Neoclassical Growth Model

I. Introduction

As discussed in the last chapter, there are two standard ways to analyze the consumption-savings decision. They are

1. The long but finite-lived people who leave their children no bequests.
2. The infinitely lived families with parents caring about their children's utility.

Having introduced the first construct in the previous chapter, we now present the infinitely-lived family construct. The model that is studied in this chapter is the Neoclassical Growth Model. Effectively, the Neoclassical Growth Model is just the Solow Model but where savings is determined by the utility maximizing choices of households.

Although the Neoclassical Growth Model is the key measuring device used in the study of business cycles, we postpone this application to a later chapter. In this chapter, we study its steady state properties with the purpose of considering alternative tax policies. More specifically, we will use the model to evaluate alternative ways of generating tax revenues to pay for a necessary level of government expenditures. The question we will be answering is whether eliminating or lowering of the capital gain tax will make the average person in the US better off.

II. Model

Recall, that the Neoclassical growth model is just the Solow model but where we drop the assumption that people save a fixed fraction of their income. Instead, they choose the optimal level of savings based on maximizing their utility. Although the most general form of the Neoclassical Growth model allows for exogenous population change and technological change, we will shut down both forces in this chapter. This greatly simplifies the notation and analysis. This simplification, however, does not come at the expense of meaningful answers. For business cycle analysis and fiscal policy related questions, the answer that the calibrated simple Neoclassical Growth model gives to questions is not much different than the calibrated more complex version with technological change and population growth.

In what follows we will develop a set of necessary conditions for a competitive equilibrium by considering the maximization problems facing families and firms. We also add a financial sector of the model and consider the maximizing problems of that sector. This set of conditions will be used to determine the unique balanced growth competitive equilibrium for a parametric set of model economies. We consider the problem facing each sector in turn.

Households

We will assume that the size of families is constant and equal to one in order to simplify the notation and analysis. The household's preferences are defined over infinite streams of consumptions $\{c_t\}_{t=0}^{\infty}$ and leisure $\{l_t\}_{t=0}^{\infty}$. Households' preferences are ordered by

$$u(c_0,l_0)+\beta u(c_1,l_1)+\beta^2 u(c_2,l_2)+\beta^3 u(c_3,l_3)+.....$$

For shorthand, this infinite series is written as

$$\sum_{t=0}^{\infty}\beta^t u(c_t,l_t)$$

The parameter $0<\beta<1$ is the subjective time discount factor. The smaller it is the more people prefer consumption now relative to consumption in the future. The functional form for within period utility is the same as in Chapter 9, namely,

$$u(c_t,l_t)=\ln(c_t)+\alpha\ln(l_t)$$

In our set-up, we will have the household having one unit of time in each period to split between work and leisure. Thus, in any period

$$l_t+h_t=1$$

The household will earn wage income in each period equal to $w_t h_t$. In addition, the household will earn interest income on its deposits held at banks. We let d_t denote the deposit of the household has in his bank account at the start of period t. In this period, he will earn wage income, $w_t h_t$, and he will consume c_t. The amount by which he will add deposits or subtract deposits to his bank account will be the differences between wage income and consumption. Thus, at the end of period t, the household will have the amount $d_t + w_t h_t - c_t$ in the bank. Between this period and the start of next period, the household will earn interest on this deposit. We let i_t denote the real interest rate on the deposit of the household. Hence, the deposit the household begins with in period t+1 is

$$d_{t+1}=(1+i_t)[d_t+w_t h_t-c_t]$$

This is the household's budget constraints in each period.

Two sets of necessary first order conditions for maximization are

(H1) $$\frac{\partial u / \partial c_t}{\partial u / dl_t} = \frac{1}{w_t}$$

(H2) $$\beta \frac{\partial u / \partial c_{t+1}}{\partial u / dc_t} = \frac{1/(1+i_t)}{1} = \frac{1}{1+i_t}$$

(H1) describes consumption-leisure substitution in equilibrium: the marginal rate of substitution between consumption and leisure must equal their price ratio. Since one unit of leisure today costs w_t consumption today (that is how much consumption you forego by deciding to enjoy this one unit of leisure instead of working), we have (H1). Similarly, (H2) describes the optimal intertemporal substitution of consumption. The marginal rate of substitution between consumption today and consumption tomorrow must equal their price ratio. Since one unit of consumption today costs $1+ i_t$ units of consumption tomorrow (that is how much consumption tomorrow you forego if you decide to enjoy one unit of consumption today instead of saving it till tomorrow), we have the ratio of prices on the right hand side of (H2).

Firms

The firms face a sequence of static maximization problems, one for each date. The date t problem is

$$\max\{F(k_t, h_t) - w_t h_t - r_t k_t\}$$

where $F(k_t, h_t)$ is a neoclassical aggregate production function at date t. We will continue to use the same functional form for production that was used in the

Solow Model in Chapter 3, namely, $F(k_t, h_t) = Ak_t^{\theta} h_t^{1-\theta}$,

The necessary and sufficient first order conditions for a maximum are

(F1) $\quad w_t = F_h(k_t, h_t)$

(F2) $\quad r_t = F_k(k_t, h_t)$

Given that F is a neoclassical production function, and therefore displays constant returns to scale, payments to factors exhaust the product and there are no dividends.

Banks

To establish a key relationship between the rental rate of capital and the interest rate, we add a banking sector to the model. The addition of this sector is not essential to the results; we would get the same results if we had the household own the capital and rent it directly to firms. Instead, we shall have the banks own the capital and rent it to the firms.

In renting a unit of capital to firms, a fraction δ of the capital wears out, i.e. depreciates, in the period. In addition to buying capital and renting it to firms, banks also accept deposits from households and pay interest *i*. At the end of period *t* the households deposit whatever income they did not consume, namely, $w_t h_t + d_t - c_t$ The banks take these deposits and buy capital to be rented to firms in the next period, t+1. Thus, the capital rented to firms in period t+1 is exactly how much the household deposits in the bank at the end of the period, namely,

$$k_{t+1} = w_t h_t + d_t - c_t$$

In period *t+1* banks rent k_{t+1} units of capital to firms and earn rental income equal to r_{t+1}

k_{t+1}. The undepreciated capital $(1-\delta)k_{t+1}$ is then sold by the banks. In effect, the undepreciated part of capital $(1-\delta)k_t$ is reversed engineered at no cost back into the final good and becomes part of the supply. This will become apparent in the market clearing condition described later on in this section.

Banks must pay households interest on their deposits. As the deposits of households made at the end of period t are equal to $w_t h_t + d_t - c_t$, and this is exactly the amount capital bought by banks, k_{t+1}, it follows that the interest and principal payments to households in period t+1 are

$$(1+i_t)[w_t h_t + d_t - c_t] = (1+i_t)k_{t+1}$$

Additionally, since the balances in the household's banking account at the start of period t+1 are $d_{t+1} = (1+i_t)[w_t h_t + d_t - c_t]$, it follows that

(B1) $$d_{t+1} = (1+i_t)k_{t+1}$$

We have only one more important result to derive from the banking industry. We assume that the banking industry is perfectly competitive and the intermediation technology has constant returns to scale. An implication of these assumptions is that in equilibrium banks neither make a profit nor suffer a loss. They break even in equilibrium. Hence, payments must equal he receipts. Recall payments in period t+1 are $(1+i_t)k_{t+1}$ and receipts are $r_{t+1}k_{t+1}+(1-\delta)k_{t+1}$. The zero profit condition thus requires that

(B2) $$r_{t+1} = i_t + \delta$$

This just says that the rental rate on capital must cover both interest costs and depreciation costs if the bank is to break even.

A final assumption is that initially balances are

$d_0 = (r_0 + 1 - \delta) k_0$.

The reason for this assumption is that it results in the net worth of banks being zero and, therefore, zero dividends paid by banks.

Goods Market Clearing

At time t the supply of goods is given by output in period t and the undepreciated capital $(1-\delta)k_t$ sold by banks. The demand for goods is given by household consumption and bank purchases of capital for tomorrow's use. Hence,

(M) $$c_t + k_{t+1} = F(k_t, h_t, t) + (1-\delta) k_t$$

Definition of Competitive Equilibrium

A Competitive Equilibrium is a sequence of household choices $\{c_t, h_t, l_t, d_{t+1}\}_{t=0}^{\infty}$, a sequence of bank variables $\{k_t, d_{t+1}\}_{t=0}^{\infty}$, a sequence of firm choices $\{k_t, h_t\}_{t=0}^{\infty}$, and prices $\{w_t, r_t, i_t\}_{t=0}^{\infty}$ that satisfy

(i) The family maximizes utility subject to its budget constraints

(ii) Banks maximize profits subject to their technology constraints

(iii) Firms maximize profits subject to their technology constraints

(iv) Markets clear

Market clearing means the following: (1) deposits of households equal deposits held by banks; (2) labor sold by households equals labor purchased by firms; (3) goods market clearing (M) and (4) capital services sold by the banks equal capital services bought by the firms at every date.

Solving for the Steady State Equilibrium

In the previous section we developed seven necessary and sufficient conditions for the competitive equilibrium. Using the specific functional forms for utility and the production function, these 7 conditions are

(H1) $$\frac{1-h_t}{\alpha c_t} = \frac{1}{w_t}$$

(H2) $$\beta \frac{c_t}{c_{t+1}} = \frac{1}{1+i_t}$$

(B1) $$d_{t+1} = (1+i_t)k_{t+1}$$

(B2) $$r_{t+1} = i_t + \delta$$

(F1) $$w_t = (1-\theta)A\left(\frac{k_t}{h_t}\right)^{\theta}$$

(F2) $$r_t = \theta A\left(\frac{h_t}{k_t}\right)^{1-\theta}$$

(M) $$c_t + k_{t+1} = Ak_t^{\theta} h_t^{1-\theta} + (1-\delta)k_t$$

Recall that in the Solow model, there was no savings/consumption decision by the households; households were assumed to always save a fraction s of their income. This made the model trivial to solve, not only the balanced growth path equilibrium but the non-balanced growth path equilibrium, i.e., the transitional dynamics. In fact, we could mechanically solve the model using excel: given K_t and N_t in period t, $K_{t+1}$was easily determined by the capital stock law of motion equation $K_{t+1}=(1-\delta)K_t+sY_t$ and N_{t+1} was given by the assumption that population grew exogenously at rate n.

With the family chosen the optimal amount of savings, solving this model becomes a challenge. Even though the law of motion of the capital stock is the same, the fact that the household is maximizing over an infinite horizon means that there is no clear link between K_t and K_{t+1}. It is not generally possible to solve the equilibrium path for any initial K_0 and N_0 with pen and pencil. The exception is the steady state where if we start with the right amount of capital, the equilibrium prices and quantities each period never change.

The following steps allow us to solve for the steady state equilibrium of the model. We start with (H2). In the steady state, consumption is constant and so it follows from (H2) that

(i^{ss}) $$i^{ss} = 1/\beta - 1$$

Now that we have solved for the steady state real interest rate use (B2) to solve for r^{ss}

(r^{ss}) $$r^{ss} = i^{ss} + \delta = 1/\beta - (1-\delta)$$

Next use (F2) and the solution for r^{ss} to solve for k^{ss}/h^{ss}. As

$r = \beta^{-1} - (1-\delta) = A\theta\left(\frac{h}{k}\right)^{1-\theta}$ it follows that

$$(k^{ss}/h^{ss}) \qquad \frac{k^{ss}}{h^{ss}} = \left[\frac{\theta A}{\beta^{-1} - (1-\delta)}\right]^{1/(1-\theta)}.$$

This allows us to solve for h^{ss} as a function of k^{ss}, namely,

$$h^{ss} = \left[\frac{\beta^{-1} - (1-\delta)}{A\theta}\right]^{1/(1-\theta)} k^{ss}.$$

Next use (F2) with our solution for (k^{ss}/h^{ss}) to solve for w^{ss},

$$(w^{ss}) \qquad w^{ss} = A(1-\theta)\left[\frac{\theta A}{\beta^{-1} - (1-\delta)}\right]^{\theta/(1-\theta)}$$

Next use (H1) to solve for c^{ss}.

$$(c^{ss}) \qquad w^{ss}(1-h^{ss}) = \alpha c^{ss}.$$

Next, rewrite (M) to be a function of h^{ss} using our expression for (c^{ss}) and (k^{ss}/h^{ss}). This is

$\frac{w^{ss}(1-h^{ss})}{\alpha} + h^{ss}\left(\frac{k^{ss}}{h^{ss}}\right) = Ah^{ss}\left(\frac{k^{ss}}{h^{ss}}\right)^{\theta} + (1-\delta)h^{ss}\left(\frac{k^{ss}}{h^{ss}}\right)$, which simplifies to

$\frac{w^{ss}(1-h^{ss})}{\alpha} + \delta h^{ss}\left(\frac{k^{ss}}{h^{ss}}\right) = Ah^{ss}\left(\frac{k^{ss}}{h^{ss}}\right)^{\theta}$. We want to solve for h^{ss}.

This is

$$h^{ss} = \frac{w^{ss}}{\alpha A(k^{ss}/h^{ss})^{\theta} - \alpha\delta(k^{ss}/h^{ss}) + w^{ss}}$$

We can now substitute the solutions for w^{ss} and (k^{ss}/h^{ss}) into (h^{ss}) and have the solution for

h^{ss} in terms of the parameters. Instead of doing this, we shall go through a numerical solution using the following parameter values A=1, β=.96, δ=.10, α=1.0 and θ = .30.

Step 1: Use (i^{ss}) and the value of β=.96 to arrive at $i^{ss} = (1/.96) - 1 = .04$.

Step 2: Use i^{ss}=.04 and δ=.10 together with (r^{ss}) to arrive at $r^{ss} = .04 + .10 = .14$

Step 3: Use r^{ss} =.14, θ =.30, and A =1.0 together with (k^{ss} /h^{ss}) to arrive at $k^{ss}/h^{ss} = (.3/.14)^{1/.7} = 2.91$

Step 4: Use k^{ss} /h^{ss}=2.91 with (w^{ss}) to arrive at $w^{ss} = .7(2.91)^{.3} = .96$

Step 5. Use w^{ss}=.96 and α=1.0 with (c^{ss}) to arrive at $c^{ss} = .96(1 - h^{ss})$

Step 6: $h^{ss} = \dfrac{.96}{.10(2.91) + (2.91)^{.3} + .96} = .37$

Calibration

The numerical example above started with parameter values and then solved for the equilibrium of the model. In Step 4 of the calibration exercise, we effectively do the reverse. We have the equilibrium values for the model economy; these are just those from Step 3 of the calibration exercise obtained from adjusting the NIPA accounts. Step 4 then finds the parametric values that guarantee the equilibrium outcomes.

Although we do not have a question in mind here, it is nevertheless to go through the model calibration. Conveniently, the adjustments to the NIPA for the Neoclassical growth model are the same as the ones we made in Chapter 3 for the Solow model. The NIPA readjustments we made in Chapter 3 implies x/y=.25, k/y=2.75, and rk/y=1/3. Additionally, given that we have leisure in the model we need to add an observation for hours work per period. For the US, the average hours worked per week by the working age population is roughly 25, assuming that people have 100 hours of non-sleep and non-personal care available. As a model period is one year, and our time endowment has been normalize to 1, the corresponding observation for hours worked is h =.25.

There are five parameters of the model: A, β, δ, α and θ. As we did in the Solow model, we are free to normalize the TFP parameter, A=1. The assignment of the technology parameter, θ, proceeds along the same logic as in the parameter assignments in the Solow model: because in the model θ=rk/y and rk/y in the data is 1/3, the calibrated value of θ=1/3. Next, we find the value for δ. Here we use the capital stock law of motion together with observations k/y=2.75 and x/y=.25. The capital stock law of motion in the steady state reduces to $\delta \frac{k}{y} = \frac{x}{y}$. Using the observations for k/y and x/y, we can solve for δ=.25/2.75=.09. Next we find the subjective time discount factor, β. This we do by first imputing the rental rate for capital. Since 1/3=rk/y and k/y=2.75, r=.12. Next we use (B2) to solve for the real interest rate, i=r- δ=.12-.09=.03. From here we use (H2) to solve for the value of β using the result that in the steady state c is constant. (H2) implies β=1/(1+i)=1/(1+.03)=.97. This leaves the leisure preference parameter, α. Here we use condition (H1) together with the observations that c/y=.75 and h=.25 and the model result that labor share, wh/y=2/3. In particular, divide both sides of (H1) by y, which implies

$\frac{1-h_t}{\alpha}\frac{w_t}{y_t}=\frac{c_c}{y_t}$. Next we multiply the left hand side by h/h so that $\frac{1-h_t}{\alpha h_t}\frac{w_t h_t}{y_t}=\frac{c_c}{y_t}$. Now we can solve for α, $\alpha=\frac{1-h_t}{h_t}\frac{w_t h_t}{y_t}\frac{1}{c_t/y_t}=\frac{.75}{.25}\frac{2}{3}\frac{1}{.75}=2.67$. This completes the assignment of parameter values.

III. Adding Government Policy

Here we use the growth model presented above but appropriately modified to include a government to study questions related to public finance. We introduce three different tax rates into the model: a tax on labor income, τ_{ht}; a tax rate on capital income, τ_{kt}, and a tax rate on consumption, τ_{ct}.

Additionally, we shall assume that the government purchases some of the economy's goods, g_t, and makes lump-sum transfers back to the household, T_t. Recall that the empirical counterpart of government transfers includes any purchase by the government that is a substitute for private consumption whereas government expenditures provide no utility. This means that government expenditures on education, health, police and the judiciary are included in government transfers. Military expenditures, in contrast, are included in g_t because they do not substitute for private consumption.

The tax on capital is paid by the bank. Following the tax code, we assume that the tax on capital income is net of depreciation. With these taxes, the zero profit condition of the banking sector is

(B2) $$r_{kt+1}k_{t+1} - \tau_{kt+1}(r_{kt+1}-\delta)k_{t+1} + (1-\delta)k_{t+1} = (1+i_t)k_{t+1}$$

This simplifies to

(B2G) $$(1-\tau_{kt+1})(r_{kt+1}-\delta) = i_t$$

The household pays taxes on labor income and its consumption. This leads to the following modification of the household budget constraint.

$$d_{t+1} = (1+i_t)[d_t + (1-\tau_{ht})w_t h_t - (1+\tau_{ct})c_t + T_t]$$

The other key change to the model is the inclusion of the government budget constraint. As we are going to deal with steady state comparisons, we impose that the government run a balanced budget every period so that tax receipts equal outlays. The government budget constraint is thus

(GBC) $$g_t + T_t = \tau_{kt}(r_{kt}-\delta)k_t + \tau_{ht}w_t h_t + \tau_{ct}c_t$$

The introduction of government into the model changes several of the equilibrium conditions. First it changes the household optimization conditions (H1) and (H2). The new conditions are

(H1′) $$\frac{1-h_t}{\alpha c_t} = \frac{1+\tau_{ct}}{(1-\tau_{ht})w_t}$$

(H2′) $$\beta \frac{c_t}{c_{t+1}} = \frac{1}{1+i_t}$$

Because banks pay the taxes, the equation (B1) does not need to be modified. This is

(B1′) $$d_{t+1} = (1+i_t)k_{t+1}$$

Additionally, the goods market clearing condition (M) needs to be modified to include the purchase by the government. This is

(M′) $$g_t + c_t + k_{t+1} = Ak_t^{\theta} h_t^{1-\theta} + (1-\delta)k_t$$

Definition of a Competitive Equilibrium. Given parameter values for (A,β,δ,α,θ) and the sequence of government policies $\{g_t,T_t,\tau_{kt},\tau_{ht},\tau_{ct}\}$, the competitive equilibrium consists of household variables $\{c_t, h_t, l_t, d_{t+1}\}_{t=0}^{\infty}$, a sequence of bank variables $\{k_t, d_{t+1}\}_{t=0}^{\infty}$, a sequence of firm choices $\{k_t, h_t\}_{t=0}^{\infty}$, and prices $\{w_t, r_t, i_t\}_{t=0}^{\infty}$ that satisfy

(i) The family maximizes utility subject to its budget constraints

(ii) Banks maximize profits subject to their technology constraints

(iii) Firms maximize profits subject to their technology constraints

(iv) The Government budget constraint is satisfied every period.

(v) Markets clear

To illustrate, we solve for the competitive equilibrium with a particular set or parameter values and policy parameters. The parameters are A=1, β=.96, δ=.10, α=1.0 and θ = .30. The government policy is given by g_t=.10F(k,h), τ_{kt}=1/3, τ_{ht}=1/3 and τ_{ct}=1/5. We do not specify the transfers because given values for g_t, τ_{kt}, τ_{ht} and τ_c, its value is determined by the government budget constraint (GBC).

We again start with (H2-). In the steady state, consumption is constant and so it follows from(H2) that

(i^{ss}) $$i^{ss} = 1/\beta - 1 = .04$$

Now that we have solved for the steady state real interest rate use (B2G) to solve for r^{ss}

(r^{ss}) $$r^{ss} = \frac{i^{ss}}{1-\tau_k} + \delta = \frac{.04}{2/3} + .10 = .16$$

Next use (F2) and the solution for r^{ss} to solve for k^{ss}/h^{ss}. As

$r == A\theta\left(\frac{h}{k}\right)^{1-\theta}$ it follows that

(k^{ss} /h^{ss}) $$\frac{k^{ss}}{h^{ss}} = \left[\frac{\theta A}{r}\right]^{1/(1-\theta)} = \left(\frac{.3}{.16}\right)^{1/.7} = 2.45\,.$$

Next use (F2) with our solution for (k^{ss} /h^{ss}) to solve for w^{ss},

(w^{ss}) $$w^{ss} = A(1-\theta)\left[\frac{k^{ss}}{h^{ss}}\right]^{\theta} = .7(2.45)^{.3} = .92$$

Next use (H1) to solve for c^{ss}.

(c^{ss}) $$w^{ss}(1-\tau_h)(1-h^{ss}) = \alpha c^{ss}(1+\tau_c)\,.$$

Next, rewrite (M) to be a function of h^{ss} using our expression for (c^{ss}) and (k^{ss}/h^{ss}). This is

$\frac{w^{ss}(1-\tau_h)(1-h^{ss})}{(1+\tau_c)\alpha} + h^{ss}\left(\frac{k^{ss}}{h^{ss}}\right) = .9Ah^{ss}\left(\frac{k^{ss}}{h^{ss}}\right)^{\theta} + (1-\delta)h^{ss}\left(\frac{k^{ss}}{h^{ss}}\right)$, which simplifies to

$\frac{w^{ss}(1-\tau_h)(1-h^{ss})}{\alpha(1+\tau_c)} + \delta h^{ss}\left(\frac{k^{ss}}{h^{ss}}\right) = .9Ah^{ss}\left(\frac{k^{ss}}{h^{ss}}\right)^{\theta}$. We want to solve for h^{ss}.

This is

$$h^{ss} = \frac{(1-\tau_h)w^{ss}}{\alpha(1+\tau_c)[-\delta(k^{ss}/h^{ss}) + .9A(k^{ss}/h^{ss})^{\theta}] + w^{ss}(1-\tau_h)} = \frac{(2/3).92}{1(1.2)[-.1(2.45)+.9(2.45)^{.30}]+(2/3).92} = .35$$

IV. Calibration

In this section we use the growth model to evaluate tax policy. We shall go through the five steps of the calibration in detail and in turn.

Step 1: Pose a Question: The question we shall address involves a change in the current the tax system, with a high reliance on capital and labor income taxes with one that taxes only consumption. The tax change we envision must be revenue neutral. It must ensure that the government is able to generate the same amount of tax revenues in order to continue to buy the same amount of goods, g, and provide the same amount of lump sum transfers. Specifically, the question we ask is: What is the effect of replacing the current tax system based on income with one that taxes consumption only?

Step 2: Choice of measuring device: This is just the model of Section 4.

Step 3. Define Consistent Measures: Introducing the government policy requires that we make several adjustments to the NIPA in order for the data to conform to our model economy. The first adjustment is that consumption in the model is not consumption in the NIPA. Let C be NIPA consumption expenditures and c model consumption. Because of the consumption tax, their relation is $C=(1+\tau_c)c$. In effect, the model counterpart is NIPA C less consumption taxes.

What are consumption taxes in actuality? Consumption taxes are sales taxes, excise taxes and value added taxes, even though in the latter case, part of the value added tax is collected on the business that generated the value added. In the NIPA, these consumption taxes are included in the income category that goes under the heading of *Taxes on production and imports*. The Taxes on Production and Import category also includes property taxes. Property taxes will affect the price the consumer pays, and so such taxes are likewise considered as consumption taxes.

Of course, these taxes apply to a business when they purchase an investment good, so not all the taxes in the Taxes on Production (Tax_P) category fall on consumption. However, since we do not have a tax on investment goods, we will simply assume that 75% of the taxes on production on tax category apply to consumption. With this rule, $\tau_c c = .75 \times Tax_P$. In the NIPA (Table B-27 2012 ERP), Taxes on Production are 7.0% of GDP over the 1963-2011 period. Using 75% rule, consumption taxes average 5% of GDP over the period.

There are two important additional adjustments that we must make on account of reducing consumption by taxes paid on consumption. Since expenditures equal output, and we have reduced consumption by the consumption tax, then we must reduce NIPA GDP by the consumption tax to be comparable to model output. This means that GDP-.75 x Tax_P equals model y.

Additionally since income equals output we must subtract this component from the income side. Hence, we must subtract out this amount from the income side. Recall, that in calibrating the Solow model and the Neoclassical growth model, the Taxes on Production category is an ambiguous category in that it is not clear whether to attribute it to labor income or capital income. Given that we are attributing 75% of this category to the consumption tax, it means we only have to assign 25% of this category to labor income and capital income. We will maintain the assumption that the ambiguous category is split equally. With this assignment rule, we still maintain the ratios of this 25% being split between labor income and capital income. Making these adjustments, we still arrive at capital share of income equal to 1/3.

Another key adjustment involves government expenditures. In the model, government expenditures provide no value to households or firms. In the NIPA, these are best approximated by the defense expenditures, G_{mil}. Those government consumption expenditures that are of value to consumers such as education, health, and judicial correspond to the model lump-sum transfers, T_t. In the 2012 Economic Report of the President, G_{mil}/GDP averaged 6% over the 1963-2011 period (Table 21). Government consumption, not including defense expenditures 12% of GDP and Government Investment is 3%. This government consumption is part of lump-sum transfers in the model of GDP over this period.

With these adjustments, the empirical counterpart of model, c, is

$c = C + G_c - .75\, Tax_P$

and

$y = GDP - .75Tax_P = GDP - .05GDP = .95GDP$ since $.75Tax_P = .05GDP$

Using the data in the 2012 ERP and averaging over the period, we find that c/y=.75, g/y=.05 and x/y=.20. Also, since K/GDP in the US is 2.75 and y=.95GDP, we arrive at k/y=2.90 [1]

Step 4: Assign Parameters. The preference and technology parameters of the model are β, θ, α, δ and A. In addition, there are the policy parameters, g, τ_c, τ_h, τ_k and Tr. For the policy parameters, we use the observation that g/y in the NIPA (after adjustments) equals .05. For the taxes on productions, we use the observations that c/y=.75 as well as that $\tau_c c/GDP = .05$ as well as y=.95GDP. This yields $\tau_c = .05/(.95x.75) = .07$. For the labor income tax and the capital income tax, we appeal to estimates in the literature. From the OECD Taxing Wages 2014, the marginal tax rate on labor income for the United States is $\tau_h = .31$ and from the Tax Foundation the tax rate on capital income $\tau_k = .29$. The final policy parameter is the lump sum transfers to households. We do not specify a value for this policy parameter because we require that the government's budget be balanced each period. Given the other policy parameters, and the implied tax revenues they generate, the lump-sum taxes are equal to the value that guarantees that (GBC) is satisfied.

[1] Actually, the adjustments imply that g/y=.063, but since we like round numbers we use .05.

Turning to the technology and preference parameters: Starting with the TFP parameter, we continue to follow the convention of setting A=1. For the capital share parameter, it is set to match capital share of income in the NIPA (after our adjustments. Thus, θ=1/3. The capital stock law of motion in the steady state reduces to $\delta \frac{k}{y} = \frac{x}{y}$. Using the observations for k/y and x/y, we can solve for δ=.20/2.75=.07. Next we find the subjective time discount factor, β. This we do by first imputing the rental rate for capital. Since 1/3=rk/y and k/y=2.90 it implies r=.11. Next we use (B2G) using τ_k=.29 to solve for the real interest rate, i=(1-τ_k)(r- δ)=.71(.11-.07)=.3. From here we use (H2) to solve for the value of β using the result that in the steady state c is constant. (H2) implies β=1/(1+i)=1/(1+.03)=.97. This leaves the leisure preference parameter, α. Here we use condition (H1′) together with the observations that c/y=.75 and h=.25 and the model result that labor share, wh/y=2/3. In particular, divide both sides of (H1) by y, which implies $\frac{1-h_t}{\alpha}\frac{w_t}{y_t} = \frac{(1+\tau_c)c_t}{(1-\tau_h)y_t}$. Next we multiply the left hand side by h/h so that $\frac{1-h_t}{\alpha h_t}\frac{w_t h_t}{y_t} = \frac{(1+\tau_c)}{(1-\tau_h)}\frac{c_t}{y_t}$. Now we can solve for α, $\alpha = \frac{1-h_t}{h_t}\frac{w_t h_t}{y_t}\frac{(1-\tau_h)}{(1+\tau_c)}\frac{1}{c_t / y_t} = \frac{.75}{.25}\frac{2}{3}\frac{.69}{1.07}\frac{1}{.75} = 1.71$. This completes the assignment of parameter values.

Step 5: Test Theory/Policy Evaluation: To evaluate alternative policies, we need to solve for the steady state equilibrium using the calibrated preference and technology parameters determined in Step 4. We just use the new policy parameters and repeat the steps that are outlined on pages 15-16.

One issue in evaluating policy is that we will want the new policy to be revenue neutral, namely, to generate the same amount of tax revenue as the current US tax policy. Additionally, we will require that the government spending be the same absolute amount as under the current policy. The idea here is that under the alternative tax policy, the government still needs to provide the same level of public consumption.

Although there is nothing difficult with numerically solving for the steady state given the tax rates, and government spending, it is not trivial to find the new tax rates that are revenue neutral. This is because for any set of tax rates, there is a different steady state capital stock, labor hours, and prices.

For this reason, it is more convenient to automate the solution and let the computer find the tax rates that are revenue neutral. The course webpage contains an interactive python code where you can do just this. The code is interactive in two parts. First, it asks you to input the observations to be used in Step 4, including the tax rates. Then it asks you to input the new tax rates on capital income and labor income. The program finds the tax on consumption that leaves tax revenues unchanged for the calibrated policy in Step 4. Importantly, it takes the spending from Step 4 and assumes that level in computing the new steady state equilibrium.

IV Conclusion

In this chapter, we have presented the Neoclassical Growth model. This is an example of an economy where the consumer lives forever. No one lives forever, at least not yet. The idea of an infinitely lived consumer is really that of a dynasty- where the current generation cares about its childrens' utility and takes that into account in its decisions. In doing so, it will leave bequests to its children.

We have used the model to perform a number of comparative static exercises where we compare steady states under alternative government policies. This is an application of the model to the area of public finance. In the next chapters we will apply the model to the study of the business-cycle.