Lecture, January 21, 2015

Chapter 7 Basic Elements of Noncooperative Games

A game requires the specification of the players, the actions available to each player, and the order in which actions are taken. We might consider this to be the *rules* of the game. Games are interesting when some outcome affects the well-being of the different agents (i.e., what any player receives depends upon the actions of all players). This is the *interdependence* referred to in MWG.

perspective of MWG: dwells on the process of *modeling* a situation with a game rather than simply taking the standard economic models as reality. It's deliberate for a reason. Be patient if you already know some game theory.

Extensive Form Representation of a Game

decision nodes vs. **terminal nodes**

Example 1 *Matching Pennies and Tic-Tac-Toe*

verbal description of the two games, vs. their extensive form representations

games of **perfect information** (all decisions of all players are publicly observable) vs. game of **imperfect information** (some decisions are only observed by a proper subset of players)

information set in Matching Pennies to denote that Player 2 does not observe the choice of Player 1 at the time he makes his choice (2 versions of matching pennies)

Strategies and the Normal Form Representation of the Game

strategy sets, outcome sets, outcome mapping

noncooperative: assumes that agents act independently of each other

Strategy: complete contingent plan that specifies an agent's actions at each information set assigned to him

Example 2 *Normal form representation of the two forms of Matching Pennies (perfect and imperfect information)*

diagram of a game

Normal (or strategic) form representation of an extensive form game

Randomized choices (behavioral strategies)

Example 3 *battle of the sexes*

F/M	B	F
B	$2,1$	$0,0$
F	$0,0$	$1,2$

Example 4 *Meeting in NY:*

$1/2$	ESB	GCS
ESB	$100,100$	$0,0$
GCS	$0,0$	$100,100$

Chapter 8: Simultaneous-Move Games

solution concept: A story or theory of what happens when the game is played – a notion of equilibrium. Solution concepts typically depend upon what the players know and the different senses in which an action can be interpreted as in the best interest of a player (e.g., does the optimality of an agent's action depend upon some specification of actions for the other agents?).

game theory: precise language of incentives.

Dominant and Dominated Strategies

Example 5 *Example: prisoner's dilemma (p. 236)*

$1/2$	DC	C
DC	$-2,-2$	$-10,-1$
C	$-1,-10$	$-5,-5$

definition of a dominant strategy

strictly dominate, (weakly) dominate, (weakly) dominated, strictly dominant, strictly dominated

I'd like to change the definition of a (weakly) dominant strategy from the way in which it is defined in MWG.

MWG: A strategy $s_i^{'}$ is *weakly dominant* for player i if it weakly dominates every other strategy $s_i \neq s_i^{'}$, i.e.,

$$u_i(s_i^{'}, s_{-i}) \geq u_i(s_i, s_{-i})$$

for all s_{-i}, *with strict inequality for at least one* s_{-i}.

This definition is unusual because of the clause "*with strict inequality for at least one* s_{-i}". One consequence of this definition is that a player can have at most one weakly dominant strategy. If he instead had two dominant strategies $s_i^{'}$ and $s_i^{''}$, then

$$u_i(s_i^{'}, s_{-i}) > u_i(s_i^{''}, s_{-i})$$

for some at least one s_{-i} because $s_i^{'}$ weakly dominates $s_i^{''}$, but

$$u_i(s_i^{'}, s_{-i}) \leq u_i(s_i^{''}, s_{-i})$$

because $s_i^{''}$ weakly dominates $s_i^{'}$.

The more commonly used definition of a dominant strategy is as follows: $s_i^{'}$ is a dominant strategy iff for s_i and s_{-i},

$$u_i(s_i^{'}, s_{-i}) \geq u_i(s_i, s_{-i})$$

The clause "*with strict inequality for at least one* s_{-i}" has thus been dropped. A player can have more than one dominant strategy with this definition, though any two of his dominant strategies must be *payoff-equivalent* in the sense of providing him with exactly the same utility for all choices of his opponents' strategies: if $s_i^{'}$ and $s_i^{''}$ are both dominant strategies, then

$$u_i(s_i^{'}, s_{-i}) = u_i(s_i^{''}, s_{-i})$$

for all s_{-i}.

Consideration of Exercise 8.B.2. (p. 262) suggests that MWG in fact use this more conventional definition of a dominant strategy.

Example 6 *A simple model of an auction of an indivisible item. n bidders, n real valuations, quasilinear utility. Discuss randomized allocation in the case of ties.*

$u_i(v_i, p) = v_i - p$

Example 7 *Dominance and the First-Price Auction*

Consider bidder i with value v_i who considers the bid b_i. Let $\overline{b}$ denote the maximal bid of the other $n-1$ bidders.

No bid is strictly dominated. In comparing bids b_i and b_i', suppose $b_i, b_i' < \overline{b}$, i.e., one of the other bidders bids so much that bidder i loses with either bid b_i or b_i'. In such a case, both bids produce a utility of 0 and so neither bid can strictly dominate the other.

Bidding one's value or more than one's value is dominated by any bid less than one's value. We compare the consequences of bidding $b_i < v_i$ with bidding $b_i' \geq v_i$ in the table below:

bid\value of $\overline{b}$	$\overline{b} < b_i$	$\overline{b} = b_i$	$b_i < \overline{b} < b_i'$	$\overline{b} = b_i'$	$b_i' < \overline{b}$
$b_i < v_i$	$v_i - b_i > 0$	$\delta(v_i - b_i) > 0$	0	0	0
comparison	$>$	$>$	$\geq$	$\geq$	$=$
$b_i' \geq v_i$	$v_i - b_i^{'} \leq 0$	$v_i - b_i^{'} \leq 0$	$v_i - b_i^{'} \leq 0$	$\delta\left(v_i - b_i^{'}\right) \leq 0$	0

Here, $\delta \leq 1/2$ reflects the randomization in the case of ties and depends upon the number of bidders who submit the bid $\overline{b}$.

Example 8 *Dominance and the Second Price Auction*

Bidding one's valuation weakly dominates any other strategy. It is the unique dominant strategy for each bidder.

Consider bidder i with value v_i who considers the bid b_i. Let $\overline{b}$ denote the maximal bid of the other $n-1$ bidders. Bidder i's utility as a function of v_i, b_i, and $\overline{b}$ is

$$u_i(v_i, b_i, \overline{b}) = \begin{cases} v_i - \overline{b} \text{ if } b_i > \overline{b} \\ \delta\left(v_i - \overline{b}\right) \text{ if } b_i = \overline{b} \\ 0 \text{ if } b_i < \overline{b} \end{cases}$$

We see here the utility consequences of bidder i being the high bidder, tying as the high bidder, or losing the auction. Here, $\delta \leq 1/2$ reflects the randomization in the case of ties and depends upon the number of bidders who submit the bid $\overline{b}$. For any given bids of the other bidders (which bidder i doesn't know when he chooses his bid), bidder i's choice of bid b_i doesn't change the 3 possible utilities that he might receive: it only affects which of the 3 values that he receives. We can order the possible utilities as follows:

- *if $v_i > \overline{b}$, then $v_i - \overline{b} > \delta\left(v_i - \overline{b}\right) > 0$.*
- *if $v_i = \overline{b}$, then $v_i - \overline{b} = \delta\left(v_i - \overline{b}\right) = 0$.*
- *if $v_i < \overline{b}$, then $0 > \delta\left(v_i - \overline{b}\right) > v_i - \overline{b}$.*

The bids of the other bidders present bidder i with a choice among 3 possible utilities. The key point to notice is that choosing $b_i = v_i$ insures that bidder i receives the largest of these 3 payoffs for all possible profiles of bids by the other bidders: if he happens to be the high bidder with $v_i = b_i > \overline{b}$, then $v_i - \overline{b}$ is the maximal possible utility available to him; if he ties as high bidder with $v_i = b_i = \overline{b}$, then he is indifferent between winning and losing (0 is the best that he can do); if he loses with $v_i = b_i < \overline{b}$, then he's glad to lose because the bids of the other bidders simply do not present him with an opportunity to make a profit. For all possible profiles of bids by the other bidders, bidding his value maximizes i's utility. It is therefore a dominant strategy.

It is interesting that a bidder would not benefit from learning the bids of the other bidders in advance, for bidding his value is optimal regardless of the bids of the other bidders.

How does one argue that it is bidder i's unique dominant strategy? Any other bid is not optimal for some choice of $\overline{b}$. Consider for instance, $v_i < b_i$. If $v_i < \overline{b} < b_i$, then bidder i wins the auction and suffers a loss of $v_i - \overline{b}$. Consider $b_i < v_i$. If $b_i < \overline{b} < v_i$, then bidder i loses the auction and forgoes the positive profit he could have won by bidding between $\overline{b}$ and v_i. Bidding one's value is the only bid with the property that it maximizes one's payoff against all possible specifications of the bids of the others.