Endogenous Growth: An Alternative Measuring Device?

I. Introduction

Facts are essential in economics as they allow us to test theory. However, they are also useful in the calibration procedure for the purpose of choosing a good measuring device. In Chapter 3, we presented the Solow Growth model, an example of exogenous growth model on account that the driver of economic growth, technological change, is determined outside the model. Exogenous growth is only one type model that researchers have developed.

Another branch of growth theory comes under the heading of *Endogenous Growth*. Endogenous Growth began with the Ph.D. thesis of Paul Romer at the University of Chicago in 1986, and has flourished ever since. The heading, endogenous growth refers to the result that economic growth does not rely on some exogenous increase in some parameter or variable in this class of models. Although this seems like a huge advantage of endogenous growth relative to exogenous growth, this does not mean that endogenous growth is a better measuring device for the question we have in hand, namely, why are some countries so much poorer than others?

The types of models that fall in this category are varied. There are really two branches of endogenous growth models. Both share the feature that economic growth is the result of decisions made by firms and households in the model. The two branches are distinguished by whether they assume perfectly competitive markets or imperfectly competitive markets. The imperfect competition based models focus on the decisions of firms to create new ideas through research and development. Paul Romer was one of the founders of this sub-branch of models, publishing a 1990 paper. His 1986 paper established the other sub-branch of this literature.

To understand the intuition behind the endogenous growth, at least the variety that maintains the perfect competition assumption, it is instructive to review the reason for the failure for the Solow model to generate sustained growth in the absence of technological change. Recall, that the growth cannot be sustained in the Solow model because the marginal product of capital decreases and in the limit approaches zero. Thus, to break this result we need for the marginal product of capital to not go to zero as the capital stock is increased. This is effectively what is done in the AK model, as the marginal product of capital is constant and equal to A.

There are other important differences between Exogenous and Endogenous growth. Recall the Solow growth model is a model of relative income levels. Country factors such as TFP, savings rates, or population do not affect the economy's growth rate along the balanced growth path. They do, however, determine a country's income level relative to other countries; a country with a higher savings rate will have a higher level of per capita output at each date along the balanced growth path. For this reason, exogenous growth is often referred to as a *level effect model*. This is not the case with endogenous growth models of the AK variety. Here any difference between countries result in permanent differences in growth rates along the balanced growth path. Moreover, as we shall see there is no convergence of per capita GDP for countries that are identical in every way except for their initial per capita capital stock.

We begin by studying the endogenous growth models that assume perfect competition. As we shall see, the analysis for the simplest of these models, the Ak model, is actually less involved then the analysis for the Solow model.

II. Perfectly Competitive Models: The AK Growth Model

The model's structure is the same as the Solow growth model with the exception of the production function. The aggregate production function is now

(Y) $$Y_t = AK_t .$$

The corresponding per capita production function is

(y) $$y_t = Ak_t .$$

Consequently, the aggregate capital stock evolves according to the following equation

(K) $$K_{t+1} = (1-\delta)K_t + sAK_t .$$

The per capita production function then is

(k) $$(1+n)k_{t+1} = (1-\delta)k_t + sAk_t .$$

Balanced Growth Path

The easiest way to determine if there is a steady state equilibrium or a balanced growth path equilibrium is to use the above equation (k) and solve for the growth rate of the per capita capital stock. This is done by first dividing each side of the above equation by k_t

$$(1+n)\frac{k_{t+1}}{k_t} = (1-\delta) + sA .$$

The growth rate is thus,

(gr-rate) $$\frac{k_{t+1}}{k_t} = \frac{sA+1-\delta}{1+n}.$$

Since (y) is linear in k, it follows that per capita output increases at this rate as well. Now for there to be positive growth in must be the case that

$$\frac{sA+1-\delta}{1+n} > 1.$$

This requires that $sA > n+\delta$. In effect, the productivity of capital and savings rate must be large enough relative to the depreciation rate and population growth rate.

Notice that the growth rate is affected by the economy's savings rate, s, TFP, A, and population growth rate, n, as well as the depreciation rate. Here then is the result that country specific variables affect the its growth rate – the so called "growth rate effect."

No Transitional Dynamics

Having solved for the growth rate along the balanced growth path, we can ask whether there is any convergence type property associated with the AK model. In contrast to the Solow growth model, there is no convergence in this model; initial differences in per capita capital stocks between otherwise identical countries will be forever maintained. Income gaps will never be eliminated between otherwise identical countries.

This result should not be surprising in light of the algebra used to solve for the balanced growth path. At no step in the analysis did we invoke the balanced growth condition that the growth rate of the capital stock equals $1+g_k$. This is in contrast to the derivation of the balanced growth path growth rate for the Solow model.

Intuitively, the failure of economies that differ only in their initial capital stocks to converge is related to the fact that there are no diminishing returns to capital in the Ak economy. As such there is no advantage with starting out with a lower capital stock. The marginal product of capital in the poorer economy is the same as the marginal product of capital in the richer country. Adding a new machine results in the same increase in output, A, in each country. This is in contrast to the Solow model where the new machine in the poor country brings about a much larger increase in per capita output than the new machine in the rich country.

III. Monopolistic Endogenous Growth

The second type of endogenous growth model has markets that are characterized by imperfect competition. The need for deviating from the assumption of perfect competition is related to the special properties of technology or ideas. Paul Romer (1990) pointed out that although technology can be viewed as an input in the production process, it has very different properties than other inputs such as labor and machines. The best way to think of a technology is a recipe for combining inputs to produce output. (It is useful to think about cooking or baking some type of food, such as chocolate chip cookies) What makes the recipe unique from other inputs such a labor or capital, is that its use by one person does not preclude its use by another. I can be using the recipe for making chocolate chip cookies in my house and another 10 people could be doing so at the same time. When one person's use of an input does not prevent someone else from using it at the same time, we say that it is *non-rival* in nature. Technology is thus a non-rival input. This is in contrast to labor or machines, which are rival inputs. The non-rival property implies that no one would ever undertake costly R&D unless there is a patent system, as someone could just copy what you expensively created, and because they did not have to spend any resources on R&D, could undercut your price, effectively putting you out of business. A patent of course creates a monopoly, which means that markets are not perfectly competitive. This is the source of imperfect competition in this second category of Endogenous Growth models.

> *The Case Against Patents*
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> *David Levine and Michele Boldrin have recently challenged the Schumpetrian Growth Literature's view that patents are necessary for innovation. They argue that the lag between when an innovation is made and when it is copied by a competitor is sufficiently long that the innovator can earn enough rents on his innovation to more than compensate for R&D expenditures. In effect, copying is not a costless and instantaneous activity. They point to empirical evidence that shows that despite the large increase in the number of patents and the increase in patent protection, there has been no notable increase in the rate of technological change. They also review historical evidence that shows that the creation of most industries is the result of a burst of discoveries that are done within highly competitive conditions.*

Another property of technology identified by Paul Romer is that it is hard for its creator to extract payments from everyone in a society that uses his or her idea. For sure, patents prevent someone from directly copying your idea, but they do not prevent someone from using your idea to create a new idea. The ability to collect payment for the use of a good or resource you own goes under the heading of excludability. The ability to collect fees for the use of something you own is a function of both the physical qualities of the asset as well as the legal system.

Because the patent system does not prevent someone from using your idea to create a new one, technology is only partially excludible. The implication here is that there are likely to be important externalities that imply that market is not doing enough R&D from a social perspective. At the same time, a line of research under the endogenous growth literature with monopolistic elements known as Schumpeterian Growth argues that as new ideas replace old ideas, they destroy the value of the resources used to discover those old ideas. There is, thus, a destructive element associated with the creation of an idea, something which the famous Austrian American economist, Joseph Schumpeter referred to as *Creative Destruction*. This destruction of the value of past investments suggests that an economy may be doing too much rather than too little from the standpoint of maximizing social welfare.

To show how these properties affect the modeling and analysis, we provide an outline of Paul Romer's 1990 article titled *Endogenous Technological Change.* In doing so, we do not attempt to solve out the general equilibrium. Again, our aim is to show how the special properties of knowledge/ideas are reflected in the model, or better said, how they change the model's structure. Interestingly enough, the model we sketch out has become the building block of business cycle analysis associated with New Keynesian analysis. Reviewing the monopolistic structure here will prove useful when we study the New Keynesian model in our last chapter.

For Romer's (1990) model we shall continue to assume the same consumption equation and capital stock equation

(C) $$N_t c_t = (1-s)Y_t$$

(K) $$K_{t+1} = (1-\delta)K_t + sY_t$$

The model differs from the ones we have studied on the production side. Here there are three levels of production: the Final Goods Sector, the Intermediate Goods Sector, and the R&D Sector. We discuss each in turn.

Final Goods Sector

The Final Goods sector is perfectly competitive. It combines intermediate goods, x_{zt}, to produce the final good, Y_t, according to the following production function

(Y) $$Y_t = \left(\sum_{z=1}^{Z_t} x_{zt}^{\sigma} \right)^{1/\sigma}$$

The above production function is generally referred to as a Spence-Dixit-Stiglitz function or aggregator for its originators: Michael Spence, Avenish Dixit and Joseph Stiglitz. It is characterized by constant returns to scale. The variable, Z_t, denotes the number of intermediate goods. The parameter, σ, is restricted to be less than one, and determines how substitutable the intermediate goods are in the production function. If σ=1, then intermediate goods are perfect substitutes making the elasticity of substation infinite.

This specification of the production function actually captures the idea of increasing returns to specialization, an idea that harkens back to Adam Smith and the *Wealth of Nations*. To see this, let us assume that all intermediate goods are equal in quantities. Then (Y) reduces to $Y_z = Z_t^{1/\sigma} x$. Thus, the more intermediate goods produced by a country, the larger its TFP is.

Being perfectly competitive, the final goods producers are price takers and maximize profits

(Y-Prof) $$\max\left(\sum_{z=1}^{Z_t} x_{zt}^{\sigma}\right)^{1/\sigma} - \sum_{z=1}^{Z_t} p_{zt} x_{zt}$$

This leads to the following profit maximization problem

(Y-FONC) $$\left(\sum_{z=1}^{Z_t} x_{zt}^{\sigma}\right)^{\frac{1}{\sigma}-1} x_{zt}^{\sigma-1} = p_{zt}$$

Using the definition of (Y), the left hand side can be rewritten as

(x_z-Demand) $$Y^{1-\sigma} x_{zt}^{\sigma-1} = p_{zt}$$

This is the Final Good Sector's demand for intermediate good z. Deriving this demand is necessary for the problem of a monopolist intermediate good producer.

Intermediate Goods Sector

The intermediate Goods Sector is monopolistically competitive. Each intermediate good z is produced by a unique monopolist. This monopoly position is the result of the firm buying a patent that lasts forever from the R&D sector. It is competitive in the sense that in equilibrium a monopolist effectively makes enough to cover its fixed costs. As new ideas can be created, and hence new intermediate goods, there is effectively free entry into the intermediate good sector. Even though each producer fixes its own price, this entry drives the price of intermediate goods and patents up so that firms are only making enough profits to cover the cost of buying a patent.

An intermediate good producer hires workers to produce the intermediate good. It pays a competitively determined wage rate. w_t. However, being a monopolist it sets the price for its own product, p_{zt}, based on the demand for its product given by (x_z Demand). For simplicity, we shall assume that one unit of labor produces one unit of the intermediate good.

Using (x_z Demand), the profit of the monopolist are

(x_z Profit) $$Y_t^{1-\sigma} x_{zt}^{\sigma-1} x_{zt} - w_t x_{zt} = Y_t^{1-\sigma} x_{zt}^{\sigma} - w_t x_{zt}$$

We can differentiate with respect to x_z and set the derivative to zero. This yields the following profit maximization condition

(x_z- FONC) $\sigma Y_t^{1-\sigma} x_{zt}^{\sigma-1} = w_t$

If we use (x_z- FONC) with (x_z-Demand), we obtain the important result in these models of monopolistic competition that the price is a constant mark up over the marginal cost, w_t. Namely,

(Price Mark Up) $$p_{zt} = \frac{w_t}{\sigma}.$$

If each worker produced A units of the intermediate good, then the Price mark up equation would include A in the denominator. Effectively, we have set A=1.

<u>The R&D Sector</u>

Ideas are produced by the R&D sector. To produce new ideas, the R&D sector rents capital from the households. Let Z_t be the stock of ideas that exist at date t. Then new ideas are (Z_{t+1}-Z_t). The amount of new ideas depends on the capital used in the R&D

sector. Following Romer's point that ideas are partially excludible, the existing stock of ideas is assumed to impact the number of new discoveries. A simple formulation of this production function is

(R&D) $$Z_{t+1} - Z_t = K_t Z_t$$

The important point here is that although the R&D Sector pays for the capital input, it does not pay for the Z input. This is on account that old ideas can be used to produce new ideas, and need not be compensated for this use- the notion of partial excludability. The profits of the R&D sector are thus

(R&D Profit) $$p_{Pt}(Z_{t+1} - Z_t) - r_t K_t$$

where p_{pt} is the price of the patent. This is a one-time cost that is incurred by the buyer of the patent, an intermediate good producer. Indeed, the existence of this one-time cost, means that we must have a monopolistic structure to the intermediate good sector. If a producer were a price taker and if there are constant returns to production in labor as there is in the intermediate good sector, then a firm that paid its inputs their marginal product would earn zero profits. However, the monopoly structure guarantees enough profits to pay for the patent. Indeed, in these models the monopolist must make profits each period over their infinite lifetimes so as to just pay for the one-time cost of the patent. This is a necessary condition of the general equilibrium, which we do not develop any further in this chapter as it takes too much time and knowledge than we have a this point.

III. Choice of Measuring Device

How useful is Endogenous Growth theory for understanding the current huge disparity in international incomes? Can either branch of this literature account for the late starts of some countries? Let us start with the endogenous growth models with monopolistic elements. Recall, that this literature emphasized the decisions of firms to expend resources to generate new ideas. The problem with this type of theory for understanding the growth and development facts is that late starters are not in the business of creating new ideas. They need not reinvent the wheel. Instead, they can use ideas that have been discovered in the rich countries.

Indeed, imitation rather than innovation is the critical element in the early catch-up of many country. Japan did not begin its growth miracle in the 1950s producing automobiles or other high tech goods. Textiles and assembly of toys and electronic equipment were large parts of its early miracle days. Nathan Rosenberg in his 1983 book, Inside the Black Box, documents that the textile industry in Japan started by using obsolete machines discarded from the UK. They then modified these machines to realize productivity gains. Eventually, they started to produce their own textile machines. More recently, the spectacular growth of China has mimicked this type of process, with perhaps a greater reliance on multinational firms for setting up manufacturing sites in special export zones. Multinationals have been extremely important in transferring knowledge from the rich to the poor countries.

For this reason, this type of endogenous growth model does not seem particularly relevant for the purpose of understanding the evolution of international income differences. This is not to say that these types of models have no value. Indeed, if we were trying to understand growth in the stock of knowledge over time, or why the US growth rate of per capita output has been roughly 2 percent per year for the last century, this is exactly the type of model that we should use as a measuring device. However, we are not studying these questions in this course.

This leaves us with the first type of endogenous growth model to consider. Is this a better measuring device than the exogenous growth model we have developed that combines Solow with Malthus? Your initial reaction probably is that the AK model is a more reasonable choice of measuring device precisely for the reason that it does not require technology to fall from the sky. Additionally, you might think that its prediction that country specific factors have a permanent effect of a country's growth rates is a virtue of the model. It is certainly the case that over the last three centuries, growth rates of per capita GDP have differed dramatically across countries. The Ak model, thus, can easily explain this difference in growth rates, and by doing so, can account for the widening of international income differences to their current size.

However, the increase in income disparities since 1700 is only one property of the data. Before we conclude that the AK model is a good measuring device, it is instructive to examine whether the model is qualitatively consistent with the remaining set of facts. As it turns out, it is not. In particular, the Ak model cannot account very well for the fact that growth models are a very recent phenomenon limited to countries that were poor at the time their miracles began. Indeed, as we shall see, the model predicts that growth miracles are every bit as likely to occur in rich countries, and were every bit as likely three centuries ago.

How would one explain the growth miracle of Japan within the context of the AK model? Recall, that an increase in either the country's savings rate or TFP or a decrease in its population growth rate leads to a permanent increase in a country's growth rate. One could then explain the large surge in Japan's growth rate starting in 1950 to a change in any of these factors. Japan's savings rate did in fact show a large increase in the post war period. A change in 1950 along these lines would change the path of (log) per capita GDP as shown below.

Figure 1: AK Model and Japanese Growth Miracle

Now Japan has not managed to maintain an average annual growth rate equal to its average over the 1950 to 1960 or the 1960 to 1970 period. In fact, as Japan's income gap with the United States has decreased, its growth rate has fallen. In the 1960s, this growth rate was close to 10 percent; in the 1970s, it was approximately 5 percent on average; and in the 1980s, 4 percent. The 1990s, referred to as the Lost Decade, was disastrous with a growth rate close to 0 percent.

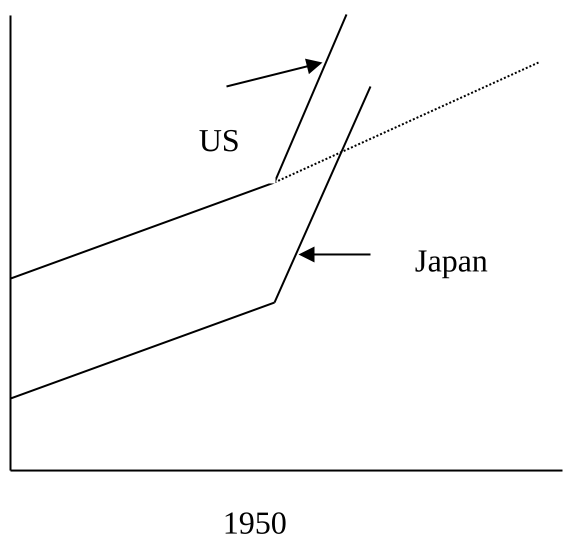
How could we explain this decline in Japan's growth rate within the Ak model? One way is to argue that Japan's savings rate or its TFP fell continuously after the big increase experienced in 1950. Savings rates and TFP are in the end determined by a country's policies, and so this really is a story about a continual deterioration of Japanese policies after 1950. This is a rather complicated story, but not an implausible one. We cannot dismiss the AK model for this reason alone.

However, we can dismiss it for the reason that it predicts that growth miracles are as likely in rich countries as in poor ones. Let us ask what would the AK model predict if the United States were to now adopt Japanese policies that were in place during the 1950's? The following diagram shows the path predicted by the Ak model if the United States adopted Japanese policies in 1950. Since policies have growth rate effects, and these policies led to 6 percent growth in Japan over the 1950-2000 period, then the

United States according to the Ak model would grow at 6 percent per year starting in 2000. The United States would therefore experience a growth model.

We could also ask the question of what would happen to the US economy if it had in year 1800 adopted Japan's 1950 policies. The effect would be identical as to when the United States adopts the policy in 2000: the United States would thereafter grow at 6 percent per year. The Ak model thus predicts that growth miracles are every bit as likely to happen in the leader and are every bit as likely to have happened two centuries ago. This is completely at odds with the pattern of growth and development. For these reasons, we conclude that the Ak model is not a good measuring device for understanding today's huge differences in incomes across countries.

Figure 2: Growth Miracles are Just as Likely in the US



Before concluding the chapter, it is worthwhile to ask what the Solow Model predicts for the path of the United States if it were to adopt Japan's 1950 policies. In the Solow model, a change in the savings rate or TFP results in a change in a country's income level along the balanced growth path. Following the change, the model predicts a transition from one balanced growth path to another. In the case that the United States adopted Japanese policies, the new balanced growth path of the United States would probably lie somewhat lower than its current path. We say this because Japan even before the 1990s appeared to be converging to a level of income around 90 percent of the US level. Thus, if the United States becomes like Japan, the Solow model predicts that the United States would be worse off and would experience lower growth until it converges to its new balanced growth path. This is a very different conclusion than the Ak model.

IV. Conclusion

Endogenous growth is a rather recent development in macroeconomics. Rightfully so, it has attracted a lot of attention. Although it has rather limited value in understanding why some countries started the growth process so much later, there are many other important questions for which it is ideally suited to address. For example, a current area of research tries to understand why the *Industrial Revolution* was primarily a Northern European phenomenon. This literature has found that starting dates seem to be determined by what happened to countries much earlier in history. For example, countries that underwent a Neolithic Revolution earlier in time (i.e., switched from hunter/gatherer to agriculture) were far more likely to industrialize earlier. We will come back to this issue and some similarly related ones in our last chapter on economic growth and development, Chapter 8.

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Problems

1. In the Ak model explain why a growth miracle is every bit as likely to occur in a rich country such as the United States as it is in a poor country. In other words, what would happen in the model if the U.S. adopted Japanese policies? Compare this prediction with that of the Solow Model.

2. Compare the path of an economy's per capita output in both the Solow Model with exogenous technological change and the Ak model before and after the following changes. Assume that the prior to the change, the economy is on its balanced growth path. (Note: your answer should consist of a graph of the $\ln(y_t)$ on the vertical axis, and time on the horizontal.

   a. An increase in the population growth rate.
   b. A decrease in the economy's savings rate
   c. An increase in TFP
   d. A earthquake that destroys half of the nation's capital stock but not its population
   e. A disease that kills off half of the population.

3. Consider the AK model. Suppose you were to assign parameter values so that the model matches the US long run growth experience. List the Kaldor Growth facts that you would use in Step 4 of the calibration exercise. Use these facts with the model equilibrium results to assign the parameter values.

4. Suppose we asked the question: Do differences in Savings Rates account for the difference in average annual growth rates observed across countries over the 1960-2000 period? Further suppose that we chose the AK model as our measuring device and that we calibrated the model parameters in Step 4 to the US experience as in Question 3 above.

a. Explain how you would test the theory being careful to list the variables in the Penn World Tables that you would use.

b. Execute the test you have outlined above. Hand in a plot of actual average annual growth rates or the 1960-200 period versus growth rates predicted by the model