing that matters is the real or relative price of good 2 in terms of good 1. For this reason, we can set the price of good 1 to 1, and express all prices in terms of good 1.

Although it is natural to think of the two goods in this example as apples and oranges, we shall see that we can interpret the two goods as consumption and leisure, or consumption today and consumption tomorrow. The consumption/leisure interpretation is the one we will use in Part III of the book when we look at work hour differences across countries and in Part V when we study the business cycle. The consumption today/consumption tomorrow interpretation will be used in Part IV when we study the savings decision and pension reforms as well as in Part V when we study business cycles. This reinterpretation of goods to cover different utility maximization problems associated with different economies is one of the key insights that grew out of a series of papers by Kenneth Arrow, Gerard Debreu and Lionel McKenzie in the 1950s. Although micro economists, these researcher developed the general equilibrium theory that is the cornerstone of modern macroeconomics.

IV. General Equilibrium

Kenneth Arrow, Gerard Debreu

Kenneth Arrow is an American economist. He was born in 1921 and earned his Ph.D. from Columbia University in 1951. He received the Nobel Prize in Economics in 1972. He spent most of his academic career at Stanford University. In addition to general equilibrium theory, Pr. Arrow made important contributions to the theory of social choice.

Gerard Debreu was born in 1921. He passed away in 2004. He studied mathematics at École Normale Supérieure in Paris in 1946. While an assistant in the Centre National de la Recherche Scientifique between 1946 and 1948 he made the switch from Mathematics to Economic. From there his path brought him to the United States. He first worked for the Cowles Commission at the University of Chicago, where Kenneth Arrow was also employed. He was a faculty member at Yale University, Stanford University and University of California. He received the Nobel Prize in economics in 1983.

The other key characteristic of modern macroeconomics is that it makes use of general equilibrium analysis. General Equilibrium requires that all markets must clear simultaneously. Effectively, the profit maximization gives us the supply of outputs and demand for inputs whereas utility maximization gives us the demand for outputs and the supply of inputs. Market clearing amounts to equating the supply and demand for each good or input. General equilibrium requires that we do this for all goods and inputs. We know that we have solved for the general equilibrium of our model when prices and quantities are expressed as function of the model's parameters and exogenous variables. That is the key; the solution must be only in terms of parameters and exogenous variables.

To illustrate the general equilibrium approach in the simplest way possible, let us consider a model economy without firms. Such an economy is known as a pure exchange or endowment economy, where households are endowed with goods and trade amongst themselves. Thus, for some goods, some householders are net suppliers whereas for other goods, they are net buyers. Let us suppose that there are a large number, N, of type A households and an equal number of type B households. Both type A and type B households derive utility from two goods, c_1 and c_2 according to the following utility function

$$u(c_1, c_2) = \ln(c_1) + \alpha \ln(c_2)$$

Type A households are endowed with (e_1^A, e_2^A) of goods 1 and 2 whereas type B households are endowed with (e_1^B, e_2^B) of goods 1 and 2.

Let P_1 be the nominal (money) price of good 1 and let P_2 be the nominal price of good 2. Then the budget constraint of the type i=A,B household is

(BC Example) $$p_1 c_1^i + p_2 c_2^i \leq p_1 e_1^i + p_2 e_2^i$$

To derive the type i household's demand for each good, we first use the condition (Utility Max), i.e., the marginal rate of substitution in consumption equals the marginal rate of exchange (MRSC=MRSE). With our specific utility function, this is $\frac{\alpha c_1^i}{c_2^i} = \frac{p_2}{p_1}$, which can be rewritten as

(Utility Max- Example) $$p_2 c_2^i = \alpha p_1 c_1^i$$

The next step is to use this rewritten (Utility Max- Example) condition and substitute it into the budget constraint (BC Example). This yields

$$p_1(1+\alpha)c_1^i \leq p_1 e_1^i + p_2 e_2^i .$$

Now solve for c_1. This gives us household i's demand for good 1

(Good 1 Demand) $$c_1^i = \frac{p_1 e_1^i + p_2 e_2^i}{p_1(1+\alpha)} = \frac{1}{1+\alpha}(e_1^i + \frac{p_2}{p_1} e_2^i) .$$

Notice that we have replaced the less than or equal sign with the equal sign since an optimizing consumer would always choose to spend all of its income. We could use

(Good 1 Demand) and the (Utility Max- Example) to derive an analogous equation for the demand for good 2.

The next step is to use these demand functions (Good 1 Demand) with the market clearing conditions. There are two markets: the market for good 1 and the market for good 2. Thus, the market clearing conditions are:

(Good 1 MC) $$Nc_1^A + Nc_1^B = Ne_1^A + Ne_1^B$$

(Good 2 MC) $$Nc_2^A + Nc_2^B = Ne_2^A + Ne_2^B$$

Let us start with good 1. Begin by inserting the demand for good 1 by the two types of households into (Good 1 MC). This yields

$$\frac{1}{1+\alpha}[(e_1^A + e_1^B) + \frac{p_2}{p_1}(e_2^A + e_2^B)] = e_1^A + e_1^B .$$

With a little algebra, we arrive at the following expression for the relative price

(GE price) $$\frac{p_2}{p_1} = \alpha \frac{(e_1^A + e_1^B)}{e_2^A + e_2^B} = \alpha \frac{e_1}{e_2} .$$

where e_1 and e_2 are the total endowments of each of the two goods. This is the solution to the model as the prices are in terms of the parameters and endowments of the model. Notice that the more households value good 2, (i.e., larger α), or the smaller the total endowment of good 2 is relative to good 1, the more valuable good 2 is relative to good 1. This is intuitive.

Now one might think that we can take the above result together with the market clearing condition for good 2 (Good 2 MC and solve for the nominal prices, but we cannot. If we use the demands for good 2, inserting them into the market clearing condition, we arrive at the exact term as above (GE price). This should not be surprising as the relative price or real price is what people care about in making their decisions.

Let us use the letter, p, to denote the relative price. Using the (Demand Good 1), we can now solve for the equilibrium consumption allocation for both type of consumers. This is

(GE C1) $$c_1^i = \frac{e_1^i}{1+\alpha} + \frac{\alpha}{1+\alpha} e_2^i \frac{e_1}{e_2}$$

This is the equilibrium consumption of good 1 by the type i={A,B}. The equilibrium consumption of good 2 can be computed by using (19) and (20) with (13). These four consumption quantities and the relative price constitute the model's equilibrium solution.

Dynamic General Equilibrium: Steady State and Balanced Growth Path

The example given above involved a static model, with only a single period. As modern macro models are dynamic, the solutions are more involved. For this reason, dynamic analysis often focuses on a particular type of equilibrium, which is referred to as a steady state equilibrium or in the case of growth, a balanced growth path. The steady state equilibrium is the easiest one to characterize as it requires that variables either never change or if they change, change by a constant percentage every period. For the economy to be at the steady state equilibrium or its balanced growth path, it must be the case that the economy starts off with the "right" initial conditions.

Utility maximization and profit maximization are not all that different in dynamic models. However, rather than tackle those issues here, we postpone them until later and consider a dynamic model that is based on demography rather than economics. Here we just want to illustrate the notion of a steady state and balance growth path.

The simple demographic model we study is based on the following three assumptions:

1. People live two periods and are either young or old.
2. Young people each have n children.
3. Old people die at the end of the period; young people all survive into old age.

At the start of time, there are a given number of old people and young that are alive. This represents the initial condition, a sort of endowment. We will consider different initial old and young populations and consider different birth rates, n, among models to illustrate the concepts.

Example 1. Steady State.

Here we assume that each young person has one child and that initially there are 50 young people and 50 old people alive. This gives rise to the population dynamics shown in the below table.

Steady State Example

	t=0	t=1	t=2
Young	50	50	50
Old	50	50	50

Example 1 is clearly a steady-state equilibrium- the population is constant in each period at 100. Moreover, there are always 50 young people and 50 old people in any period. Nothing ever changes. A snapshot of the world every period is the same. Hence, we have a steady state.

Example 2. No Steady State, but Convergence

Remember to have a steady state or balanced growth path, the initial conditions have to be just right. To illustrate, in this example we assume that each young person has one child and that initially there are 50 young people and 25 old people alive. This gives rise to the population dynamics shown in the below table.

Non-Steady State Example

	t=0	t=1	t=2
Young	50	50	50
Old	25	50	50

This is not a steady state equilibrium as the total population and the distribution of young and old agents is not the same in all periods starting with the first t=0. Although the economy is not in its steady state in period 1, it reaches it in the second period. Hence the economy converges to its steady state equilibrium. It does not matter if the economy reaches the steady state in period 1, 2 or 500. As long as one period is different from the others, we call this a non-steady state path.

Example 3. Balanced Growth Path

Next, we illustrate the concept of a balanced growth path. Here we assume that each young person has two children and that initially there are 100 young people and 50 old people alive. This gives rise to the population dynamics shown in the below table.

Balanced Growth Example

	t=0	t=1	t=2
Young	100	200	400
Old	50	100	200

Reinforcement of Concepts*: If you understand these concepts, can you say what the equilibrium would look like in the above example if we started with an equal number of young and old people? Does this world converge to the steady state where population doubles? What if we allowed people to live 3 periods where each person has one child in the middle period?*

This is an example of a balanced growth path. What we have is that the total population as well the number of young people and old people all double each period. Everything looks balanced in this sense.

V. Conclusion

This chapter provides the microfoundations that you will need to successfully study all the models in this book. The key relations are: marginal product is equal to the marginal cost (from profit maximization) and the ratio of marginal utilities (Margrinal rate of substitution in consumption MRSC) equals the relative price (marginal rate of exchange). These give us the supplies and demands for goods. The final step is to set supply equal to demand using the market clearing conditions. Because the equilibrium is often hard to compute in dynamic models, we often focus on the steady state or balanced growth path equilibrium.