Introduction to Quantitative Macroeconomic Theory

by

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I. Book Overview

This book grew out of lecture notes that Prescott developed for an advanced undergraduate course at Arizona State University and that Parente developed for a Masters course at the University of Illinois at Urbana-Champaign. Our goal in these courses was to give students the opportunity and experience of using models to develop sound intuition regarding various macro questions and puzzles. By being able to develop sound economic intuition, it is hoped that tomorrow's decision makers will formulate good economic policy, leading to a richer society, and a happier world!

In many ways, the book grew out of a frustration with the way macroeconomics has been taught at the undergraduate level, something which is inherently difficult on account that frontier research in economics is based on mathematical models most of which are far too technical for most undergraduates to handle. Graphical analysis, in contrast, which was the basis for almost every undergraduate macro textbook prior to 1980, is for the most part inadequate for the purpose of gaining insight into frontier research. Effective teaching of macroeconomics at the undergraduate level today requires presenting models at an appropriate level, one not devoid of mathematics, but not so involved that the student requires an advanced degree in mathematics. A basic understanding of calculus is sufficient for mastering this book.

Another reason graphical analysis is insufficient is that most questions that are of interest to macroeconomists are quantitative in nature. It is simply inadequate to tell policy makers that they should decrease the tax rate on capital gains, or increase the amount of infrastructure in the country. Policy makers need to know by how much taxes should be raised or lowered, or how much additional infrastructure will add to economic growth. It i not enough to say what will be the direction of the change. Every action will have a cost, or opportunity cost, and good policy formulation requires that costs be compared to benefits.

This book takes a quantitative approach to macroeconomics. Specifically, it provides quantitative answers to many important macroeconomic questions that confront us, without requiring that students be expert statisticians. The quantitative analysis used in this book is based on model calibration, an approach that views the model as a sort of measuring device. In calibration, the model is treated as an input in the analysis that the researcher uses to derive the implications of theory. Calibration does not seek to find the model that is most likely to have generated the data we observe. This is the goal of model estimation. All models, by being abstractions of reality, are necessarily false. In this sense, the calibration procedure is radically different from model estimation.

Another difference between this book and most other undergraduate textbooks in macroeconomics is that most of the chapters investigate different questions and economic puzzles. For this reason, the different chapters present different models, each tailored to the question at hand. This is consistent with the frontier research. There is no single

macro model that is used to answer every question. There is no pretense in this book to develop such a model, an approach that dominated macro textbooks written in the 20th Century.

II. The Evolution of Macroeconomics

The book reflects the dramatic transformation in macroeconomics over the last 30 years, particularly in terms of methodology. A watershed in this evolution is the 1982 paper published by Finn Kydland and Edward Prescott in *Econometrica*. This paper truly revolutionized macroeconomics, and completed a transformation that began in the early 1970s. That paper made two important contributions. First, it introduced real business cycle theory- the theory that productivity shocks accounted for a large part of the fluctuations of output over the US business cycles since the end of World War II. Second, it introduced to economics the methodology known as model calibration- a methodology that allows one to derive the quantitative implications of theory.

Over the last 30 years, model calibration has proven to be an extremely powerful tool allowing macroeconomists to test and develop all sorts of theories and to evaluate alternative policies. For example, this methodology has been used to study the problem of economic development. Specifically, researchers have employed this methodology to determining the factor or sets of factors account for the huge differences in living standards that currently exist across countries, and that have caused some countries such as S. Korea, Taiwan, and China to experience unprecedented growth episodes. This methodology has been also used to answer the question why Americans on average work

currently more than their European counterparts whereas in the 1970's the opposite was true. It has also been used to answer whether the stock market in the 1990's was overvalued. It has been used to address the rise in the skill premium, the ratio of the wages of those with a college education to those of those with a high school degree or less, that started in the 1980s. It has been applied to a large range of issues that are not entirely macro in nature. For example, it has been used to address the rise in female participation in the labor force that started in the 1970s, and trends in marriages and divorce.

Some of the questions you will address in this course are as follows: Why are some countries so much richer than others?; Why did hours worked per adult fall by one-third in Western Europe, and not in Canada and the United States, in the 1970–1995 period?; Would a higher labor tax rate in the United States raise or lower government revenues? How much of the business cycle is accounted for by productivity shocks? The models and tools we develop will allow us to analyze the sustainability of social security systems in countries such as the United States, Italy, and France, and evaluate the feasibility and welfare effects of alternative systems.

Macroeconomics Before the Transformation

Prior to the transformation, macroeconomics was really a distinct field, separate from the rest of economics, with little hope of being integrated. It was mostly concerned with the short-run behavior of the economy, namely, business cycles. Business cycles refer to the upward and downward movements in output that are commonly identified with economic

expansions and recessions. In the data, they correspond to the upward and downward spikes around a smooth trend in output. These can be seen for the US economy in Figure 1. US Output, which is the economy's Gross Domestic Product, is plotted below on quarterly basis (i.e. every 3 months). The shaded areas represent the major downturns, i.e., recessions, experienced by the US economy.

Figure 1:US real GDP, 1947-2014 by quarter
in Billions of Chained 2009 dollars
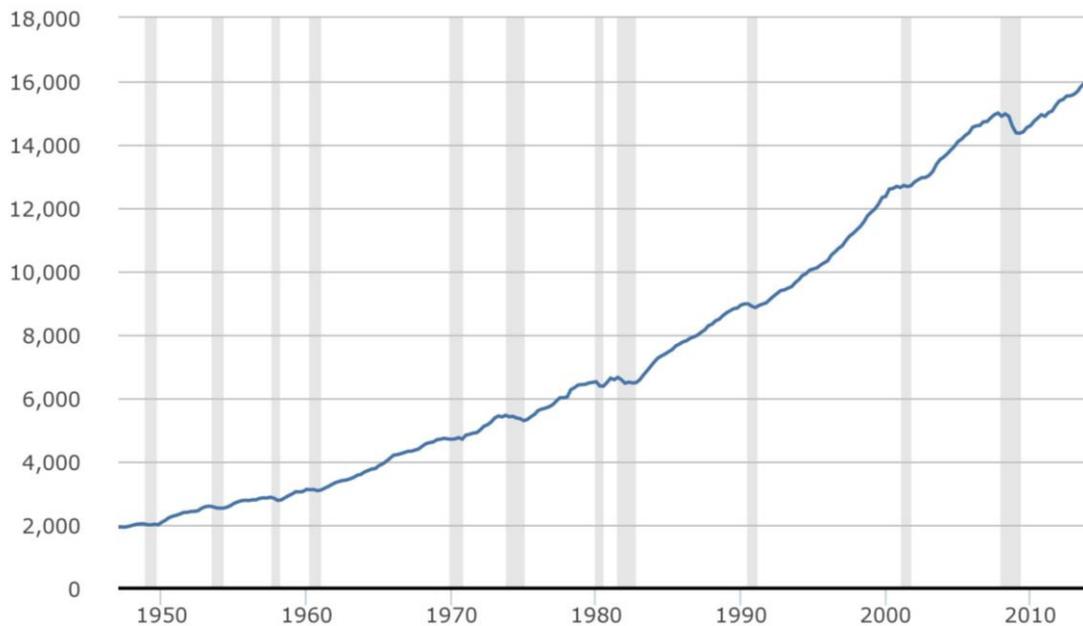
Source: U.S. Department of Commerce: Bureau of Economic Analysis
Shaded areas indicate US recessions - 2014 research.stlouisfed.org

Prior to the transformation, business cycles theory was devoid of capital theory, and separate from growth theory. The theory of the business cycle that dominated in this period was attributed to John Maynard Keynes. His 1936 book, *The General Theory of*

Employment, Interest, and Money, shaped the direction of macroeconomic research and thinking over the next 40 years. Substantively, Keynesian economics viewed fluctuations of output, namely, the business cycle as a demand driven phenomenon. A key feature of this theory is that not all prices were free to adjust clearly so as to clear markets. On account of this "price stickiness", a situation of involuntary unemployment – at the current prices, the quantity supplied of a labor exceeded the quantity demanded by firms- could arise and persist. The implication of this market failure was the need for government intervention.

Keynesian models were represented by a system of *structural equations*. They are called structural for the reason that they describe the structure of the economy. Mathematically, a structural equation states the relation between a set of *endogenous* variables and a set of *exogenous* variables. An *endogenous variable* is one whose value is to be determined within the system. In contrast, an *exogenous variable* is one whose value is taken to be determined outside the system. In addition, a structural equation contains parameters, which relate endogenous and exogenous variables. Parameters differ from exogenous variables in that their values are viewed to be invariant to any changes in the model. In contrast, the modeler will often change the value of the exogenous variables so as to see how the system adjusts to this new value.

An example of a structural equation in the Keynesian model is the consumption function, which states that the total amount that private citizens eat in the economy, C, is a linear function of aggregate income, *Y*, less taxes, *Tx*. More specifically,

$$C = a + b(Y - Tx),$$

The letter *a* is a structural parameter and equals the amount that people would consume in the case they had no income. It is often referred as *autonomous consumption*. The letter, *b,* is another structural parameter of the model and represents the increase in consumption associated with an additional increase in after tax income, the so called marginal propensity to consume. The endogenous variables in this equation are consumption and income whereas the exogenous variable is taxes.

With the system-of-equations approach, each equation in the system is determined up to a set of parameters. The simple prototype system-of-equations Keynesian macro model has an investment equation, i.e., the amount of machines, and structures, and houses firms and household's buy, a money demand function, and a Phillips curve, (i.e., a negative relationship between unemployment rate and the inflation rate first noted by William Phillips in 1958) in addition to a consumption function. Behind each of these

equations, there was a rich empirical literature and, in the case of the consumption function, money demand function, and investment equation, some serious theoretical work.[1] The theoretical work, however, examined each equation separately. This is to say, for example, that the theoretical underpinnings for the consumption function were neither considered nor derived in the context of money demand.

The final step in the Keynesian macroeconomic paradigm was to use the tools of statistical estimation theory to select the parameters that precisely define the structural equations.   In model estimation, the output of the exercise is the model itself. In effect, the macroeconomist's job was to "identify" the model that was the one most likely generated by the data. Using econometric techniques, macroeconomists estimated the parameters of the model. [2] Lawrence Klein at the University of Pennsylvania, who was awarded the Nobel Prize in economics in 1980, was important in pioneering this approach and developing the Keynesian macroeconometric models of this period.

There were numerous problems with this approach. First, the models even in the short-run performed poorly in forecasting the path of the economy. Another problem was identified by Robert E. Lucas of the University of Chicago, another Nobel Prize recipient in economics. As Lucas pointed out, a key assumption in the system-of-equations approach is that the equations are *policy invariant*. In the context of the consumption

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[1] Milton Friedman and Franco Modigliani were each awarded the Nobel Prize for their theoretical work on the consumption function, Friedman in 1976 and Modigliani in 1985. James Tobin was awarded a Nobel Prize in 1981 for his work on money demand, and Trygrve Haavelmo was awarded the Nobel Prize in 1989 for his work on the investment equation.

[2] As explained by Lucas and Sargent (1979), the macroeconometrician did not actually estimate the structural parameters. Instead, they estimated the reduced form parameters, which were complicated functions of the structural parameters. In general, it was not possible to back out the values of the structural parameters from the estimated reduced form values.

function written above, this meant that the autonomous consumption parameter, *a*, and the marginal propensity to consume parameter, *b*, do not change in response to policy. Lucas's insight was that this assumption was inconsistent with dynamic economic theory. His criticism, which has come to be known as the *Lucas Critique,* states that it is impossible to define arbitrary decision rules, such as the consumption function or the investment function that remained unchanged to changes in the rest of the economy's structure. Lucas's concluded that the Keynesian approach could not be saved with a few alterations, and that an entirely different approach was called for.

Macroeconomics after the Transformation

The transformation of macroeconomics started in the 1970's and initially was termed the "Rational Expectations Revolution" on account that it introduced the assumption of rational expectations into macroeconomics. At a non-technical level, the assumption of rational expectations is that people take all available information into account in making decisions, and act in a way that is consistent with those decisions. The concept of rational expectations was introduced by John Muth in a finance paper he wrote in 1961 on price movements in a single market.

Initially, this revolution was seen as the start of a new school of economic thought – *New Classical Economics* – that was based on the assumptions of market clearing and rational expectations. Today this revolution is seen simply as the start of an alternative approach to macroeconomics, one that advocates dynamic general equilibrium models with strong

microeconomic foundations, and that uses the calibration methodology. This approach is heavily used today in macroeconomics.

Micro-Based Models: Economic models after the transformation are grounded in strong microeconomic theory. Models after the transformation are dynamic, fully articulated model economies with optimizing consumers and firms. Model people maximize utility given the price system, policy, and their consumption possibility set; firms maximize profits given their technology set, the price system, and policy; and markets in some sense clear.

Preferences or utility, on the one hand, determine what people choose from a given choice set. Technology, on the other hand, specifies what outputs can be produced given the inputs. In Macroeconomics after the Transformation, *preferences and technology are policy invariant*, and empirical knowledge is organized around preferences and technology. This is in sharp contrast to the system-of-equations approach, which organizes knowledge about equations that specify the behavior of aggregations of households and firms. For example, in the modern approach it is often assumed that people have preferences over consumption and leisure and that these preferences are given by a log-log utility function of the form, $\ln(c) + \alpha \ln(\ell)$. The letter α is a preference parameter that determines how much the individual enjoys leisure. Although policy may affect the consumer's choice of how much to work, it does not affect how much they like leisure, i.e., policy does not affect the value of α. By starting at the level

of preferences and technology, modern macroeconomics avoids a major problem that plagued the Keynesian systems of equations model.

General Equilibrium- Modern macroeconomics employs general equilibrium analysis. This type of analysis requires that all markets in the economy must simultaneously be in equilibrium. This is in contrast to partial equilibrium analysis that considers a single market and examines how a change in the economy, possibly policy related, changes the equilibrium in the market, ignoring all other markets and feedbacks between them. With general equilibrium, these feedbacks are taken into account. This is important, especially when examining the effects of policy changes on an economy. In general, the effect of any change will be smaller when considered in a general equilibrium context.

Model Calibration – Modern economics employs model calibration as an important tool for quantitative analysis. The calibration procedure views a model as an input in the analysis. More specifically, a model is seen as a measuring device to derive the quantitative implications of theory. In a certain sense, a model is like a thermometer; just as a

thermometer measures the temperature of some location or some organism, a model measures the effect of a policy or the implications of some feature of the economy.

There are five steps to the Calibration methodology. They are

1.   *Pose a question.* Questions can be of two varieties. First, they can be of the type that is used to test and develop theory. The second variety is of the type that explores the consequences of various policies.

Type I Question Examples: Test and Develop Theory

    a. What accounts for the huge disparity across countries?

    b. Was the stock market overvalued in the 1990s?

    c. Why do Americans work more than Europeans?

    d. What accounts for the business cycle?

Type II Question Examples: Policy Evaluation

    a. What would be the effect lowering the tax rate on capital gains?

    b. What would be the effect on welfare of making the social security system a fully funded system rather than a pay as you go system?

2. *Choose a good measuring device* – The second step in the process is to choose a model, or measuring device. In this step, both theory and data interact to choose the device. In most of the quantitative studies taken up in this book, the

measuring device is the Neoclassical Growth (or Solow Growth model, with the difference being that in the Solow model, people do not choose savings optimally). The reason for this choice is two-fold. First, it is simple enough to be presented at the undergraduate level, yet rich enough to address a large number of questions. Second, it, qualitatively and quantitatively, matches the stylized growth facts pertaining to the United States and other similarly industrialized countries since the turn of the 20th Century.

3. *Define Consistent measures*. All models are abstractions of realities and hence necessarily false. The point of a model, after all, is to strip away those features of the world that the researcher does not deem as important in order to concentrate on those features that are believed to be potentially important. The consequence of this is that a model will not have predictions for all the variables that are listed in the *National Income and Product Accounts*. And yet we will need to make comparisons between the model and the data. For instance, for many questions and those addressed within this course, the assumption that is made is that the model is closed. Hence, there is no international trade. Yet, this is clearly counterfactual; some output of the each economy is sold to foreigners, and some goods bought by domestic citizens and firms are produced abroad. A closed model economy has no exports or imports. We want to be able to make comparisons between the model economy and the actual economy, however. Given these disparities, we must therefore make certain adjustments to the data. In this example, we would need to adjust the net export category in the National

Income and Product accounts. Specifically, we need to move the amount of output in the Net Export Category into another category such as Personal Consumption or Investment. For this reason, it is critical that we understand both the income and product side of the National Accounts.

4. *Assign Parameter Values*- Here, we restrict parameters of the model so that the equilibrium properties of the model match observations. Having chosen a model with a utility function and production functions, we need to assign parameters of the model so as to derive its quantitative implications. This we do so that the equilibrium quantities and prices of the model match their counterparts for the actual economy being studied. In effect, we know the values we want for the equilibrium outcome. What we do in Step 4 is to assign values to the parameters that give us that equilibrium outcome.

In the context of the Neoclassical growth model, we will assign the parameter values so that the equilibrium of the model matches the US long-run growth observations over the twentieth century. With the model restricted in this way, we will explore its quantitative implications for a host of other issues such as international income differences, business cycles, and changes in labor hours.

Sometimes Step 4 uses econometric analysis. Calibrators sometimes use econometrics, but they do so for the purpose of estimating parameters, not the model. The calibrator is not interested in the likelihood that the model

generated the data. This is a key difference between model calibration and model estimation.

5. *Compare data generated with model with actual data* − As many of the applications of the calibration method in these notes are by way of tests of theories, we need to compare the predictions of the model to the data. What is important in this stage is that the dimensions of the data that are being compared to the model's predictions cannot be the ones used in Step 4 of the calibration procedure. By assignment of the parameters, the model trivially matches the set of observations used in Step 4 of the calibration. The test has to be along dimensions of the data not used in step 4 to assign parameter values of the model.

For deterministic environments, we compare the actual path of the model with actual data. For stochastic environments, that is model worlds where there is some uncertainty and randomness, the process is more involved, requiring us to simulate the economy over and over again to generate a sample of equilibrium outcomes, and then compare the moments of the sampling distribution with the data. What is important is that we apply the same procedures to the data generated by the model as we do the actual data.

Calibration and Mercury Thermometers.

The analogy of a model as a thermometer is really not such a stretch in the calibration procedure. A thermometer can be used to answer a number of questions, such as does my child have a fever, or should I wear a jacket? Most thermometers today are of the digital

variety, but earlier ones used mercury liquid. You might not have realized this, but those old mercury thermometers had been "calibrated" to the freezing points and boiling points of water. The maker of these thermometers simply indicated on the glass tube a line with the number 0 degree Celsius/32 degree Fahrenheit by determining the height of the mercury liquid when it was stuck in a bowl of ice water. The 100 degree Celsius/212 degree Fahrenheit point on the glass tube was marked by sticking the thermometer in a pot of boiling water and noting the line of the mercury. The temperature marks in between were based on the theory that mercury expands linearly with temperature. Note how uninteresting it is to use the thermometer to measure the temperature at which water freezes or boils; by the way the points are marked on the thermometer water freezes at 0 degrees and boils at 100 degrees Celsius. The thermometer, just as the economic model, by Step 4 of the calibration exercise trivially matches the boiling and freezing points. This is not a test of the thermometer.

III. Book Outline-

The book is divided into five parts. The first part of the book, which includes this chapter, provides a review of consumer and producer theory, as well as introduces students to the concept of general equilibrium analysis. This is Chapter 2. This is followed by a chapter that demonstrates the calibration method with an example based on the Solow Growth Model.

The next part of the book deals with several macroeconomic issues that are generally identified with the long-run. As solving most dynamic, general equilibrium micro founded macroeconomic models is challenging, we begin with a model that consists of a single period. Here the key economic decision is how households decide how much to eat and how much to work. Despite the single period abstraction, the model is useful for understanding work differences across G-7 countries. The average working age person living in the G-7 nations used to work a lot more than the US counterparts. Today, this pattern has been reversed. This is Chapter 4 of the book.

Chapters 5 – 7 proceed to study the household savings decision and how it relates to important macroeconomic issues such as social security, tax reform, and government debt. These chapters consider dynamic models, i.e., models in which there are multiple periods so as to study the savings decision of households. A key component of the household savings problem is the intertemporal budget constraint, which is determined by a household's wealth. Chapter 5 derives the household's intertemporal budget constraint. It also derives an intertemporal budget constraint for the government, and uses this constraint to make sense of the current sovereign debt crises faced by many nations, most notably, Portugal, Italy, Ireland, Greece and Spain, the so called PIGs. Having derived the household's intertemporal budget constraint, it derives the utility maximizing conditions for savings. Chapter 6 considers the saving decision and government policy using the overlapping generations model developed by Paul Samuelson (1958). In particular, the chapter uses this model to evaluate alternative pension systems. Consumers in the overlapping generations model do not live forever. Importantly, at any

point in time people of different ages are alive. This mix of young and old consumers can lead to an equilibrium with some less than desirable properties. Chapter 6 also introduces money in the overlapping generation model to derive a quantity of money type result. Chapter 7 then considers how government policy affects the saving decision using a dynastic construct where the consumer lives forever. In doing so, the chapter presents the neoclassical growth model. In this chapter we study how a reduction in the capital gains tax would change welfare.

The next part of the book takes up the study of business cycles. This is the most difficult topic to teach at this level. We begin with a chapter that documents the business cycle regularities. This is Chapter 8. We then follow this with a presentation of the real business cycle theory of Kydland and Prescott (1982) in which the importance of productivity shocks for accounting for the business cycle are examined within the context of the neoclassical growth model. The last chapter, Chapter 10, presents a second type of business cycle theory, the New Keynesian Dynamic stochastic General Equilibrium model (DSGE). Being dynamic, and using a general equilibrium approach, it is similar to the Real Business Cycle theory. The main difference is that the New Keynesian models introduce some type of market failure, often in the form of wage and price rigidity. They also include a monetary policy component, and hence have become particularly popular among central bankers. Although they are quantitative in nature, it is hard to say that they adhere to the rigors of the calibration methodology. They are far more involved than the RBC models, and hence far more challenging to solve. For this reason, Chapter 10 differs from all the others in that it does not have attempt to have the student go through

the steps of solving the model. In effect, it simply provides a basic understanding of the model set-up and an elementary outline of the solution. It also summarizes the general predictions of the models, noting failures and successes.

The last part of the book concentrates on the problem of economic development and growth, namely, why some countries are so rich and others are so poor. These are Chapters 11-15. The models presented in these chapters are not fully modern in that they do not derive consumer savings by solving a household utility maximization problem. Instead, the savings rate is treated as a model parameter. The models do, however, include profit maximizing firms and are general equilibrium in their analysis. The advantage of treating the savings rate as a parameter is twofold. First, the analysis is extremely simple. Second, despite not being fully micro-founded, these models nevertheless offer much insight into one of the most fascinating questions in macroeconomics-why some countries are so much richer than others.

References

Keynes, John Maynard. (1936). The General Theory of Employment, Interest, and Money. London: Macmillan.

Kydland, Finn. and Edward C. Prescott (1982) "Time to Build and Aggregate Fluctuations." *Econometrica* 2 (November): 1345-1370.

Lucas, Robert E. Jr. (1976) "Econometric Policy Evaluation: A Critique." *Carnegie-Rochester Conf. Ser. Public Policy* 1: 19-46.

Lucas, Robert E. Jr. and Thomas Sargent (1979) "After Keynesian Macroeconomics", *Federal Reserve Bank of Minneapolis Quarterly Review* 321:1-16.

Mortensen, Dale T. and Christopher Pissarides (1994) "Job Creation and Job Destrucition in the Theory of Unemployment." *Review of Economic Studies* 61: 397-415.

Muth, John F. (1961). "Rational Expectations and the Theory of Price Movements", *Econometrica* **29:** 315–335.

Samuelson, Paul. (1958) "An Exact Consumption-Loan Model of Intereste with or without the Social Contrivance of Money." *The Journal of Political Economy* 66: 467-482.