Savings, Wealth and Ricardian Equivalence

I. Introduction

In the previous chapter we studied the decision of households to supply hours to the labor market. This decision was a static decision, being done within the same period. We now turn to another important decision of households, but one in which the decision is dynamic or intertemporal as it involves a comparison across time. This is the household's saving decision. How much one saves today affects how much one consumes today and in the future.

This chapter consists of three parts. The first is the microeconomic analysis of the household consumption/savings behavior. As we shall see, the optimization problem and conditions are very similar to those developed in the previous chapter on the household's labor/leisure decision. In studying this problem, we define the notion of present value, and wealth.

The second part of the chapter analyzes the effect of a certain types of taxes on the household's savings decision. The main result is that tax changes of a lump sum variety have no real impacts on the economy. This is what is known as Ricardian Equivalence. Recall that a lump-sum tax is fixed amount that is not affected by anything the household does.

The analysis of this tax change brings the government and its budget constraint into play. This we study in third part of the chapter. In studying the government's financing needs through time, we will arrive at an intertemporal budget constraint. This has important implications particularly for the recent events in some European Monetary Union countries such as Greece, Italy, Spain, Ireland, Portugal, the so called *PIIGS.* The plight of the PIIGS is the subject of the third section of this chapter.

II. Microeconomic Foundations

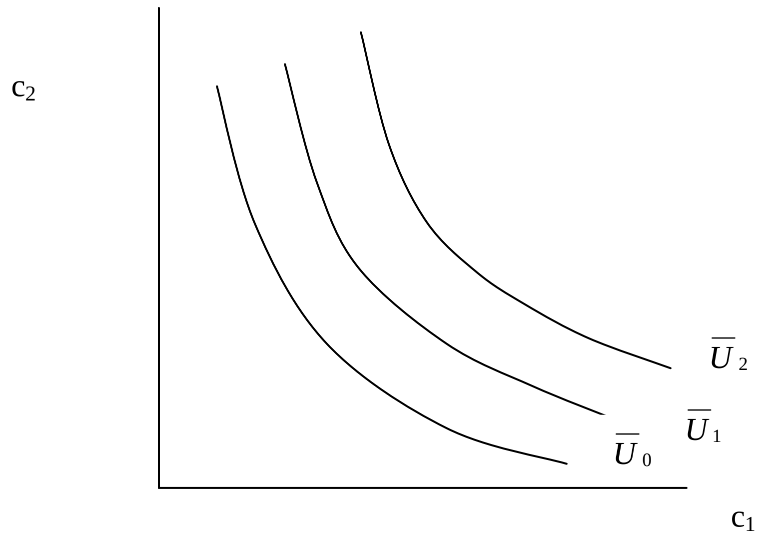
Consumer Theory

We return to our study of microeconomics and the theory of the consumer. Currently, we are interested in the choice of savings, namely, how much to eat today versus how much to eat tomorrow. For this purpose, we shall begin with an assumption that our consumer lives for only two periods, Periods 1 and 2, and that he/she derives happiness or utility from consumption in Period 1 and consumption in Period 2. For now, we will assume that the household does not derive any utility from leisure. We will relax this assumption in the next chapter. Given these assumptions, the household' utility function is denoted by $U(c_1, c_2)$ where c_1 is the quantity of the good consumed in the first period and c_2 in the quantity of the good consumed in the second period.

Indifference Curves

Given that the household's utility is defined over two goods, we can still represent it via indifference curves and show the optimal saving choice graphically. Figure 1 shows the indifference curves. On the horizontal axis is Period 1 consumption, c_1, and on the vertical axis is Period 2 consumption, c_2.

Figure 1: Indifference Curves



As we saw in our study of the household labor decision, each indifference curve describes all the combinations of consumption in period 1 and consumption in period 2 that gives the household a certain level of utility. The consumer, hence, is indifferent between any of the consumption bundles along a given curve. Again, as long as more consumption of each good is desirable, the indifference curve must be downward sloping. Additionally, as long as people like variety, the indifference curve, is convex, namely, bowed in toward the origin.

Budget Constraints

Because the consumer lives two periods, he has two budget constraints. Let us assume that the consumer does not begin his life with any inherited debt, and let us also assume that he must pay all his debt back before he departs the world. Thus, in the first period, the consumer is able to save or borrow, and in the second period, he must pay back any loans plus interest, or in the case that he saved, be paid back the principal and interest.

To gain some intuition, let us think of the consumption good in each period as corn, measured in liters. Savings and borrowing are thus done in terms of liters of corn. In the first period say you lend 10 liters. In the second period, you are repaid the 10 liters plus interest on those 10 liters, also paid to you in liters. Because the interest payment is made in terms of corn, it is a real rate, not a nominal rate. To distinguish between borrowing and lending, we use $s > 0$ to indicate savings, and $s < 0$ to indicate that an individual is borrowing. We are now ready to write down the household's budget constraints in Period 1 and Period 2.

<u>Period 1:</u> Assume that the household has some labor income in the period measured in liters of corn. Because he does not value leisure (i.e., by assumption, leisure does not enter the household's utility function), he will supply his entire time endowment to the labor market. In the previous chapter we thought of a period representing a week. Here we are viewing a period of being far longer. To make things simple as possible, let us assume that the household is endowed with one unit of time in each period. Let w_1 be the wage rate of labor in Period 1 measured in liters of corn. Thus, the consumer's labor income in the first period is just w_1.

The consumer can do two things with this income: he can save (s) and consume corn, (c_1). (If he eats more corn than his income, then his savings is negative and is borrowing.) The Period 1 budget constraint is thus

(1st Per BC) $$c_1 + s \le w_1.$$

Period 2: In Period 2, the household again supplies labor to the economy and receives labor income. The wage rate in Period 2, is w_2. This is the liters of corn the household receives in the second period for working an hour in that period. The household also has some savings and interest on the savings that he can use for consumption in the second period. We shall denote the net interest rate by the letter, i. This is in real terms, so it is the number of liters a lender must pay you for each liter he borrowsu. Thus, if you lent s=1 in the first period, you would have (1+i) liters of corn in your savings account in the second period. In the second period, the household will eat all of his income, as he does not live past that period.

In light of this, the second period budget constraint is:

(2nd Per BC) $$c_2 \le w_2 + (1+i)s$$

Present Value, Wealth, and the Intertemporal Budget Constraint

We now present the consumer's maximization problem via the indifference curve diagram. We have depicted household preferences for consumption (corn) in Period 1 and consumption (corn) in period 2 via the indifference curves. To characterize the optimal savings decision, we must consider the budget constraints. In terms of depicting the optimal consumption choices via indifference curves this presents a dilemma because we typically only have a single budget constrain to plot. With a 2-Period lived household, there are two such budget constraints. The question is how we should proceed in light of the fact that there are two budget constraints.

The trick is to convert the two single period constraints into a single budget one, which is called the *intertemporal budget constraint*. This can be done in three steps. First, from Equation (2^{nd} Per BC), we solve for savings:

$$s \geq \frac{c_2}{1+i} - \frac{w_2}{1+i} \tag{1}$$

Equation (1) is next substituted into Equation (1^{st} Per BC) so as to eliminate the savings term from the first period budget constraint. This gives us

$$c_1 + \frac{c_2}{1+i} - \frac{w_2}{1+i} \leq w_1$$

The final step is to rearrange terms as follows

(IBC) $$c_1 + \frac{c_2}{1+i} \leq w_1 + \frac{w_2}{1+i}$$

Equation (IBC) is the consumer's **intertemporal budget constraint**. It says that the present value of consumption expenditures over the consumer's life is equal to the present value of its time endowment. The present value of the consumer's time endowment is what economists define as a person's **wealth.**

The concept of *present value* is just as it name suggests. In effect, present value converts the value of a commodity in the future into today's consumption. In this two period model, the present value of Period 2 consumption is its quantity equivalent value today in Period 1 consumption. To understand this better, consider the following savings option.

You could save one liters of corn today, in which case you would receive (1+i) liters tomorrow. Alternatively, you could save just enough today so that tomorrow you would have 1 liters of corn in your savings account. To do this, you would need to save $s = \frac{1}{1+i}$. If you saved exactly this amount, then s(1+i) would leave you with exactly 1 liters in your bank account in the next period.

If you think about this a little more, what we have just done is compute the price of a liter of corn in Period 2 measured in Period 1 liters. That is to say, we can buy 1 liter of corn to be delivered tomorrow at the price of $\frac{1}{1+i}$ liters today. This is the relative price of the Period 2 good in terms of the Period 1 good. This relative price is the key to concept to present value. If we take c_2 liters of corn and divide by $\frac{1}{1+i}$, what we obtain is the number of liters today that those c_2 liters are worth. Its present value is $\frac{c_2}{1+i}$. If we had a 3 period world, then we would divide consumption in period 3 by $(1+i)^2$. More generally, we can convert a quantity of any good or income in period t into its present value by dividing by $(1+i)^{t-1}$. Note that this assumes a constant interest in all periods. If the interest rate in the periods differed, then we would divide by $(1+i_1)(1+i_2)\ldots(1+i_{t-1})$.

Notice that if we denote $p_2 \equiv \frac{1}{1+i}$, and use V to denote a person's wealth, namely, $V \equiv w_1 + \frac{w_2}{1+i}$, the intertemporal budget constraint given by equation (IBC) can be rewritten as

$$p_1 c_1 + p_2 c_2 \leq V \ .$$

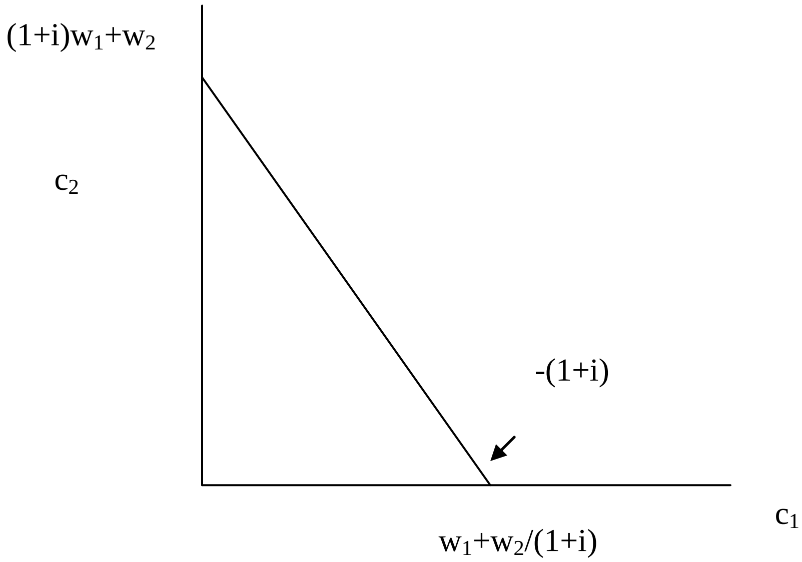
This is just what we had before, except c_2 was leisure in the previous chapter. This is informative because it shows that the intertemporal maximization problem is no different from the static one we studied earlier. The same utility maximization conditions will apply.

For the graphical presentation of the household's optimization, it is useful to solve for c_2 in the Equation (IBC). This is

$$c_2 \leq (1+i)w_1 + w_2 - (1+i)c_1 \, . \tag{6}$$

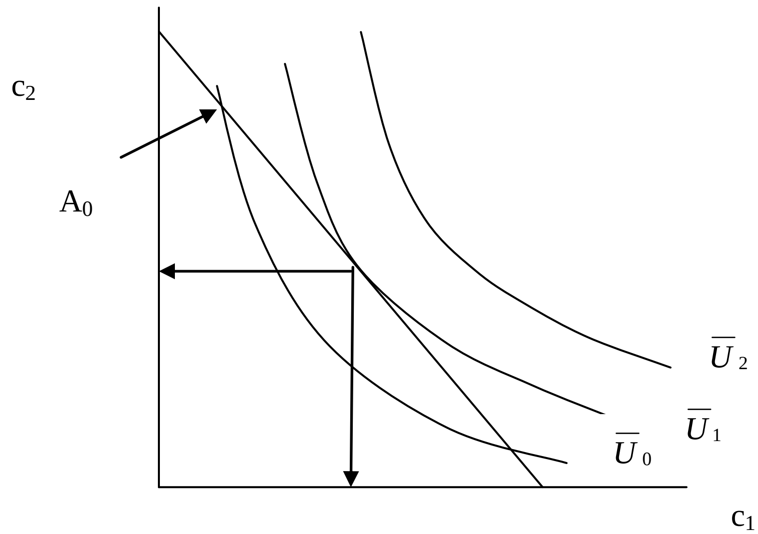
This is just a straight line with y-intercept $(1+i)w_1+w_2$ and slope $-(1+i)$. This is graphed in Figure 2. The y-intercept is the amount you could eat in Period 2 if you ate nothing in Period 1. If you ate nothing in period 1, you would save your entire wage income, w_1, and have $(1+i)w_1$ in your bank account tomorrow. Similarly, the x-intercept is $w_1+w_2/(1+i)$ and it represents the total amount that you could consume in Period 1 if you consumed nothing in Period 2. This is just your wealth. Think of it as follows: you can eat all of your income today, w_1, as well as obtain a loan from the bank which you could use to buy some of the good in Period 1. The maximum amount you could borrow is such that you could pay back with your wage in Period 2, w_2. This maximum loan is $w_2/(1+i)$.

Figure 2: Budget Constraint



As before, maximizing utility means reaching the highest indifference that the household can reach given its budget constraint/line. The highest indifference curve that can be reached is the one that is just tangent to the budget line. In Figure 3, this indifference curve corresponds to $\overline{U}_1$. The household could certainly afford to buy the consumption bundle given by point A_0 which is on $\overline{U}_0$, but clearly he is not optimizing in this case.

Figure 3: Utility Maximization



Mathematically, the tangency point is the one where the slope of the indifference curve equals the slope of the budget line, i.e. the marginal rate of substitution in consumption equals the marginal rate of substitution in exchange. This is

$$\frac{\partial U / \partial c_1}{\partial U / \partial c_2} = \frac{p_1}{p_2}$$

In our two period model, the price of the good in period 1 is normalized to 1 and the price of the good in period 2 is 1/(1+i). Thus, the optimizing condition above is

(MRSC=MRSE) $$\frac{\partial U / \partial c_1}{\partial U / \partial c_2} = \frac{1}{\frac{1}{1+i}} = 1+i$$

An example- Logarithmic Utility

We will now use a specific example to illustrate via algebra the optimal savings decision. The utility function for our consumer is

$$U(c_1, c_2) = \ln(c_1) + \beta \ln(c_2) \qquad (11)$$

You probably note that it looks identical to the utility function we used in the previous chapter in studying the labor/leisure decisions except that we have used the Greek letter β instead of α to denote the parameter on the utility from good number 2. This is on purpose. The parameter on the utility associated with good 2 has a clear and different meaning in the context of the dynamic model. Its value is restricted to be between 0 and 1. It is referred to as the *discount factor*, and it is meant to capture the psychological fact that people value today over tomorrow. For example, given the option, would you rather have a slice of pizza today or a slice of pizza tomorrow? Most people would pick the slice of pizza today on account of having a certain level of impatience. Indeed, one way to think of β is a measure of how patient a person you are.

With this utility function, the umarginal rate of substitution in consumption is,

$$\frac{\partial U / \partial c_1}{\partial U / \partial c_2} = \frac{1}{\beta} \frac{c_2}{c_1}.$$

Thus the utility maximization condition is

$$1 + i = \frac{1}{\beta} \frac{c_2}{c_1}.$$

Using this utility maximizing condition with the intertemporal budget constraint, we can derive the household's savings function. To do this we rearrange the above expression so that

$$\beta c_1 = \frac{c_2}{1+i}.$$

We know from the diagram that the household maximizes utility by being on the budget constraint. Thus, the intertemporal budget constraint holds with equality, namely.

$$c_1 + \frac{c_2}{1+i} = w_1 + \frac{w_2}{1+i}.$$

Thus, substituting condition obtained from equating MRSC with MRSE above into the intertemporal budget constraint above we arrive at

$$c_1(1+\beta) = w_1 + \frac{w_2}{1+i}$$

Solving for c_1,

$$c_1 = \frac{1}{1+\beta}\left[w_1 + \frac{w_2}{1+i}\right].$$

In words, the consumer eats a constant fraction 1/(1+β) of its wealth in the first period. This is the houshold's demand for first period consumption. It is a function of the wage in the first period, the second period and the real interest rate.

To determine household's savings, we use the definition of savings, namely the difference between first period income and first period consumption, c_1. Thus,

$$s = w_1 - c_1 = w_1 - \frac{1}{1+\beta}\left[w_1 + \frac{w_2}{1+i}\right] = \frac{\beta}{1+\beta}w_1 - \frac{1}{(1+\beta)(1+i)}w_2$$

We can graph this as a function of the interest rate. As the interest rate increases, the amount of savings increases. Notice in the special case that the consumer has no second period income its savings function is independent of the interest rate. In this case, the substitution and income effects completely cancel each other out. This is a special property of the log utility function.

II. Government Finance and Ricardian Equivalence

Ultimately, we shall encompass the saving decision studied above in a general equilibrium model with profit maximizing firms. This we will do extensively in the next chapter. For now, we can use some of the concepts we developed here to derive a few insights about the effect of government taxes on consumption and savings behavior.

What do governments do? They spend. They tax. If they have a shortfall, namely tax receipts are smaller than expenditures, they might borrow. In the opposite case, namely a surplus, they can lend. There is also another important way governments cover a deficit- printing money. While this has and continues to be an important way actual government finance deficits, it is not something we are ready to study at this point in the course. The problem is that our model is a real one- all transactions are done in terms of the economy's output, and prices are denoted in output as well, so there is no role for money in this world. Nevertheless, we can still learn an important lesson about government finance in this simple real, two-period world.

Government Seignorage

The word Seignorage has its roots in France, meaning the "right of the lord to print money. Seignorage in the days of commodity money equaled the difference between the face value of the minted coins and the cost of producing it.

With the advent of fiat money, seignorage has come to be used synonymously with the inflation tax. This is loss in the purchasing power of money that is associated with an increase in the money supply. Effectively, as the increase in the money supply causes a rise in the price level, the amount of goods one can buy with their nominal income falls. This acts like an income tax. It has the equivalent effect as an increase in an income tax which reduces the value of your paycheck.

Seen in this light, there is not much difference in financing a current deficit by printing money or by borrowing today and raising income taxes in the future.

In a two period world, the government like the consumer can borrow or lend in the first period. We will denote the government's borrowing or lending with the letter, B. Like the consumer the government cannot borrow or lend in the second period, as the world ends at that point. Thus, like the household, the government has two budget constraints, one for the first and another for the second. In each period it buys some of the economy's output (corn). We denote these purchases by G_1 and G_2 respectively.

It also taxes the household in the economy. In contrast to the last chapter, we shall consider a different type of tax here- a lump-sum tax. With this tax, the government simply takes a fixed amount of income away from the consumer in each period. This is in contrast to the marginal tax rate that we studied earlier, where the amount of taxes collected depends on the number of hours the consumer works. Let T_1 and T_2 denote the lump-sum taxes the government imposes on the consumer. The government's budget constrain in the first period is

(GBT 1) $$G_1 + B = T_1$$

and its budget constraint in the second period is

(GBT 2) $$G_2 = (1+i)B + T_2$$

With these taxes, the budget constraint of the consumer in period 1 becomes

$$c_1 + s \leq w_1 - T_1$$

and the budget constraint in period 2 becomes

$$c_2 \leq w_2 + (1+i)s - T_2 .$$

As we did before, we can convert the two single period budget constraints of the consumer into an intertemporal budget constraint. With the lump sum taxes, the consumer's intertemporal budget constraint is

$$c_1 + \frac{c_2}{1+i} \leq w_1 - T_1 + \frac{w_2 - T_2}{1+i} .$$

We can perform the same steps to arrive at an intertemporal budget constraint for the government. This is

(IBC G) $$G_1 + \frac{G_2}{1+i} = T_1 + \frac{T_2}{1+i} .$$

The intertemporal budget constraint of the government implies that the present value of government spending must equal the present value of its tax receipts. Note that if the government has a deficit in the first period, (i.e. $G_1 > T_1$), then Equation (IBC G) implies that it must have a surplus in the second period (i.e. $G_2 < T_2$).

Another important implication of this is that if the government lowers lump-sum taxes in the first period (i.e., ΔT_1=-1), then the second period tax must go up by exactly $\Delta T_1(1+r)$.

Herein lies the key to the notion of Ricardian Equivalence. By definition, Ricardian Equivalence refers to the neutral effect of change in lump sum taxes today on the consumer's consumption choices in both periods. Why is this the case? In the above example of a one liter tax cut in the first period, the government must raise the tax in the second period by (1+r). In the consumer's intertemporal budget constraint, we have

$$c_1 + \frac{c_2}{1+i} \leq w_1 - (T_1 - \Delta T_1) + \frac{w_2 - [T_2 + (1+r)\Delta T_1]}{1+i}.$$

This reduces to

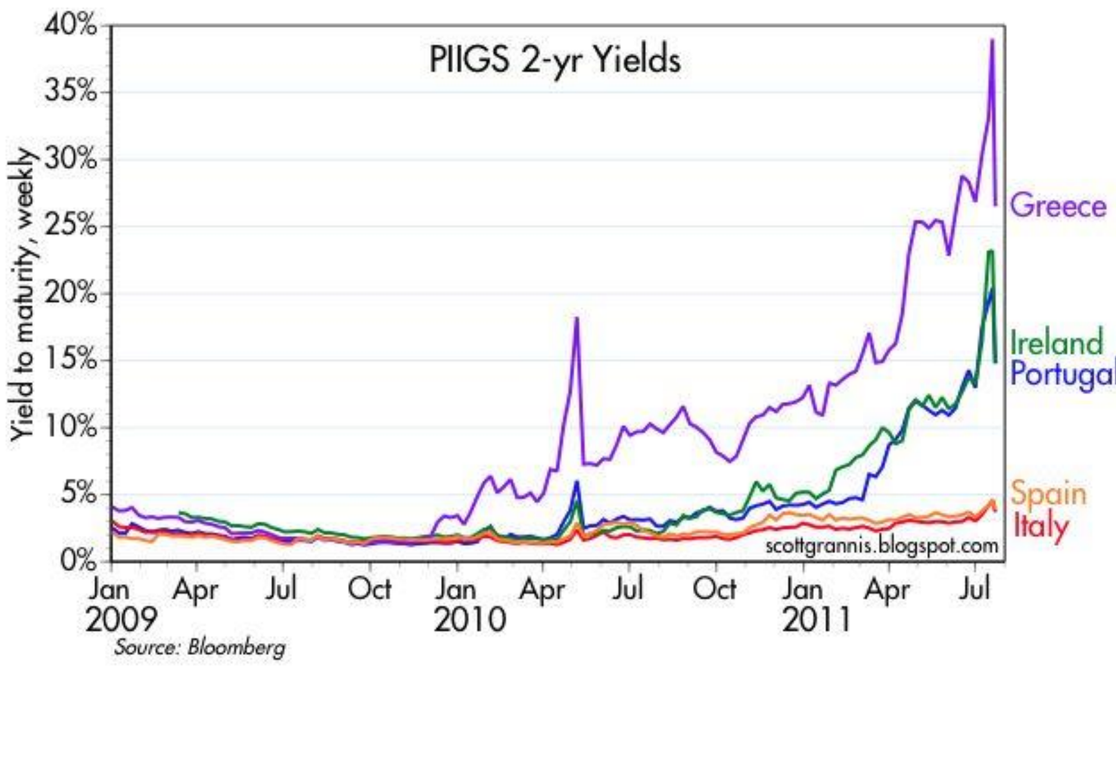
$$c_1 + \frac{c_2}{1+i} \leq w_1 - T_1 + \frac{w_2 - T_2}{1+i},$$

which is the same as before. In other words, the effect of the tax decrease in period 1 has no effect on the household's wealth. The intertemporal budget constraint is the same, and thus the optimal consumption choices are the same as before. While consumption in each period is unaffected, will savings be unaffected under the alternative tax plan?

There are a number of implicit assumptions that we have made to arrive at this result. The first is that the household is not credit constrained. If he or she is limited to how much they can borrow, then it might be the case that an alternative timing of the lump-sum taxes would affect the savings and consumption choices. Another implicit assumption is that the person that receives the tax cut today is the same one who pays the tax increase tomorrow. If this is not the case, then the wealth of individual consumers in the economy will change with the change in tax policy, and hence the policy will have real effects. And of course, Ricardian Equivalence requires that taxes be lump sum so that they do not affect the relative prices of goods and hence the utility maximizing conditions of the households. In other words, a change in the marginal tax rate of income or consumption will generally affect the equilibrium outcomes.

III. Sovereign Debt Crises: the European Monetary Union and PIIGS

We now use the intertemporal budget constraint of the government to make sense of the events in several European countries that belonged to the EMU. All of these countries experienced a crisis in their credit markets as private investors hesitated at buying new bond issues that were needed to cover revenue gaps. The result was a huge increase in interest rates on government debt. This was the market adjustment to private investor's unwillingness to hold Greek, Italian, Portuguese, Irish and Spanish Bonds.



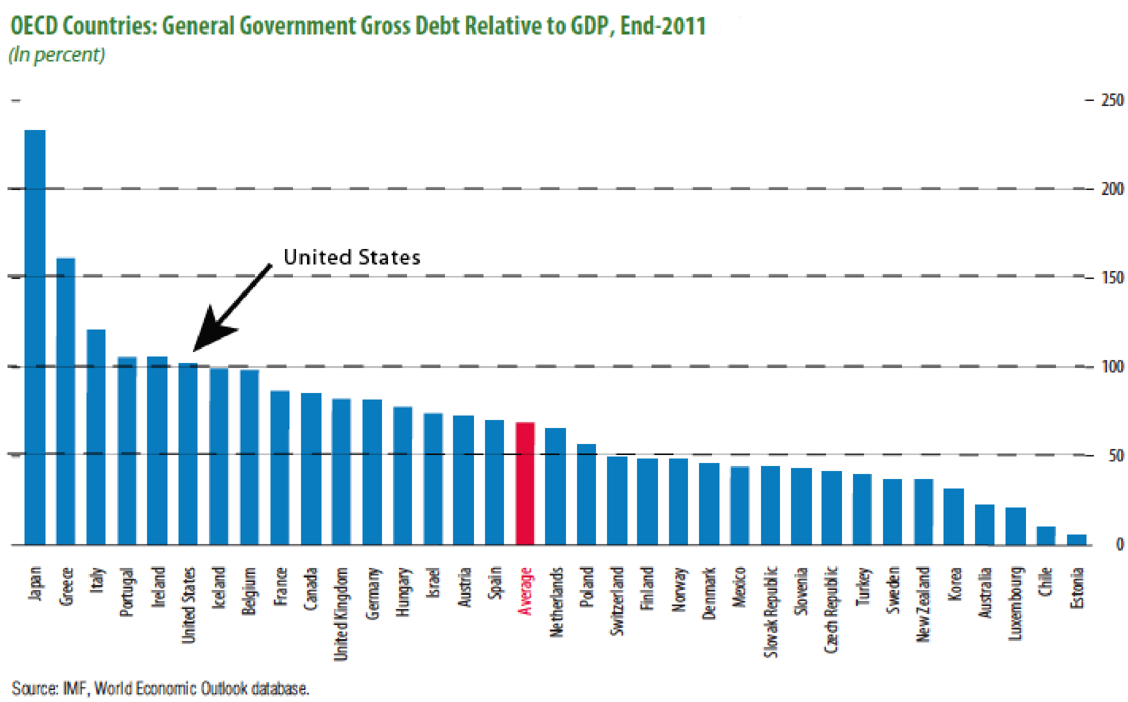
Why were private investors hesitant about holding PIIGS bonds? Clearly, private investors felt there was a reasonable risk of default on these nations on these bonds. The fear is easily understood in terms of the government intertemporal budget constrain, that we developed in the case that sovereign nations lacked the ability to print money.

Recall, that if you do not have the ability to print money, then current deficits must be met with future surpluses. This is the downside of being a member of a currency union, such as the European Monetary Union. Without this ability, people should expect that the government will at some date be unable to pay back what it owes. Private investors, seeing this potential down the road, balk at the option of buying the bonds now.

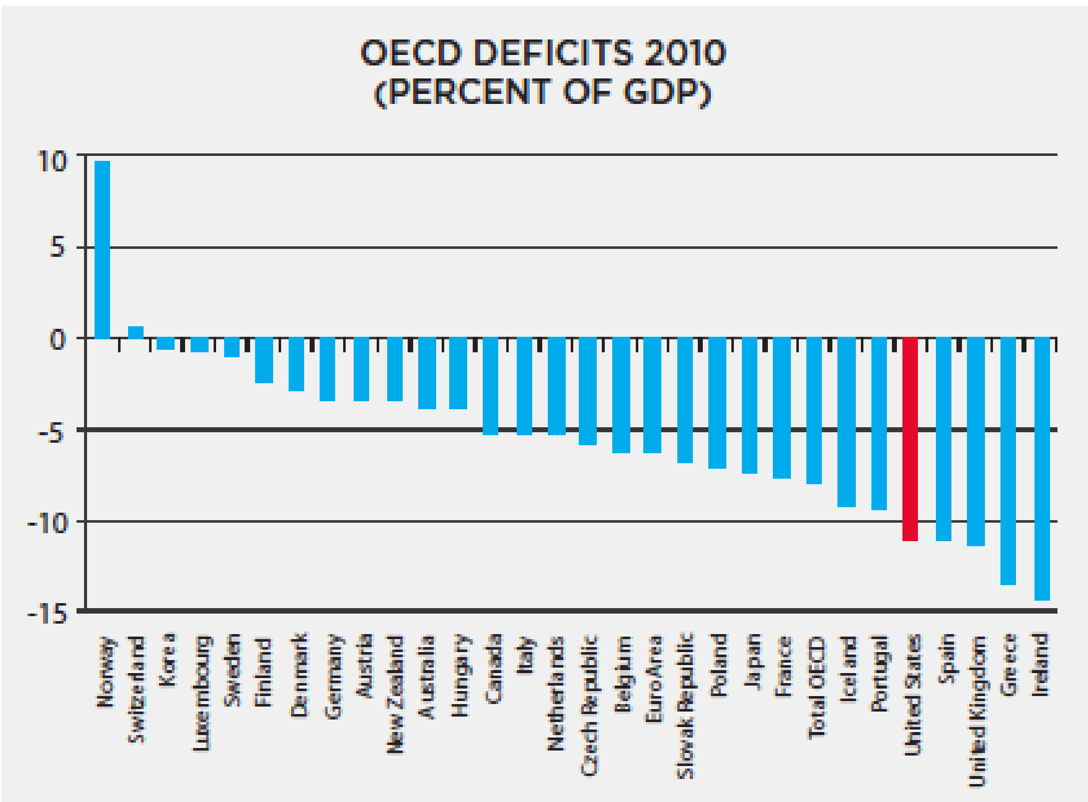
There are a number of ways to generate future surpluses. One way is to raise higher taxes in the future. Another is to reduce future outlays. Still another way is to maintain current tax rates and keep government outlays at their current level, but grow the economy. That is to say, that if we kept the tax rate at 10%, the government's tax receipts would increase just on account of growth in GDP.

A good indicator of how big a country's future surpluses must be is the size of its public debt (measured as a percentage of GDP). The below figure plots the Debt to GDP ratios for the OECD countries in 2011. Certainly, Greece, Italy, Portugal and Ireland are among the OECD countries with the largest debt to GDP ratios. However, they are not all that different from the United States, Belgium, France and Canada and none of these countries underwent a debt crisis. Spain is actually lower in its debt to GDP ratio, having a ratio that is average for this set of countries. There is also Japan which has a debt to GDP ratio almost double that of Italy, Ireland and Portugal.

So what is going on? Recall that the US, Canada and Japan are not part of any currency union. As they can print money, they do not need future surpluses to meet current deficits. However, there are those EMU countries with similar Debt to GDP ratios such as France and Belgium that did not experience a crisis, so there is still a mystery here.



One possible explanation is that despite these similar debt to GDP ratios, France and Belgium are relatively better at getting a grip on this problem. This could be indicated by smaller recent deficits. However, the evidence does not suggest that this is the case, at least for understanding Italy's situation. Whereas government budget deficits in Greece, Ireland and Spain in 2010 were among the largest among OECD members as a percentage of GDP, Italy's was not. It was a mere 5 percent of GDP, smaller than France's and Belgium's.



Source: OECD, Economic Outlook No. 87 database, Table 27. Dēmos

So why did Italy have a debt crisis? Or perhaps, better posed, Why did France and Belgium not have ones? On this, most economists think that Italy experienced this crisis on account of having a very low future growth prospect. In 2005, Italy was identified by The *Economist* as the real sick man of Europe on account of what it saw as low future growth due to the large amount of bureaucracy and high cost of doing business.

IV. Conclusion

This chapter has studied the final important micro decision, the savings decision. This is the last key building block of modern macro general equilibrium models. In the next two chapters we will consider this decision with the labor supply decision to study issues in public finance. In the section that follows, we will use it to study business cycles.

Although we did not consider the savings decision in the context of a general equilibrium model, our study gave us some important insights into the problems that several European countries have encountered recently.

REFERENCES:

PROBLEMS:

1. A consumer lives for two periods and receives labor income in both the first period of his life and the second. Let w_1 be the wage income in the first period, and let w_2 be the wage income in the second. The household pays lump-sum taxes in the first period, Tx_1 and lump-sum taxes in the second period Tx_2. The consumer does not value leisure, but does value consumption in period 1 and period 2. The time endowment of the consumer is 1 unit of time in each of the two periods. Suppose the consumer has the following utility function

$$\ln(c_1) + \beta \ln(c_2)$$

a.) Suppose that $\beta(1+i)=1$. Derive the household's demand for consumption in period 1, consumption in period 2. Describe conditions on the after tax earnings in periods 1 and 2 that ensures that the household would neither be a borrower or lender.

b.) Suppose that taxes are zero in each period. Continue to assume that $\beta(1+i)=1$. Now suppose that $w_2=5w_1$. Find the optimal savings for the consumer as well as his first and second period consumption quantities.

c.) Now suppose there is a borrowing constraint whereby the consumer can at most borrow an amount up to his first period wage income, w_1. Show via a diagram how this borrowing constraint affects the budget constraint. Find the optimal savings and consumption for this consumer faced with this borrowing constraint.

2. A two-period lived government has expenditures G_1, and G_2, and lump-sum taxes Tx_1, and Tx_2. The consumer has the same utility as above. He does not face any borrowing constraint. He has wage income w_1 and w_2 in each period. The production of the consumption good is A_1h_1 and A_2h_2 in periods 1 and 2 respectively. Assume as above that the consumer is endowed with 1 unit of time in each of the two periods.

   a.) Assume that $G_1=G_2=Tx_1=Tx_2 = 0$, $A_1=20$ and $A_2=22$ and $\beta=1$. Verify that the equilibrium for this economy is $c_1=20$, $c_2=22$ and $r=10\%$.

   b.) Now assume that $G_1=G_2= 0$, but $Tx_1= 10$ and $Tx_2 = -11$. Continue to assume that $A_1=20$ and $A_2=22$, and $\beta=1$. Verify that the equilibrium is still $c_1=20$, $c_2=22$ and $i=10\%$. How much does the consumer borrow/lend in the first period?

   c.) Now assume that $G_1=10$, $G_2= 11$, and $Tx_1= 10$ and $Tx_2 = 11$. Continue to assume that $A_1=20$ and $A_2=22$, and $\beta=1$. What are the equilibrium values for c_1, c_2, and i? Why doesn't Ricardian Equivalence hold here?