Workhour Differences: The Labor/Leisure Choice

Chapter Overview

This paper continues the development of macroeconomic models based on microeconomics. The models in these chapters that covered the issue of growth and development were not truly modern in that they did not consider they took the savings rate as exogenous. They were modern in that they included the profit maximizing decision of firms and they studied the general equilibrium of the system. Still, they failed to include the utility maximization problems of households.

This chapter moves us closer to a fully, micro founded model of the macroeconomy. There are really two key decisions that consumers make in the context of studying the macro economy. The first relates to labor supply worked, which is a trade-off between work and leisure. The second relates to savings, which is a trade-off between consumption today and in the future. This chapter considers the labor/leisure decision of households, postponing the savings decision.

Despite this abstraction, the model we develop in this section is a useful device for drawing scientific inferences, and it lays the foundation for much of what we shall do in the remainder of the book. There are a number of questions that we can address with it. One question is whether differences in tax rates on labor income across countries can account for the observed differences in hours worked per person in these countries.

Consider the following Table taken from Edward C. Prescott's (2004) article titled, "Why do Americans work so much more than Europeans?" The table reports average hours worked per week by the population aged 15-64 in the G-7 countries in both

the early 1970s and the mid-1990s. The Table shows that whereas Americans on average worked shorter work weeks in the early period, they worked longer weeks in the later period. In the early period, only the Canadians and Italians worked fewer hours per week on average than Americans. However, in the later period, things are reversed; all countries except the Japanese work shorter hours than Americans.

Hours per week per working age population ages 15-64

	1970-74	1993-96
Germany	24.6	19.3
France	24.4	17.5
Italy	19.2	16.5
Canada	22.2	22.9
UK	25.9	22.8
Japan	29.8	27
USA	23.5	25.9

Source: Prescott 2004, Table 2.

Another question, and one you will be asked to answer, is what is the tax rate on labor income that would maximize the government's tax revenue? This question is one that is particularly relevant to policy makers. Indeed, it was at the heart of the political debate in the late 1970s, which brought Ronald Regan to the White House. One of the promises that Ronald Regan made was to lower the marginal tax rate on people's labor income and increase the federal government's tax revenues. The principal idea behind this policy is known as the *Laffer Curve*, named after Arthur Laffer, (who purportedly introduced the idea on a napkin at a cocktail party). The Laffer Curve shows the amount

of government revenues that are generated for tax rates between 0 and 100 percent. As shown in Figure 1, government tax revenues are 0 when the tax rate is either 0 percent or 100 percent: if the tax rate is 0 percent, no taxes are collected; and if the tax rate is 100 percent, no rational individual will produce a thing knowing that the government is going to confiscate all of his income. As the tax rate increases from 0 percent, tax revenues must rise initially with the tax rate. Eventually, they must fall with the tax rate. Consequently, there is a tax rate that maximizes the government's tax revenues.

Figure 1: The Laffer Curve
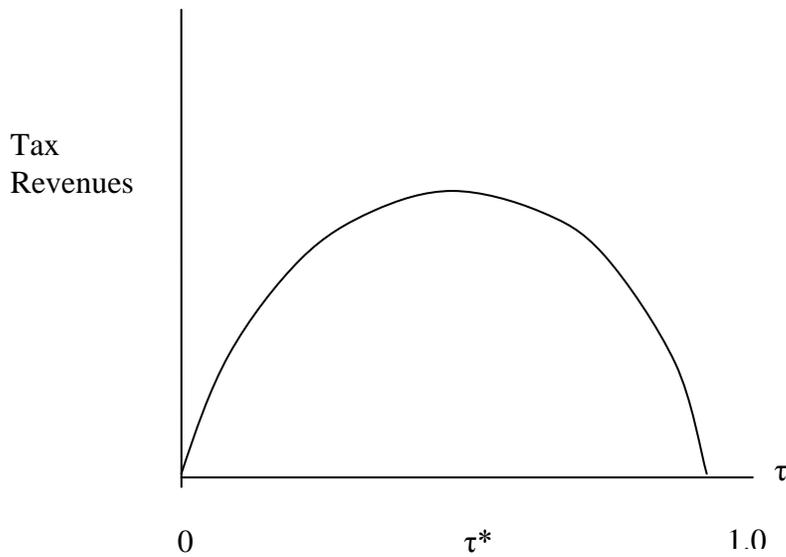
Utility Maximization: Leisure and Consumption

In the utility maximization example studied in Chapter 2 there were two-goods. The budget constraint was

$$c_1 + pc_2 \leq V \tag{1}$$

Where p is the price of good 2 in units of good 1 and V is the consumer's wealth also measured in units of good 1. The utility maximizing condition sets the Marginal Rate of Substitution in Consumption (MRSC) to the Marginal Rate of Substitution in Exchange, which is

$$\frac{\partial U / \partial c_2}{\partial U / \partial c_1} = p \tag{2}$$

Although it is natural to think of these two goods as say apples and oranges, we can think of these two goods differently. For example, we can think of good 1 as apples and good 2 as leisure. Alternatively, we can think of good 1 as apples today and good 2 as apples tomorrow. In this chapter, we will interpret the goods as apples and leisure. In the next chapter, where we study the household savings decision, we will interpret the goods as apples today and applies tomorrow. In terms of the mathematics, the interpretation we give to the two goods is irrelevant in that the same utility maximization condition (2) must hold.

To convince you that this is the case, let us examine the household's problem when utility is defined over consumption and leisure, c and l. An individual in any period will only have a limited number of hours for which he can enjoy leisure. The other use of his time is work. For now, let us fix these limited hours available to the person at 100 in the period.

The indifference curves display the same shapes as before (i.e. downward sloping and convex to the origin) and so the utility function is increasing in consumption and increasing in leisure, and which is subject to the law of diminishing utility. The more challenging issue is determining what the budget constraint for the household is in this case? Let h denote the hours the household works, and the letter w to denote the wage rate measured in units of the consumption good. (Instead of being paid in dollars, you are paid in apples). If the household works, h, hours than he generates wh in income measured in units of the consumption good. Then the amount of consumption the household can afford is

$$c \leq wh \tag{3}$$

If we use the fact that $l+h=100$, so that $h=100-l$, then the budget constraint is

$$c + wl \leq 100w. \tag{4}$$

This is just the same as before. The right hand side is not labor income, but your wealth-the value of your time endowment, which you own.

The necessary conditions for household utility maximization are just as they were before, namely,

$$w = \frac{\partial U / \partial l}{\partial U / \partial c} \quad . \tag{5}$$

2. General Equilibrium Analysis

From consumer theory, we obtain the household's demand for goods and its supply of inputs. From producer theory, we obtain the firm's supply for goods and its demand for

inputs. The intersection of the supply and the demand for a final good or an input determine its price and the quantity traded. General Equilibrium analysis simply requires that all markets clear simultaneously.

To illustrate this, let us return to the model economy in which household utility is defined over consumption and leisure. Namely, household utility is just

$$\log(c^d) + \alpha \log(\ell), \tag{6}$$

where the superscript *d* has been added to distinguish demand from supply. Without loss of generality, we will assume that there is a single representative household in the economy. The household takes prices as given.

On the producer side, we assume a production function that only uses labor as the sole input, and for which the marginal product of labor is constant, rather than decreasing. Thus, the *Law of Diminishing Returns* does not apply to the production side of the economy. The production function does exhibit constant returns to scale. Graphically, this production function is just a ray from the origin.

Mathematically, the production function is just linear in labor, namely,

$$c^s = Ah^d \tag{7}$$

where A is the Total Factor Productivity.

For this economy, we can define a competitive equilibrium.

Definition of Competitive Equilibrium

A competitive equilibrium for this economy is real wage rate, *w* a quantity of labor supplied, h^s, a quantity of labor demanded, h^d, production, c^s, and consumption, c^d, such that

i. given the real wage rate, the consumer chooses c^d and h^s to maximize utility subject to his budget constraint.

ii. Given the real wage rate, firms choose c^s and h^d to maximize profits

iii. The goods market clears; $c^s = c^d$

iv. The labor market clears: $h^s = h^d$.

We next solve out the competitive equilibrium for the household's utility and the firm's production technology assumed above. We begin with the Household's problem. The household utility is given by the following functional equations

$$\log(c^d) + \alpha \log(\ell). \tag{8}$$

The household's budget constraint is

$$c + wl \leq 100\,w. \tag{9}$$

The utility maximization condition is

$$\frac{\partial U / \partial l}{\partial U / \partial c} = \frac{\alpha\, c^d}{l} = \frac{\alpha\, c^d}{100 - h^s} = w. \tag{10}$$

We can use (10) to solve for c^d. This is

$$c^d = \frac{w(100 - h^s)}{\alpha}. \tag{11}$$

Next we substitute (11) into (9) using the fact that $l=100\text{-}h$. This gives us a single equation in h^s, for which we can solve for:

$$h^s = \frac{100}{1 + \alpha}. \tag{12}$$

This is the household's supply of labor. Notice, that the supply of labor is independent of the price of labor, *w*, so that it is a vertical line. At first this may sound strange. However, it is an implication of the choice of utility function.

To understand this, recall from microeconomic theory that when a price of a good changes, there is both an income effect and a substitution effect. The substitution effect always works to decrease the quantity demanded of a good as people will substitute away from the good as the good's price increases. Here, if we think of an increase in the real wage as an increase in the price of leisure, it is clear that the substitution effect will push the consumer to work more. The income effect works in the opposite direction. Namely, with a higher real wage, the household can work less hours and maintain the same income. Since the household likes leisure, he shall want to take more leisure. With the log utility, the two effects completely cancel each other out. This explains why the real wage rate does not appear in the household's labor supply.

Profit Maximization

For the firm, its production function is just linear in labor, namely,

$$c^s = Ah^d. \tag{13}$$

The profit maximization condition described by (19) is just

$$A = w. \tag{14}$$

Again, with this production function, the marginal product is constant so there is no law of diminishing returns. Labor demand, therefore, is a horizontal line. If the wage rate exceeds TFP, then the firm wants to hire zero workers. In the opposite case, the firm would want to hire infinitely many as every worker would add to its profits by the amount, *A-w*.

Graphically, the labor market is shown in the following diagram. The equilibrium wage rate is just A, and the equilibrium quantity is just $100/(1+\alpha)$.

Figure 8: Labor Market Equilibrium

w

Labor
Supply

Labor Demand

A

$$\frac{100}{1+\alpha}$$

h
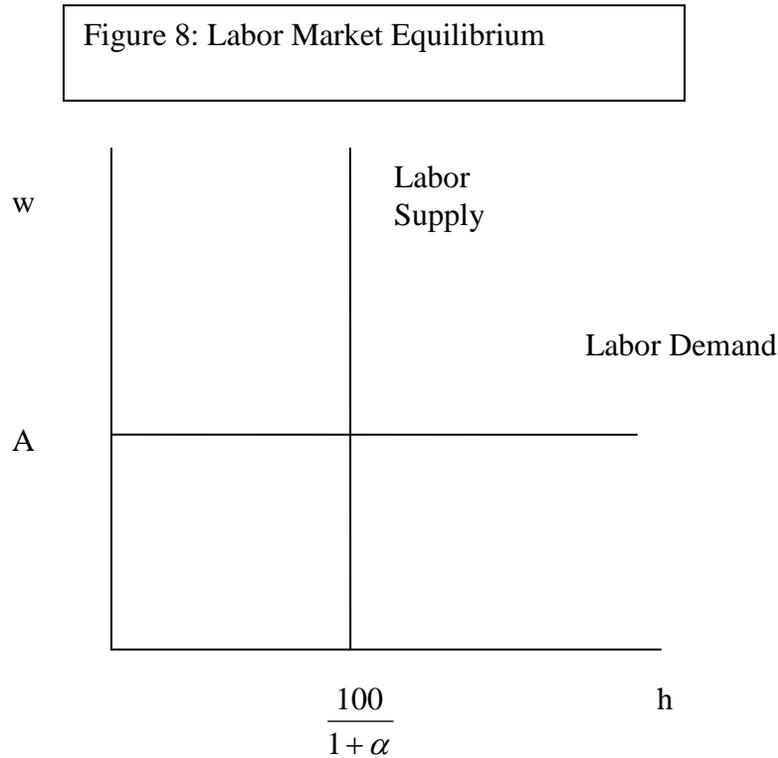
The only issue that that remains in completing the characterization of the competitive equilibrium is the quantity of consumption. This is easy, as once we have the equilibrium quantity of hours, we can substitute this either into the production function or the household's demand function given by equation (11). In either case, we obtain that the equilibrium amount of consumption/output

$$c = \frac{100\,A}{1+\alpha}\ .$$
(15)

This completes the characterization of the competitive equilibrium. Note that the solution is in terms of the parameters of the model, and the endowment of the household's time. This will always be the case in solving for a competitive equilibrium, namely, the equilibrium quantities and prices are expressed as functions of only the model parameters and the endowments.

3. Government Policy

Having laid out the basic model and solved for the general equilibrium prices and allocations, we now turn to the questions motivating this chapter. Why did Americans tend to work less hours per week relative to other G-7 countries in the 1970s but more in the 1990s. Relatedly, what is the tax rate on labor that maximizes government tax receipts? For this reason, we reintroduce government into the static model used in the previous section.

Government Budget Constraint

In general governments buy goods and services. They generate tax revenues from labor income, as well as from other forms of income such as capital income. In many countries, taxes on consumption are an important source of government revenues. They also make payments to households. These are known as transfer payments. Now, a government whose outlays are greater than its tax receipts can borrow. Of course, it will need to pay off the interest and principle on its previous borrowings. Since the model is static, there

can not be any borrowing or lending. [1]   Additionally, since our model does not have any

capital, there cannot be any capital income taxes in our static model.

Taxes and Transfers

For now, we shall consider a limited form of government policy, namely, where the

government only imposes a tax on the household's labor income and then rebates all of

the tax receipts back to the household in a single check in the form of a lump-sum

payment. The key feature of the lump-sum payment is that the household views the

payment amount to be independent of any decision it makes.

If the government taxes the household's labor income at a tax rate of τ, what this means

is that for each hour worked the household keeps $(1-\tau)w$. Additionally, by assumption

the household has a second source of income, the lump-sum transfer by the government.

If ψ denotes the lump-sum transfer, (which could be negative in which case it would be a

lump-sum tax), then the budget constraint is just

$$c \leq (1-\tau)wh + \psi \quad . \tag{16}$$

Using the time use constraint for households, we can rewrite the budget constraint as,

$$c + (1-\tau)wl \leq 100(1-\tau)w + \psi . \tag{17}$$

As the total amount of tax revenue collected equals the amount of lump-sum transfers,

the government's budget constraint is

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[1] Governments can also print money if tax revenues are deficient. This has been a common method used
by many countries, particularly Latin American countries in the post war period. As there is no money in
our model, i.e., it is a real economy, we do not consider this possibility in this chapter.

$$\psi = \tau wh. \tag{18}$$

Now that we have introduced the government into the model, we shall want to solve for the competitive equilibrium.

Competitive Equilibrium

There are still two markets in this economy: the labor market and the goods market. A competitive equilibrium for this economy is still a real wage rate, w a quantity of labor supplied, h^s, a quantity of labor demanded, h^d, production, c^s, and consumption, c^d. In addition, there is the government policy, given by the tax rate on labor income τ and the lump-sum transfers, ψ. This price, allocations, and policy must satisfy the following conditions:

    i.   Given the real wage rate, the tax rate and lump-sum transfers, the consumer chooses c^d and h^s to maximize his utility subject to its budget constraint.

    ii.   Given the real wage rate, firms choose c^s and h^d to maximize profits

    iii.   The government's budget is satisfied, namely $\psi = \tau wh$

    iv.   The goods market clears; $c^s = c^d$

    v.   The labor market clears: $h^s = h^d$.

To solve for the competitive equilibrium we start with the household's utility maximization problem. Equating MRSC=MRSE implies

$$\frac{\alpha c}{l} = (1 - \tau)w \tag{19}$$

Next, we rewrite (19) using the time use constraint of the household so that we MRSC=MRSE is expressed in terms of hours rather than leisure. This is

$$\frac{\alpha c}{(100-h)} = (1-\tau)w \tag{20}$$

Next, we shall use the goods market clearing condition c=Ah, together with the profit maximizing condition that w=A and substitute into (20). This gives
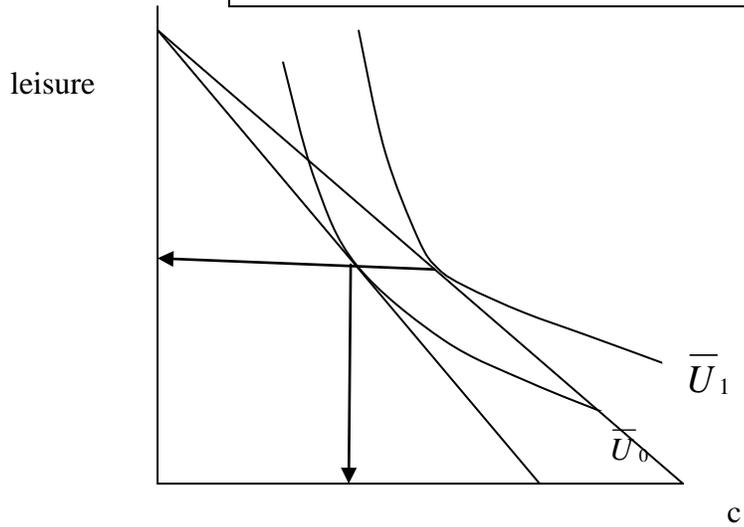
$$\frac{\alpha h}{(100-h)} = (1-\tau) \tag{21}$$

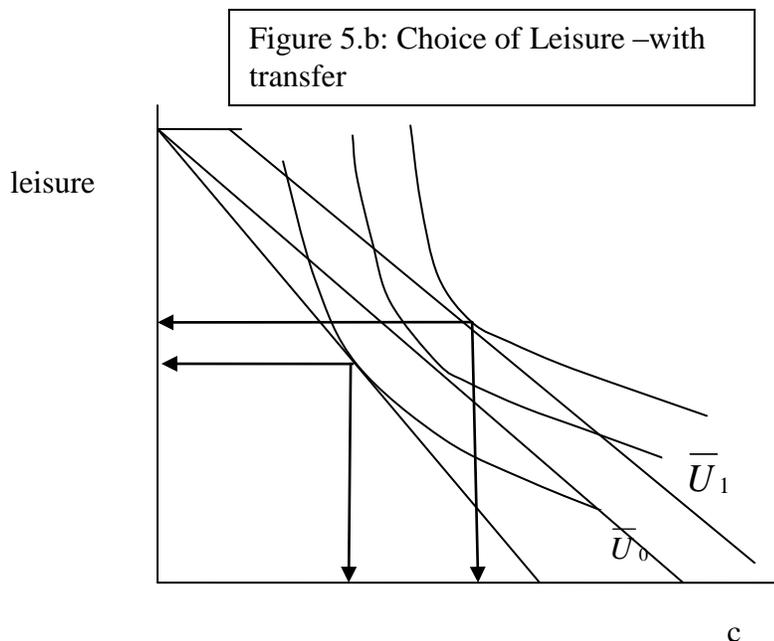From (21), we arrive at the equilibrium value for work hours

$$h = \frac{1-\tau}{(\alpha + (1-\tau))}100 \tag{22}$$

It is easy to show that h is a decreasing function of τ, so that a higher tax rate reduces work hours in the economy.

What is the intuition for this result? Recall, our utility function is characterized by a unit elasticity of substitution so that a 1% increase in the wage has no effect on work hours. It is true that putting a tax on labor income in effect represents a change in the relative price of leisure, and so in this sense the tax rate has no effect of the household's leisure choice. However, the lump sum transfer that is paid back to the households acts as type of gift to the household, that makes the feel wealthier and hence makes them want to buy more consumption and leisure. The following two figures represent the difference between a change in relative prices (without the transfer), and the change in relative prices with the transfer. As can be seen the transfer ψ allows the consumer to consume consumption up to ψ even when the household does not work.

Figure 5.a: Choice of Leisure –No transfer

leisure

$\overline{U}_1$

$\overline{U}_0$

c

Figure 5.b: Choice of Leisure –with transfer

leisure

$\overline{U}_1$

$\overline{U}_0$

c

Comparing Alternative Policies: Welfare Analysis:

For any tax rate, we can compute the competitive equilibrium. While we can easily show

how the prices and quantities are affected by tax policy, we ultimately want to know what

happens to households' welfare. As the tax rate changes, a household's consumption and

leisure will change.   An interesting question is how will the household's welfare differ

across alternative tax policies?   To compare welfare across different policy regimes, we

will determine the factor that we must scale consumption by so that the utility of the

household in the equilibrium when the tax rate is set to some value, (say $\tau = .40$) equals

the utility realized by the household in the case that the tax rate is at a lower value. This

gives the household's welfare in terms consumption differences.

Consider as an example a policy experiment whereby the tax rate on labor income

would be raised from .40 to .60. Household welfare will be lower under the higher tax

15

rate.   Thus, we would like to know by how much we would have to increase the consumer's consumption so that his utility is equal to the utility that he realizes with the lower tax rate of .40.   Mathematically, we are interested in finding the value of the scaling factor, $1+\lambda$, such that

$$\log(c(\tau = 0.40)) + \alpha \log(100 - h(\tau = 0.40)) =$$
$$\log((1 + \lambda)c(\tau = 0.60)) + \alpha \log(100 - h(\tau = 0.60)) \quad (23)$$

Therefore, $(1+\lambda)$ is the factor that consumption must be scaled if there is a change in the tax rate from 40% to 60% to leave people indifferent to the change. The welfare loss in consumption equivalents is λc.

In general, it is not possible to determine this compensation factor without assigning parameters to the model. This is the goal of the calibration procedure. We now illustrate the calibration procedure within the context of the static model with government policy.

III. Calibration Application: Work Hour Differences

We next use the model with government taxes to understand the differences in work hours per person of working age across G-7 countries by undertaking a calibration exercise that is based on Prescott (2004). We proceed in detail by going through each of the five steps. However, before doing so, it is useful to consider the issue in the context of labor market outcomes.

One reasonable question to ask about the observation that Americans in the 1990s worked more than their G-7 counterparts whereas the opposite was the case in the 1970, is whether differences in unemployment rates across countries and across time are important to understanding differences in work hours. Additionally, since work hour per person are averages over the working age population there is also the question of whether how much of these differences are the result of differences in participation rates.

A person is defined to be in the Labor Force (LF) if he is either employed (E) or looking for employment, that latter of which are defined as unemployed (U). Thus, LF=E+U. A country's unemployment rate (UR) is simply the number of unemployed divided by its labor force, namely, UR=U/LF. A country's participation rate (PR) is equal to its labor force divided by its population of people above 15 years of age. If P denotes the working age population, then a country's participation rate, PR=LF/P.

Let h be the average hours worked per worker. Then the hours, H, worked per person (above age 15) is

$$H = h\frac{E}{P} \tag{24}$$

Next, we shall use the fact that LF=E+U, PR = LF/P=(E+U)/P. and UR=s U/LF to rewrite (24) as

$$H = hPR(1-UR) \tag{25}$$

Equation (25) thus allows us to decompose the average hours per working age population into hours per worker, the participation rate, and the employment rate, or 1 less the unemployment rate.

[Insert Figures 6-7]

Although unemployment rates and participation rates have increased over the two sub-periods for most of the G-7 countries, it is not the case that either changes in PR or ER account for change in

H. The following Figures are reprinted from Rogerson and Shimer (2011) chapter in the *Handbook of Labor Economics*.   They show h, PR and (1-UR) for Germany, France, the United Kingdom and the United States. As one can see, the unemployment rates differ across countries; the unemployment rate in all countries increased dramatically between 1960 and 1983, before declining in all countries, and increases in the participation rate of G-7 countries most of which is fueled by the increase of woman in the labor force.   For each country, a decomposition based on Equation (25) makes it clear that much of the decline in H has very little to do with an increase in unemployment/decrease in employment;   Hours per worker (h) decreased dramatically in most countries.

UK

US

1.2
1.1
1
0.9
0.8
0.7
0.6
1965 1970 1975 1980 1985 1990 1995 2000 2005
FRANCE

h
1-ur
participation rate
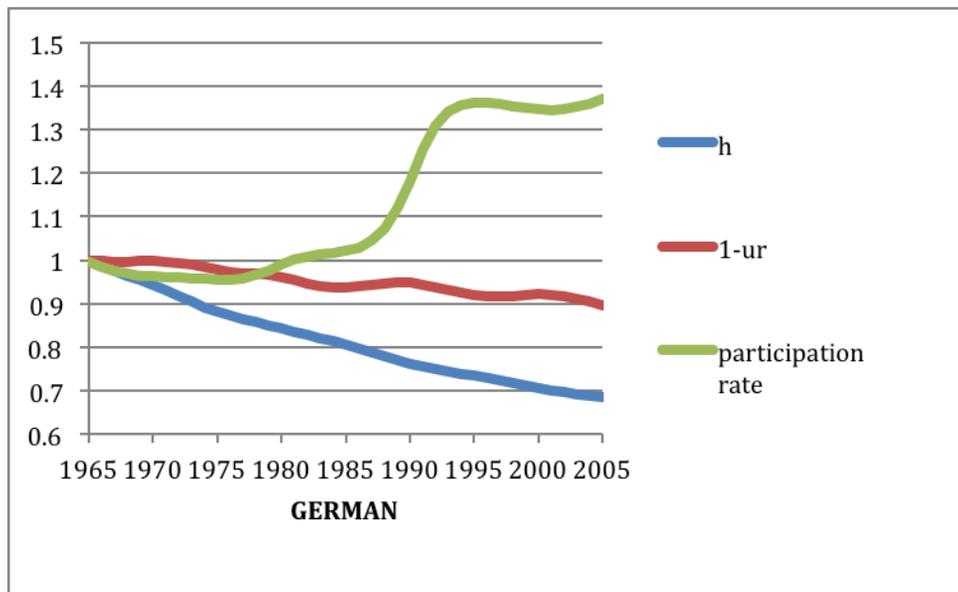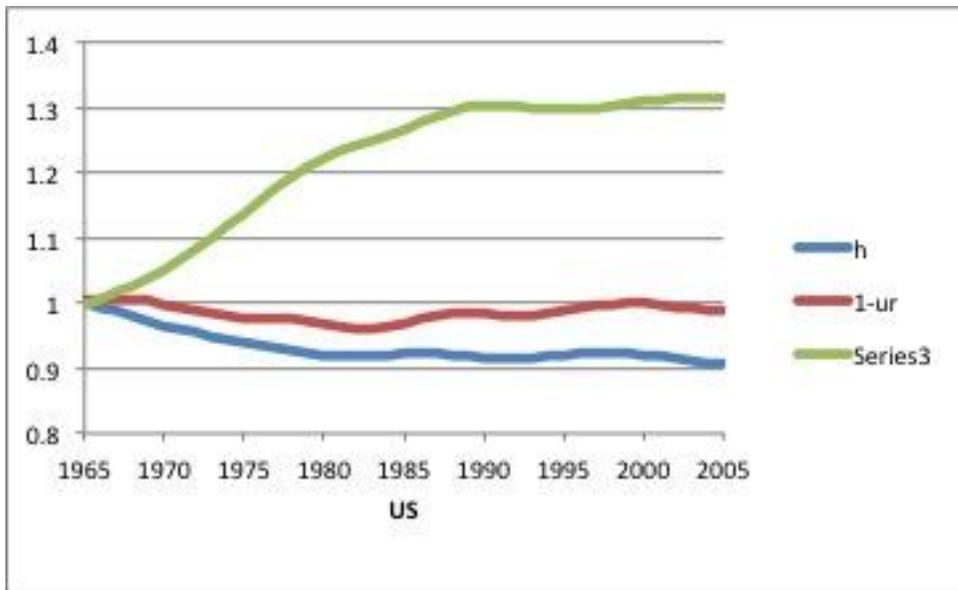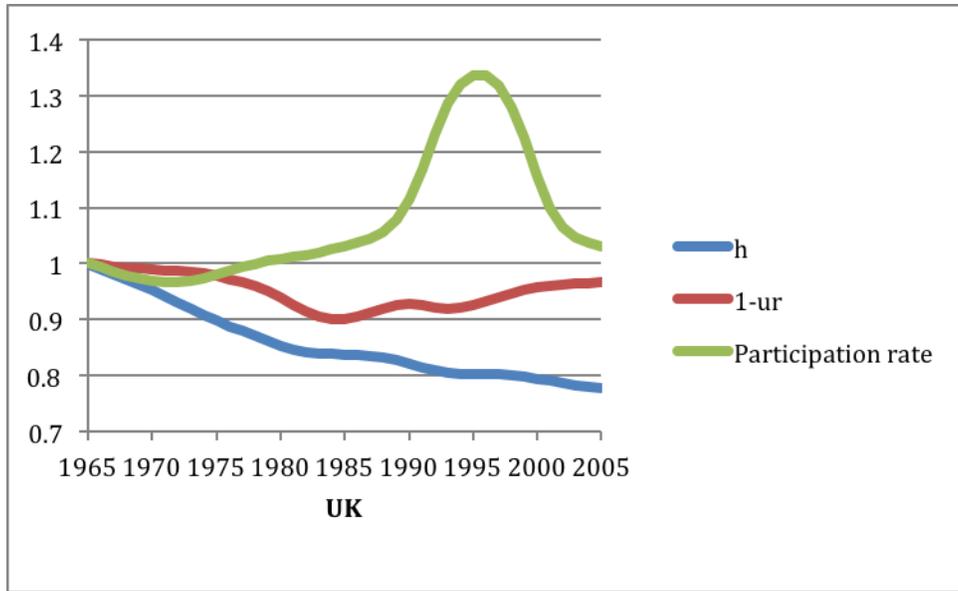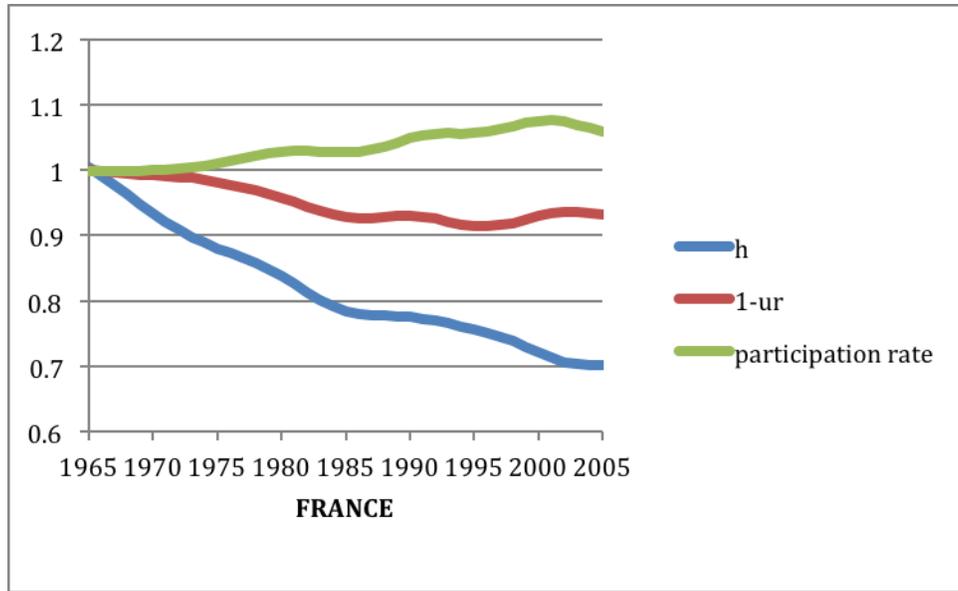
Data Resource: The annual hours, 1-unemployment rate and participation rate is relative to 1965 level. They are handled by HP filter. Data for annual hours per worker in employment are from the Groningen Growth and Development Center(GGDC), Total Economic Database. The participation rate is calculated by using data active population from OECD StatExracts. The activte population divided by the total populatioin. The active population is the sum of unemploymed population and employed population. The total population data is from GGDC. The active population data is missing for UK from 1992-1999, so these data is replaced by using participation rate from U.S Bureau of Labor Statistics: the International Comparisons of Annual Labor Force Statistics,1970-2012.

To make the point that unemployment is not an important factor in understanding work hour differences of the working age population, Rogerson and Shimer compute H for each country in Equation (25) fixing (1-UR) at the US rate and using each country's own h and PR. What they find is the difference in hours per person obtained by using the US employment rate is very similar to the actual hours per person.

These findings suggest that the issue of unemployment is not important for accounting for work hour differences. This is important as our model does not consider the issue of unemployment. Our model assumes perfectly competitive markets with a wage rate equal to its marginal product. It is classical in that given the equilibrium wage, everyone is happy about the amount of time they work or do not work. The study of unemployment, specifically of differences in unemployment rates across countries, will require a different type of model.

Calibration Steps

Recall there are 5 steps: pose a question; choose a model; define consistent measures; assign parameters, and comparison of model predictions with data. We now go through each of the 5 steps.

a. Pose a Question.

We can use the model in this section to answer a number of relevant questions. One question is "What is the tax rate that maximizes the government's tax receipts?" Another related question is "What would be the effect on household welfare if Europe adopted the U.S. tax rate on labor income equal to .40 instead of its current tax rate of .60?

b. Choice of model

Here the issues are what things to put in the model, and what functional forms should utility and production take. Obviously, the question one poses has a lot to do with what goes in the model. As we are interested in how tax rates affect work hours, it is critical

that we allow for the household to trade off between leisure and work. Thus, it is critical that we define utility over both consumption and leisure.

As for the functional forms, we want to use both theory and data to guide us in their selection. In solving for the competitive equilibrium, we used a utility function of the form $\log(c) + \alpha \log(l)$. This was not by accident. Recall, that in the model without taxes we showed that the labor supply is unaffected by changes in the real wage. That is because with the log utility function the substitution effect exactly offsets the income effect.

This utility function is consistent with the long-run record of the United States. The real wage in the United States has grown at roughly 2 percent per year over the 20th century. Despite this increase, there has been no secular change in the hours worked by households, at least in the post World War II period. What this suggests is that the entire increase in the real wage has gone to pay for more consumption by households. Alternatively, a one percentage increase in the real wage rate has been associated with a one percent increase in consumption. This is just saying that the elasticity of substitution between consumption and leisure goods is unity. This effectively limits the functional forms for household utility.

The production function was chosen so that there are constant returns to scale. With constant returns to scale, profits of firms are zero provided each factor is paid its marginal product. Zero profits is consistent with perfect competition. Additionally, with constant returns to scale, the size of firms is unimportant in the economy. In the U.S. economy, some firms in an industry are large while others are small. This is consistent with the constant returns to scale technology.

If you have been paying attention, the question you should now ask yourself is why would a change in the tax rate on labor income change the amount of time an individual works? If the log utility implies that the substitution and income effects cancel each other out, how can it be the case that the tax rate affects work hours, as the tax rate effectively determines the real wage the consumer takes home? As you shall see, a higher marginal tax rate on labor income will decrease the equilibrium work hours. The substitution and income effects still cancel each other out. So what is going on here? This is a good question, and one that you are assigned to answer on your own. A helpful hint: What happens to the tax revenues that are collected by the government?

c. Defining Consistent Measures

The model is truly abstract. For instance, there are no investment expenditures or government expenditures. All output is consumption. Additionally, there is no capital, so all income is paid to labor. Normally, we would have to rearrange the National Income and Product accounts around the categories of the model. For instance, we would need to add all investment and government expenditures in the NIPA to consumption. While we could do this, it is really not essential for completing steps 4 or 5 of the calibration exercise. This is because these steps only involve the data on hours worked, which are directly comparable between the model and the data.

d. Assignment of parameters

The next step is to assign parameters to the model. There are three parameters: the tax rate, τ, preference parameter, α and the TFP parameter, A. The value of A only

determines the choice of units for measuring output and prices. For this reason, we can proceed to normalize its value to 1. In effect, it doesn't matter if we measure the output of the dairy industry in gallons, half gallons, or pints. In the case we measure it in pints, output will be smaller, and this will be reflected by a smaller value of A. The tax rate is chosen to the value in the U.S. economy, which is equal to .40. This leaves the leisure parameter, α. This can be pinned down by using the observations for h=25 in the United States, and the household utility maximization condition

$$c^d = \frac{w(1-\tau)(100 - h^s)}{\alpha},$$

together with the result that in equilibrium c = Ah . This is left for you to do as an exercise.

Notice that if we started with this value of α and proceeded to solve for the equilibrium with the tax rate equal to .40 and A= 1.0, we would find that in equilibrium h=25. In effect, Step 4 of the calibration exercise is akin to an inverse problem to finding the equilibrium. We know the equilibrium property, namely h=.25. What we want to find is the value of α that gives us this equilibrium result.

e. Test of the Theory

The question we posed is a policy related question. If it were a test or development of theory related question, such as do tax rate differences account for the differences in work hours observed between the rich industrialized countries, we would want to compare the predictions of the model for work hours as we vary the tax rate to market hours worked in various countries with their tax rate on labor income. This is what Prescott (2004) does in trying to see if differences in tax rates between OECD countries

can account for the fact that in the 1970s Europeans worked more than Americans on a per person basis while in the 1990s the opposite was true.

For the policy evaluation at hand, we simply compute the equilibrium for alternative tax rates and determine the welfare differences between policies. What we really want to do is to determine what side of the Laffer curve the United States is on. If the current tax rate of .40 is to the left of the maximum tax revenues, than increasing the tax rate is a good idea from the standpoint of increasing government tax revenues. If the opposite holds, then increasing the tax rate will actually lower the government's tax revenues. This is also left as an exercise for you to do.

IV. The Uses of Tax Revenue: The Scandinavian Outliers

Richard Rogerson in a 2007 paper title *Taxation and Work Hours: Is Scandanavia an Outlier?* published in *Economic Theory* pointed out that the Scandinavian countries in general have higher marginal tax rates than Western Europe, yet work more hours per person. Does this mean that Prescott (2004) was wrong in arguing that differences in tax rates can account for most of the observed differences in work hours among OECD countries? Rogerson's answer is that Prescott's findings are not necessarily wrong, because households work hour responses to taxes will depend critically on the how the government spends the tax revenues.

Rogerson considers four types of government programs, including the lump-sum transfer program studied by Prescott and presented in Section 2. We now present each in turn.

Program 1: Useful government transfers

This is just the case studied in Section 3 of this chapter. The key feature of this program is the lump-sum payment of tax revenues back to the household. Although we model this as a rebate check from the government, the key aspect of this policy is that the government revenue provides something that the consumer values. This can be in the form of added purchasing power to consumers, (outright income transfers), goods that the government buys and transfers to consumers (education, judicial, health), or employs workers to produce output transferring the output to consumers.

In the case of useful government transfers, the household's budget constraint is

$$c \leq (1-\tau)wh + \psi$$

and the Government Budget Constraint is

$$0 = \tau wh + \psi$$

Examples: Government revenue is used to provide something that the consumer values. This can be in the form of added purchasing power to consumers, (outright income transfers), goods that the government buys and transfers to consumers (education, judicial, health), or employs workers to produce output transferring the output to consumers.

Solving for the Equilibrium: The utility maximization condition is:

$$\frac{\alpha c}{(100 - h)} = (1-\tau)w$$

In equilibrium c=wh, and w=A. Hence

$$\frac{\alpha h}{(100-h)} = (1-\tau)$$

This yields the familiar solution given by Equation (22)

$$h = \frac{1-\tau}{(\alpha + (1-\tau))} 100$$

Program 2: Useless government expenditures

In this case the government uses the tax revenues to buy output. The key property of this program is that the use of the government revenue yields no utility to consumers and does not enhance the productivity of firms. Here the best example is probably military services. Much of employment in the public sector could be classified as useful government expenditures.

The household's budget constraint is

$$c \le (1-\tau)wh$$

and the Government Budget Constraint is

$$g = \tau wh$$

FONCs:

$$\frac{\alpha c}{(100-h)} = (1-\tau)w$$

In equilibrium, since $g = \tau wh$, $c = (1-\tau)wh$. Hence

$$\frac{\alpha h(1-\tau)}{(100-h)} = (1-\tau)$$

It is immediate apparent that taxes rates have no effect on the equilibrium work hours in the case of useless government expenditures. We have

$$h = \frac{100}{1+\alpha}$$

Program 3: Consumption subsidy.

Another use of tax revenues is to subsidize the consumption purchases of households. Here the key is that government subsidizes consumption at the margin. Examples that Rogerson lists are subsidized child care or elderly care. In Sweden, the child care transfers are actually made conditional on both parents working.

For this policy, for every \$1 spent by the consumer on goods and services, the subsidy would rebate back s cents.   In the case of the consumption subsidy, the household's budget constraint is

$$(1-s)c \le (1-\tau)wh$$

And the Government Budget Constraint is

$$sc = \tau wh$$

FONCS

$$\frac{\alpha(1-s)c}{(100-h)} = (1-\tau)w$$

In equilibrium sc=τwh, and w=A and c=h, hence

$$\frac{\alpha A(h-\tau h)}{(100-h)} = \frac{\alpha A(1-\tau)h}{(100-h)} = (1-\tau)A.$$

Thus, equilibrium work hours are independent of taxes and are just

$$h = \frac{100}{1+\alpha}$$

Program 4: Subsidy to leisure;

The final type of use of tax revenues is a transfer that depends on how much leisure the consumer enjoys. An example of this type of transfer is unemployment compensation. In particular, for each unit of leisure enjoyed by the household, he or she enjoys a transfer of b units of the good. Note that this is not a lump-sum transfer as it depends on leisure, or how little you enjoy. The household's budget constraint is

$$c \leq (1-\tau)wh + bl$$

The Government Budget Constraint is

$$\tau wh = bl$$

FONCS

$$\frac{\alpha c}{(100-h)} = (1-\tau)w - b$$

In equilibrium c=Ah, and w=A and $\tau wh = b(100-h)$, hence

$$\frac{\alpha hA}{(100-h)} = (1-\tau)A - b$$

$$\frac{\alpha hA}{(100-h)} = (1-\tau)A - \frac{\tau Ah}{100-h} = 1 - \frac{\tau A}{100-h}$$

$$\alpha h = 100(1-\tau) - h$$

So that
$$h = \frac{100(1-\tau)}{1+\alpha}$$

Findings: Rogerson points to key differences in the use of government tax revenues between Scandinavia and Continental Europe as a way of reconciling Prescott's conclusions with the Scandinavian experiences. For example, he points out that Government spending on family services in Scandinavia is 8% of GDP whereas it is 2%

in continental Europe 2%; Additionally, he points out that government employment to total employment is 15% in the US, 18% in Continental Europe, and 28% in Scandinavia. If these expenditures are wasteful, then according to Case 2 above they do not change work hours. Rogerson argues that half of these expenditures in Scandinavia are wasteful. Rogerson concludes that these differences gets you most of the difference between Continental Europe and Scandinavia.

V. Conclusions

In this chapter we introduced the labor/leisure decision and government policy, and used the model to examine how much of the differences in work hours observed across G-7 countries in the 1970s and the 1990s could be accounted for by differences in tax rates. The conclusion of the calibration exercise is that tax rate differences across countries and across time can account for most of the differences in hours worked per week per person aged 15 to 65.

Quantitative Exercises:

1. Calibrate model under program x to US and then change the tax rates.

2. More interesting, calibrate the model with all four programs in the US and then change the tax rates.