1. Introduction

   (a) double auction: any procedure in which buyers and sellers interact to arrange trade.

      i. contrast with the passive role of the seller in most auction models

      ii. can be dynamic or one shot; most theory focuses on one-shot procedures, but most experimental work has focused on dynamic procedures

   (b) What are the main issues and motivations?

      i. experimental evidence dating from the 1960s (V. Smith, inspired by E. Chamberlin's classroom experiments at Harvard)

         A. "clearinghouse" (one-shot) versus continuous time

      ii. price discovery vs. price verification (R. Wilson, Reny and Perry (2006)): where do prices come from?

      iii. M. A. Satterthwaite: strategic behavior is a problem in small markets but not in large markets, and it doesn't require a lot of traders for a market to be large

         A. Myerson-Satterthwaite

         B. Gibbard-Satterthwaite

      iv. market design

         A. design of algorithms for computerized trading

         B. in parallel to the use of auction theory to inform the design of auctions

         C. Budish, Cramton and Shim (2014)

         D. Loertscher and Mazzetti (2014), "A Prior-Free Approximately Optimal Dominant-Strategy Double Auction"

2. Elementary model of a static double auction: independent, private values

   (a) Chatterjee and Samuelson (1983) in the bilateral case

   (b) m buyers, each of whom wishes to buy at most one unit of an indivisible good, n sellers, each of whom has one unit of the good to sell

   (c) redemption value/cost: $v_i \in [\underline{v}, \overline{v}]$, $v_i \backsim G$, $c_i \in [\underline{c}, \overline{c}]$, $c_j \backsim F$. Here, for simplicity, $[\underline{v}, \overline{v}]$=$[\underline{c}, \overline{c}] = [0, 1]$

   (d) for $k \in [0, 1]$, k-double auction in bilateral case:

      i. bid b, ask a

      ii. trade iff $b \geq a$ at price $kb + (1 - k)a$

      iii. $k = 1$: buyer's bid double auction

         A. dominant strategy of seller to submit his cost as his ask

      iv. $k \in (0, 1)$

v. FOCs, assuming increasing strategies:

$$\begin{aligned}
\pi^B(v,b) &= \int_0^{S^{-1}(b)} (v-(kb+(1-k)S(c)))f(c)dc \\
\pi^B_b(v,b) &= \left(v-\left(kb+(1-k)S(S^{-1}(b))\right)\right) f(S^{-1}(b)) \cdot \frac{1}{S'(S^{-1}(b))} - kF\left(S^{-1}(b)\right) \\
&= (v-b)\cdot \frac{f(S^{-1}(b))}{S'(S^{-1}(b))} - kF\left(S^{-1}(b)\right)
\end{aligned}$$

$$\begin{aligned}
\pi^S(c,a) &= \int_{B^{-1}(a)}^1 ((kB(v)+(1-k)a)-c)g(v)dv \\
\pi^S_a(c,a) &= -(\left(kB(B^{-1}(a))+(1-k)a\right)-c)g(B^{-1}(a))\cdot \frac{1}{B'(B^{-1}(a))} + (1-k)\left(1-G\left(B^{-1}\right.\right. \\
&= -(a-c)\cdot \frac{g(B^{-1}(a))}{B'(B^{-1}(a))} + (1-k)\left(1-G\left(B^{-1}(a)\right)\right)
\end{aligned}$$

equilibrium:

$$\begin{aligned}
0 &= (v-B(v))\cdot \frac{f(S^{-1}B(v))}{S'(S^{-1}B(v))} - kF\left(S^{-1}(B(v))\right) \\
&= -(S(c)-c)\cdot \frac{g(B^{-1}(S(c))}{B'(B^{-1}(S(c))} + (1-k)\left(1-G(B^{-1}S(c)\right)
\end{aligned}$$

vi. "linked" differential equations

vii. Chatterjee-Samuelson linear solution in uniform case:

$$\begin{aligned}
B(v) &= \begin{cases} \frac{2}{3}v+\frac{1}{12} \text{ if } v \geq \frac{1}{4} \\ v \text{ if } v \geq \frac{1}{4} \end{cases} \\
S(c) &= \begin{cases} \frac{2}{3}c+\frac{1}{4} \text{ if } c \leq \frac{3}{4} \\ c \text{ if } c \geq \frac{3}{4} \end{cases}
\end{aligned}$$

viii. in general: every system of differential equations has a geometric representation
$0 \leq c \leq \lambda \leq 1$,

$$\begin{aligned}
0 &= (v-\lambda)\cdot f(c)\dot{c} - kF(c) \\
0 &= -(\lambda-c)\cdot g(v)\dot{v} + (1-k)(1-G(v))
\end{aligned}$$

ix. Increasing strategies:
   A. probability of trading must be nondecreasing in a buyer's value and nonincreasing in a seller's cost
   B. "no flat spots" in the multilateral case

x. Sufficiency of FOC: Sufficiency of FOC evaluating the buyer's FOC at bid b, value $B^{-1}(b)$:

$$0 = \left(B^{-1}(b) - b\right) \cdot \frac{f(S^{-1}(b)}{S'\left(S^{-1}(b)\right)} - kF\left(S^{-1}(b)\right) \Leftrightarrow$$

$$\left(B^{-1}(b) - b\right) = \frac{kF\left(S^{-1}(b)\right)}{S'\left(S^{-1}(b)\right)} S'\left(S^{-1}(b)\right)$$

marginal utility with value v and bid b:

$$\begin{aligned}
\pi_b^B(v, b) &= (v - b) \cdot \frac{f(S^{-1}(b))}{S'\left(S^{-1}(b)\right)} - kF\left(S^{-1}(b)\right) \\
&= \frac{f(S^{-1}(b))}{S'\left(S^{-1}(b)\right)} \left[(v - b) - \frac{kF\left(S^{-1}(b)\right)}{f(S^{-1}(b))} S'\left(S^{-1}(b)\right)\right] \\
&= \frac{f(S^{-1}(b))}{S'\left(S^{-1}(b)\right)} \left[(v - b) - \left(B^{-1}(b) - b\right)\right] \\
&= \frac{f(S^{-1}(b))}{S'\left(S^{-1}(b)\right)} \left[(v - B^{-1}(b))\right]
\end{aligned}$$

xi. existence of "double continuum" of equilibria

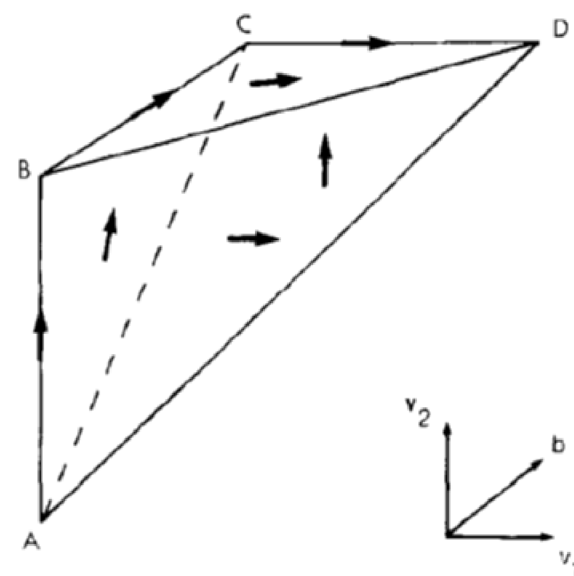


FIG. 3.1. Tetrahedron $0 \leqslant v_1 \leqslant b \leqslant v_2 \leqslant 1$ that contains solutions. The arrows ind
imit of the normalized vector field on the tetrahedron's faces and edges.

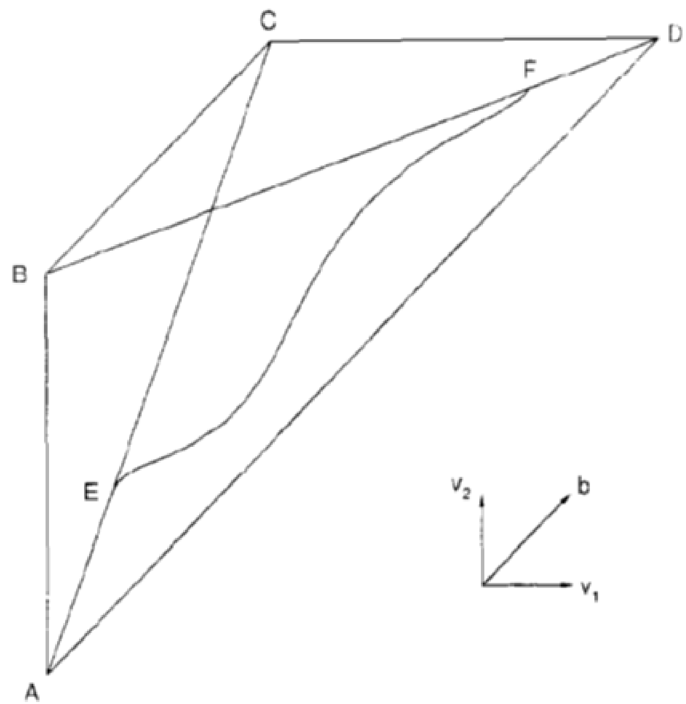


FIG. 4.1. Solution through $(v_1, v_2, b) = (0.375, 0.625, 0.45)$ shown within tetrahed
solution enters the tetrahedron at point E and exits through point F.

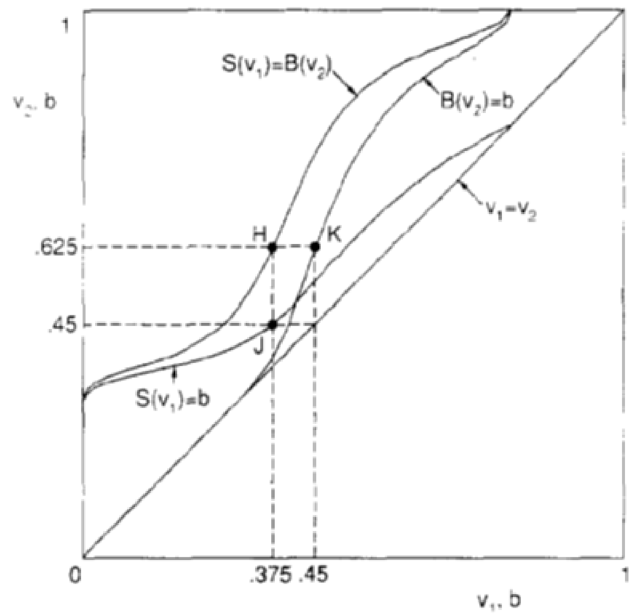


FIG. 4.2. Solution through $(v_1, v_2, b) = (0.375, 0.625, 0.45)$. Point $H = (v_1, v$
$(0.375, 0.625)$ is on the trading boundary where $S(0.375) = B(0.625) = 0.45$. P
$J = (v_1, S(v_1)) = (0.375, 0.45)$ is on the graph of the seller's strategy. Point $K = (B(v_2), v$
$(0.45, 0.625)$ is on the graph of the buyer's strategy. The ex ante expected utility is 0.065
the seller and 0.0725 for the buyer.

A. Leininger, Linhart and Radner (1989): step function equilibria

B. Linhart and Radner (1989) : minmax regret and minmax expected regret

(e) Multilateral case: m bids, n asks. Expressing market-clearing price in terms of order statistics.

$$s_{(1)} \leq s_{(2)} \leq \dots \leq s_{(m+n)}$$

Assuming $s_{(m)} < s_{(m+1)}$:

	bids	asks
$\geq s_{(m+1)}$	t	
$\leq s_{(m)}$	$m-t$	t

In the case of $s_{(m)} = s_{(m+1)}$: $s + x > 1$

	bids	asks
$> s_{(m)} = s_{(m+1)}$	t	$n-x-y$
$= s_{(m)} = s_{(m+1)}$	s	x
$< s_{(m)} = s_{(m+1)}$	$m-s-t$	y

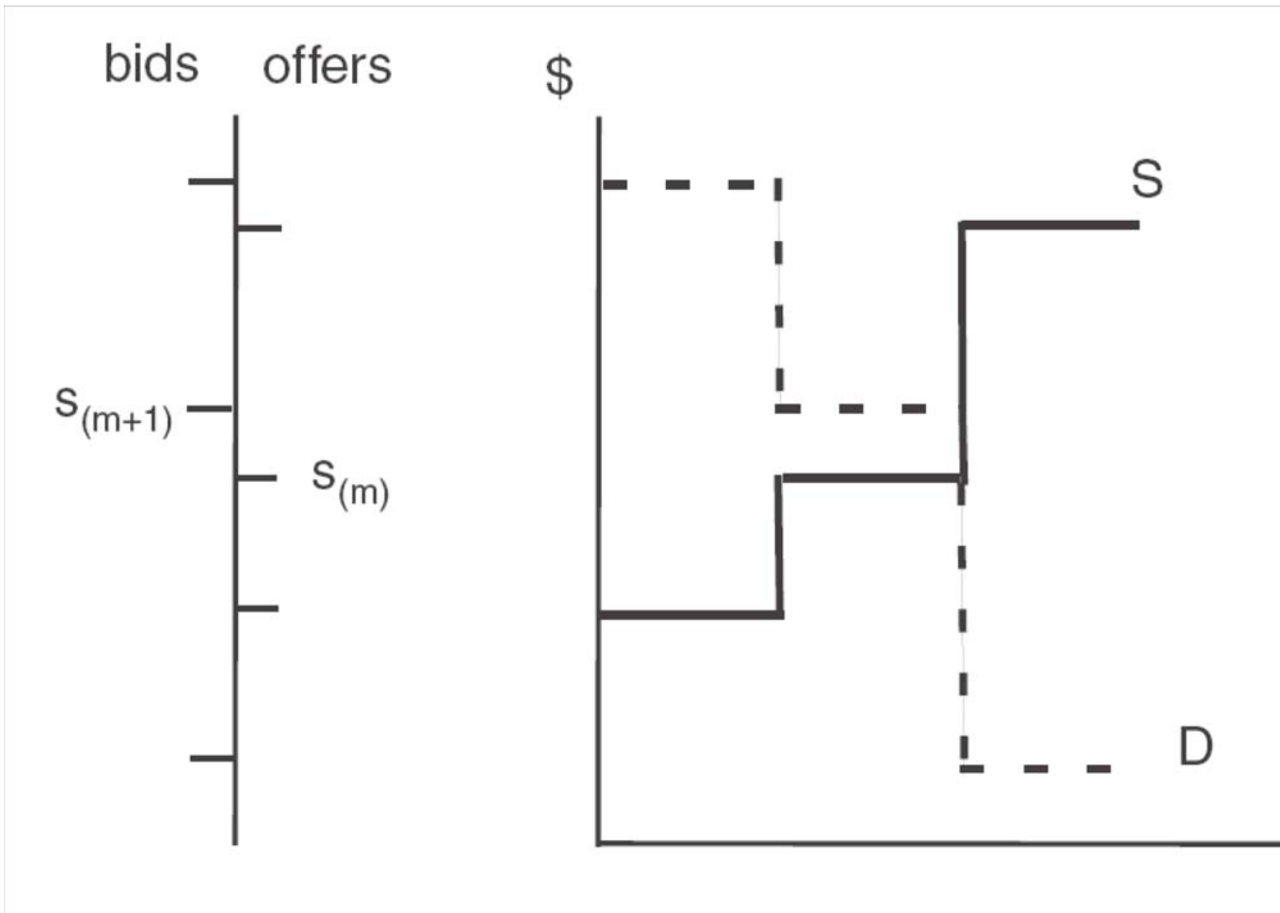


Figure 1:

Figure 2:

We have $t+(n-x-y)<n \Rightarrow t<x+y$; there are enough units for sale at $p=s_{(m)}=s_{(m+1)}$ to satisfy every buyer who is willing to pay more than this price. Similarly, $y+(m-s-t)<m \Rightarrow y<s+t$, and so there are enough buyers willing to buy at this price to satisfy every seller who is willing to accept less than this price. At issue is allocating among the $s+x$ traders who bid/ask $p=s_{(m)}=s_{(m+1)}$. If $t+s>x+y$, then there is excess demand at the price $p=s_{(m)}=s_{(m+1)}$; satisfy all of the buyers who bid more than the price and then randomly allocate the remaining supply among those buyers whose bids equaled the price. If $t+s<x+y$, then we have excess supply at the price. Allow all the sellers whose asks were below p to sell and then randomly choose among those whose asks equaled the price to determine who gets to sell.

(f) no trade equilibrium

(g) the BBDA: $p=s_{(m+1)}$ with sellers trading only if their asks are strictly less than the price

i. FOC:

$$\pi_b^B(v,b)=(v-b)\Pr(b=s_{(m)})-\Pr(s_{(m)}<b<s_{(m+1)})$$

$$\begin{aligned}\Pr(s_{(m)} &< b<s_{(m+1)})=\sum_{i=0}^{m-1}\left(\begin{array}{c}m-1\\ i\end{array}\right)\left(\begin{array}{c}n\\ m-i\end{array}\right)G\left(B^{-1}(b)\right)^i\left(1-G\left(B^{-1}(b)\right)\right)^{m-1-i}F\\ \Pr(b &= s_{(m)})=nf(b)\cdot\sum_{i=0}^{m-1}\left(\begin{array}{c}m-1\\ i\end{array}\right)\left(\begin{array}{c}n-1\\ m-1-i\end{array}\right)G\left(B^{-1}(b)\right)^i\left(1-G\left(B^{-1}(b)\right)\right)^{m-1}\\ &+(m-1)\frac{g(B^{-1}(b))}{B'(B^{-1}(b))}\cdot\sum_{i=0}^{m-1}\left(\begin{array}{c}m-2\\ i\end{array}\right)\left(\begin{array}{c}n-2\\ m-1-i\end{array}\right)G\left(B^{-1}(b)\right)^i\left(1-G\left(B^{-1}(\right.\right.\end{aligned}$$

ii. existence of equilibrium

iii. uniform on $[0,1]$: $B(v)=\frac{mv}{m+1}$

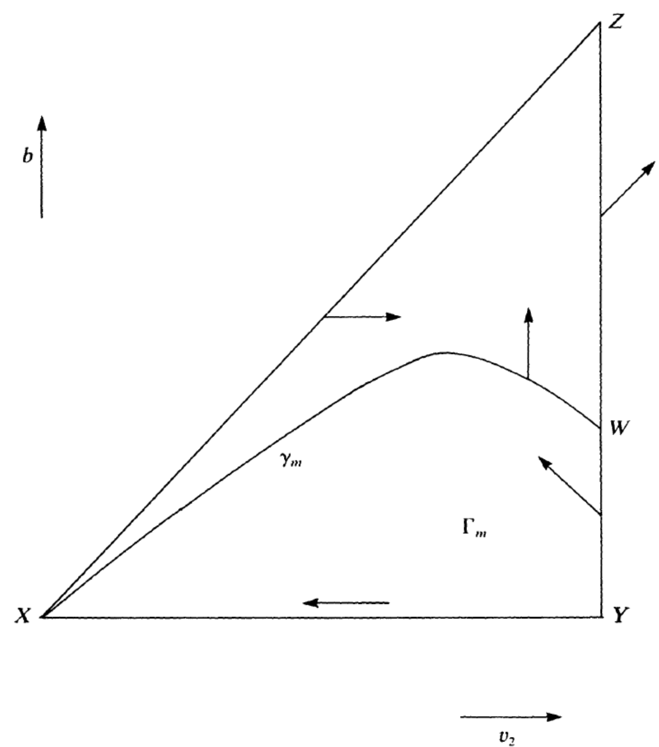


FIGURE 4.1

If $\langle \tilde{S}, B \rangle$ is an equilibrium then the graph of B lies in the triangle XYZ defined by the inequalities $0 \leqq b \leqq v_2 \leqq 1$. The arrows show the direction of the vector field $(\dot{v}_2, \dot{b})$ on the edges and at a point on γ_m

Figure 3:

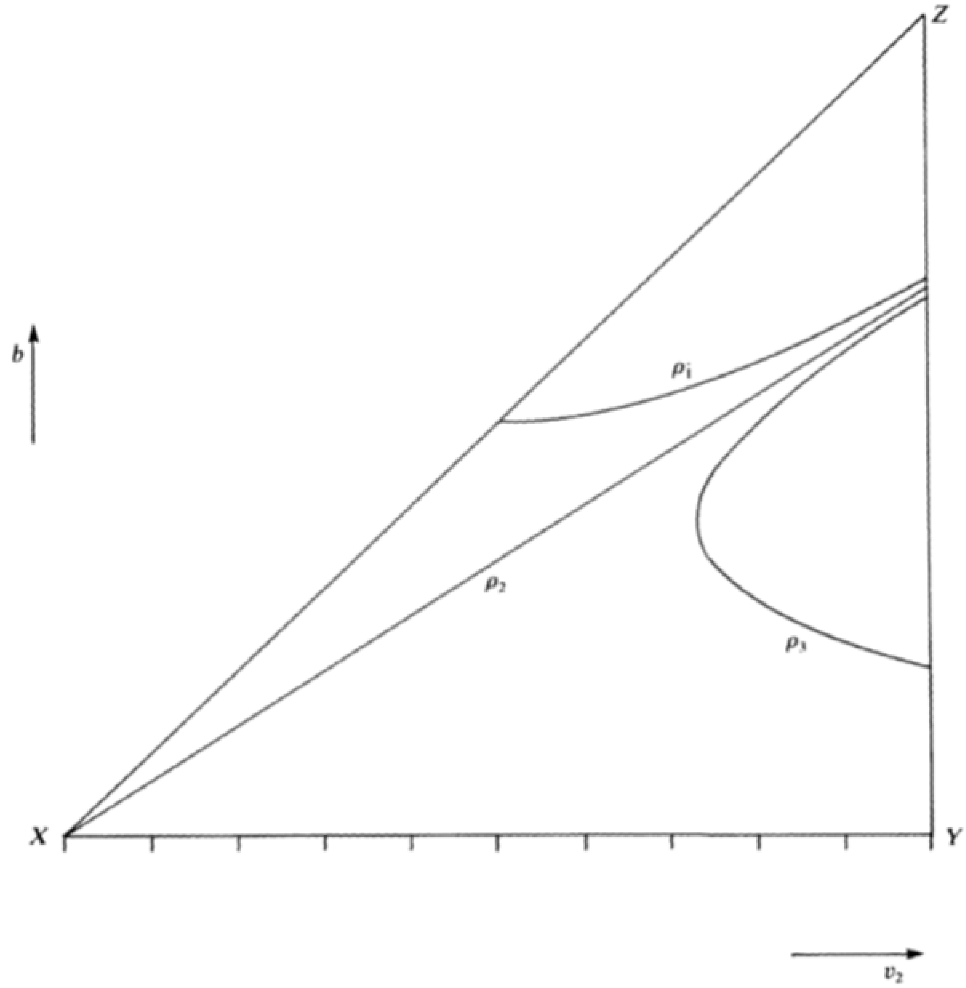


FIGURE 4.2

The curves ρ_1, ρ_2, and ρ_3 are solutions to the differential equation (3.6)–(3.7) when $m = 2$ and reservatio
are distributed uniformly. Only ρ_2 defines an equilibrium

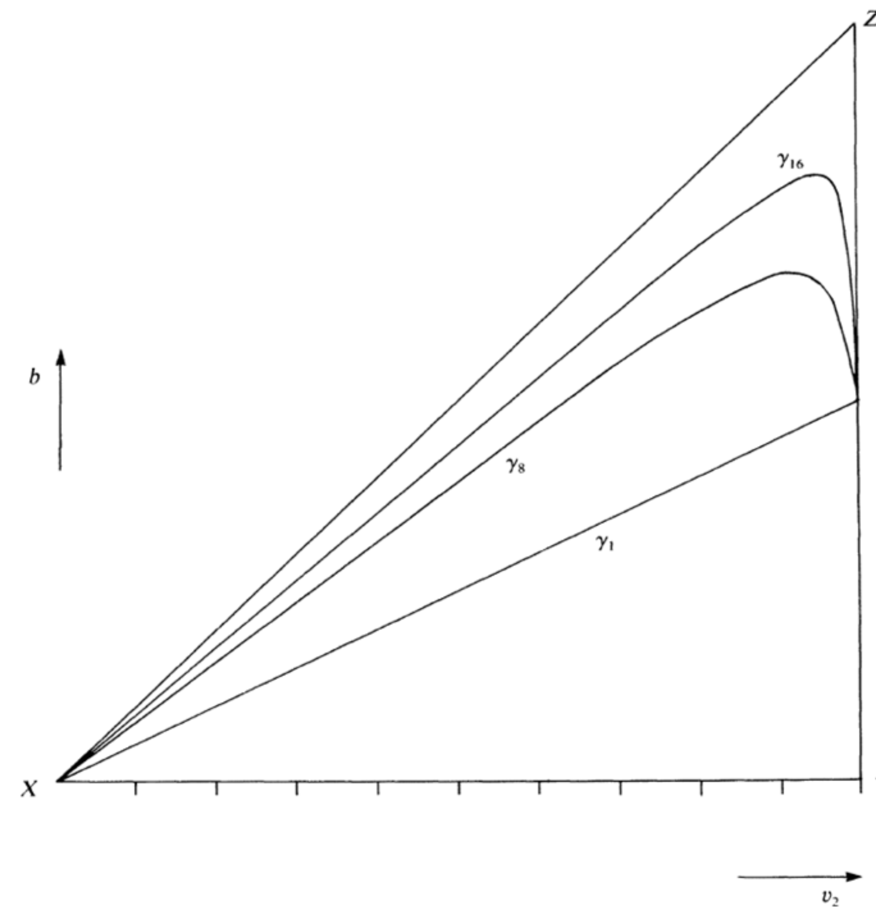


FIGURE 4.3

The boundaries γ_1, γ_8, and γ_{16} are shown for the uniform case. The graph of any equilibrium strategy B in market with $2m$ traders must lie above γ_m almost everywhere. The edge XZ corresponds to the strategy truthful revelation

iv. k-DA

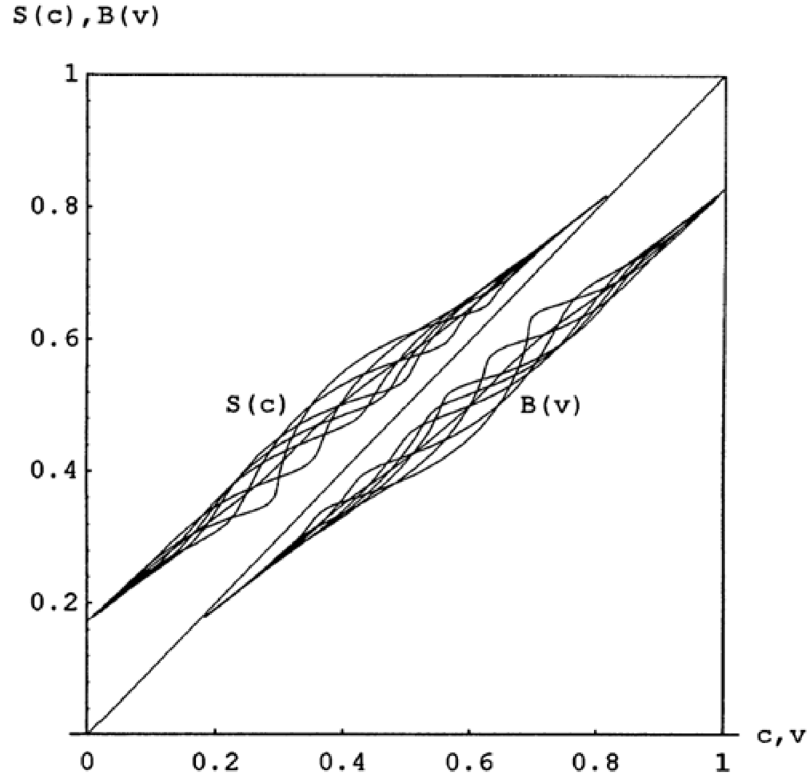


FIGURE 3.—A bundle of equilibrium strategies in the 0.5-DA for uniform F ar $m = n = 2$. Buyers' strategies lie below the diagonal, sellers' strategies lie above it, and ea strategy is paired with a particular seller's strategy to form an equilibrium.

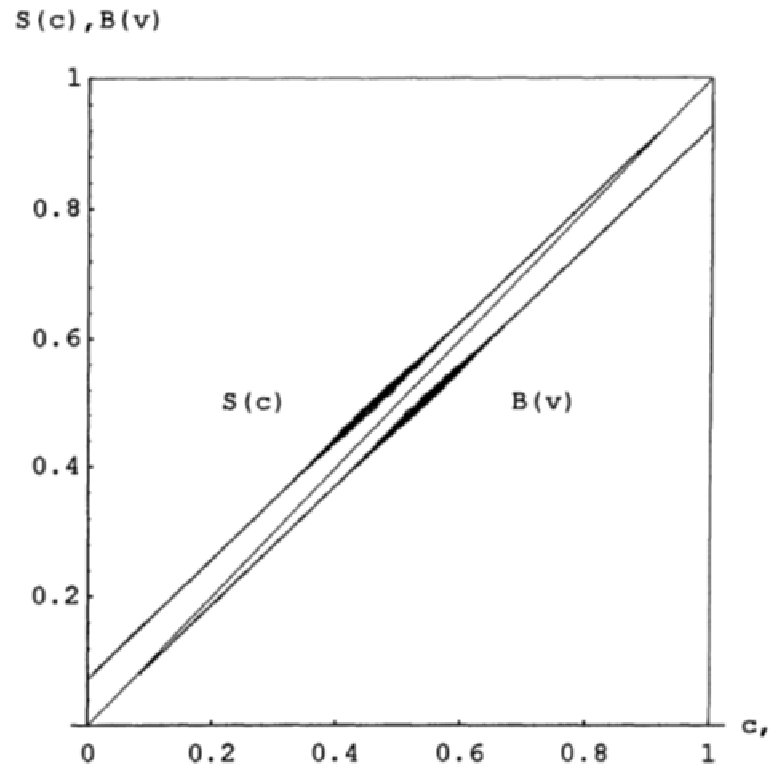


FIGURE 4.—A bundle of equilibrium strategies in the 0.5-DA for uniform F and G and $m = n = 6$.

1. (a) Convergence results: m, n satisfy the bounds

$$\frac{1}{K} \leq \frac{m}{n}, \frac{n}{m} \leq K$$

for some constant K. Let $< S, B >$ denote an equilibrium in the market with m buyers and n sellers.

i. Convergence to price-taking behavior at the rate $O(1/m)$: There exists a constant $\kappa_1 (K, F, G)$ such that

$$v - B(v), S(c) - c \leq \frac{\kappa_1 (K, F, G)}{m}$$

ii. Convergence to efficiency:

A. relative inefficiency:

$$\frac{GFT^{pt} - GFT^{e}}{GFT^{pt}}$$

B. relative inefficiency is $O(1/m^2)$, i.e., there exists a constant $\kappa_2(K, F, G)$ such that

$$\frac{GFT^{pt} - GFT^{e}}{GFT^{pt}} \leq \frac{\kappa_2(K, F, G)}{m^2}$$

iii. meaningfulness of rates of convergence: statistics

A. numerical results

EXPECTED INEFFICIENCIES OF THE OPTIMAL MECHANISM, THE LEAST AND MOST
EQUILIBRIA OF THE 0.5-DOUBLE AUCTION, THE DUAL PRICE MECHANISM, AND THE
MECHANISM FOR DIFFERENT MARKET SIZES IN THE CASE OF UNIFORM F AND G.
(For the dual price mechanism the first number listed includes in the gains from t
profits of the traders while the second number also includes the specialist's profits.)

$m = n$	Optimal Mechanism	0.5-DA Least	0.5-DA Most	Dual Price Mechanism
1	0.16	0.16	1.00	0.25 (0.25)
2	0.056	0.056	0.063	0.21 (0.18)
4	0.015	0.015	0.016	0.16 (0.075)
6	0.0069	0.0069	0.0070	0.12 (0.040)
8	0.0039	0.0039	0.0039	0.099 (0.024)

Notes: The values of the optimal and fixed price mechanisms are taken from Tables I and II respectiv
Satterthwaite (1989). We calculated the values for the dual price mechanism by numerical integration; ou
and 4 agree with the values from a simulation that McAfee (1992) reported in his Table I. Finally, the valu
were obtained by numerical integration using the equilibria that we computed employing the proced
footnote 12. Calculation of the values for the 0.5-DA posed numerical difficulties; consequently values are
two significant digits.

(b) Are these rates fast? M. A. Satterthwaite and S. R. Williams, "The Optimality of a Simple Market Mechanism". *Econometrica*, Vol. 70, No. 5 (Sep., 2002), pp. 1841-1863.

i. the k-DA is *worst case asymptotic optimal* among all Bayesian incentive compatible, interim individually rational and ex ante budget balanced mechanisms

A. asymptotic: mechanisms are compared using the rates at which relative inefficiency converges to zero

B. worst-case: each mechanism is evaluated in its least favorable environment (i.e., the choice of the distributions F and G)

ii. Result

A. *constrained efficient mechanism* in the sense of Myerson

B. relative inefficiency of the constrained efficient mechanism is at least γ/m^2 in the uniform case for some γ

C. We believe that this holds for all F, G that are reasonably well-behaved

D. relative inefficiency of any mechanism in its worst case $\geq$ relative inefficiency of constrained efficient mechanism in its worst case $\geq \gamma/m^2$

E. Applying a term from computer science, the main result of this paper is that the k-DA is worst-case asymptotic optimal among all mechanisms for organizing trade that satisfy these two constraints. "Asymptotic" refers here to the ranking of mechanisms using rates of convergence and "worst-case" refers to the evaluation of each mechanism in its least favorable environment for each value of m. Stated simply, this result means that the k-DA's worst-case error over a set of environments converges to zero at the fastest possible rate among all interim individually rational and ex ante budget balanced mechanisms.

2. Related Work

(a) T. A. Gresik and M. A. Satterthwaite, "The Rate at Which a Simple Market Converges to Efficiency as the Number of Traders Increases: An Asymptotic Result for Optimal Trading Mechanisms", *J. of Econ. Theory* 48 (1989), pp. 304-332.

i. Along with Wilson (1985), these two papers are to first to go beyond the bilateral bargaining model of Chatterjee and Samuelson (1983) to the multilateral case.

ii. markets with ηm buyers and ηn sellers for $\eta \in \mathbb{N}$

iii. relative inefficiency in the constrained efficient mechanism is $O\left(\frac{\ln \eta}{\eta^2}\right)$, a result that has been superceded by the rate established for the k-DA

iv. notable for the question that it pursues, which is the origin of all of the convergence results that I discuss above

(b) R. Wilson, "Incentive Efficiency of Double Auctions". *Econometrica*, Vol. 53, No. 5 (Sep., 1985), pp. 1101-1115.

i. If $\min\{m, n\}$ is sufficiently large, then an equilibrium of a k-double auction is interim incentive efficient in the sense of Holmström and Myerson (1983)

A. It cannot be common knowledge at the interim stage that some other equilibrium of some possibly different procedure Pareto dominates the equilibrium of the k-double auction under consideration

B. endurance of simple procedures such as the k-double auction

ii. assumes existence of increasing, differentiable equilibrium strategies with derivatives bounded uniformly for all m, n

iii. imputes welfare weights from the allocation rule in the k-DA in equilibrium

A. the issue is whether or not these imputed weights are positive as required for interim incentive efficiency

B. these imputed weights converge uniformly for all trader types to 1 as $\min\{m,n\} \to \infty$, and so for large $\min\{m,n\}$ they are positive for all trader types

C. no intuition as to why a large market is required for interim incentive efficiency; market size is a means to an end

D. no examples of small markets in which an equilibrium fails to be interim incentive efficient, except the no-trade equilibrium

iv. two aspects of the Wilson Critique:

A. procedures that are not defined in terms of the probabilistic beliefs of the agents (traders)

B. relaxing the assumption of common knowledge of beliefs

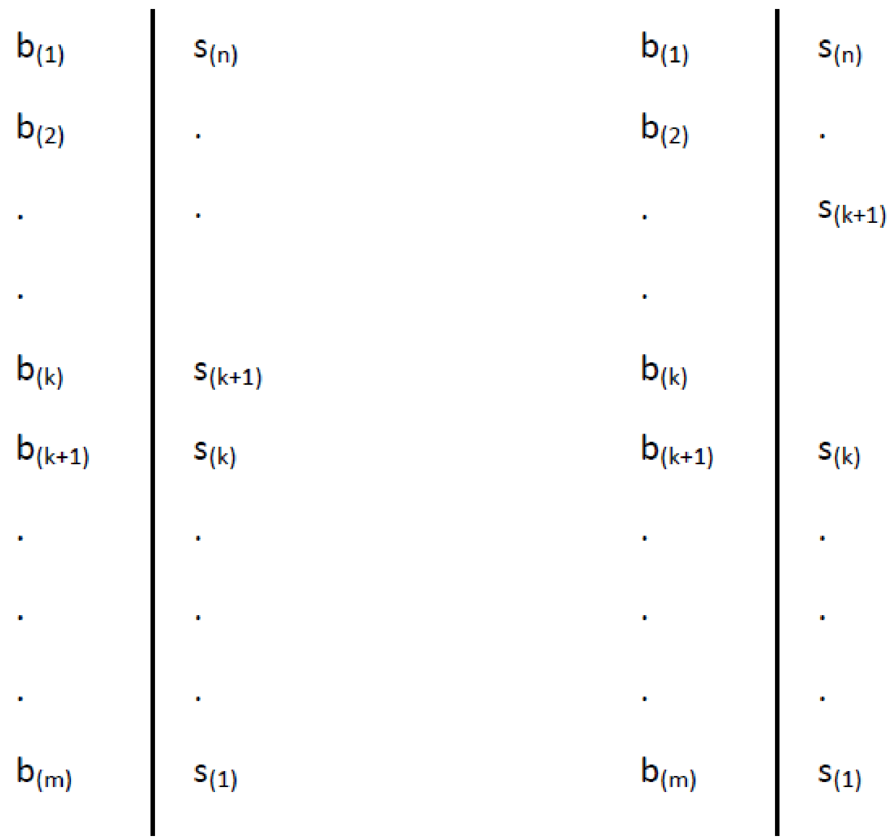
C. p. 1114: "The practical advantage of a double auction is that its rules for trades and payments do not invoke the data that are common knowledge among the agents – namely, the numbers of buyers and sellers, the joint probability distribution of their types, and the functional dependence of their reservation prices on the type parameter. Instead, the burden of coping with the complexity of the common knowledge features is assumed by the traders in the construction of their strategies."

(c) P. McAfee, "A Dominant Strategy Double Auction." *Journal of Economic Theory*, Vol. 56, No. 2 (April 1992), pp. 434-450.

i. When $(b_{(k+1)} + s_{(k+1)})/2 \in [s_{(k)}, b_{(k)}]$, $p = (b_{(k+1)} + s_{(k+1)})/2$ and k trades are made.

ii. When $(b_{(k+1)} + s_{(k+1)})/2 \notin [s_{(k)}, b_{(k)}]$, highest$k-1$ buyers pay $b_{(k)}$, lowest $k-1$ sellers receive $s_{(k)}$, $k-1$ trades made and monetary surplus of $(k-1)(b_{(k)} - s_{(k)})$

iii.



iv. virtues:

A. dominant strategies

B. if the monetary surplus of the "specialist" is counted among the gains from trade, then expected inefficiency is $O\left(1/\left(m+n\right)\right)$

v. flaw: does not converge to efficiency if the monetary surplus is treated as a cost of trading to the traders

vi. Loertscher and Marx (2015)

(d) Large Double Auctions

i. approach of my work with Satterthwaite

A. focus on the first order conditions

B. analyzed using combinatorics

ii. large double auctions: assumes a sufficiently large number of traders

A. results of probability and statistics become applicable

B. asymptotics
C. remains a model of strategic price discovery
D. rarely the production of an equilibrium or any connection to smaller markets
E. motivation typically based upon longstanding problems in microeconomic theory: strategic foundation for competitive equilibrium and for REE

iii. M. W. Cripps and J. M. Swinkels, "Efficiency of Large Double Auctions". *Econometrica*, Vol. 74, No. 1 (Jan., 2006), pp. 47-92.
   A. result: As the number fo traders grows every nontrivial equilibrium of the double auction setting converges to the Walrasian outcome. Relative inefficiency disappears at the rate $1/n^{2-\alpha}$ for any $\alpha > 0$
   B. two goods, initially unitary supply/demand, later multiple units are allowed
   C. considers sequences of markets with possibly correlated, private values in $[0, 1]$
   D. asymmetry of the distribution and across the strategies used by each side of the market is allowed
   E. Cripps and Swinkels: symmetry of strategies assumes away the problem of making sure that the units are allocated properly to each side of the market
   F. restriction on distribution: *no asymptotic gaps*, *no asymptotic atoms*
   G. for $z \in (0, 1]$, *z-independence*: bounds the amount that the distribution of any trader's value changes conditional on the values of the remaining traders
   H. asymmetry and "purification" of equilibrium strategies as the number of traders grows
   I. to establish the rate of convergence, n "quite large" is necessary. No indication of when the rate begins to be observed.

iv. P. J. Reny and M. Perry, "Toward a Strategic Foundation for Rational Expectations Equilibrium", *Econometrica*, Vol. 74, No. 5 (Sep., 2006), pp. 1231-1269.
   A. provide a strategic foundation for rational expectations equilibrium
   B. affiliated, interdependent values/costs
   C. limit market, with a continuum of traders and real-valued bids/asks: Bayes-Nash equilibrium in increasing strategies that implements a fully-revealing REE price
   D. main result is continuity as the number of traders and the number of possible bids/asks goes to infinity: for a sufficiently large number of traders, and for a discrete grid of

possible bids/asks that is sufficiently fine, there exists a BNE in increasing strategies that approximates the equilibrium of the limit market

E. all traders are fully rational and strategic: no noise traders and sellers are active (unlike auction models)

F. No indication of how large a market is required, no examples in finite markets

G. "Our main insight is that establishing the existence of a monotone equilibrium poses serious difficulties only when individual agents can have a significant impact on the price." Is this a problem of "proof", or truly of existence? They suggest the later.

(e) R. C. Shafer, "Convergence to Price-Taking by Regret-Minimizers in k-Double Auctions."

i. P. B. Linhart and R. Radner, "Minimax-Regret Strategies for Bargaining over Several Variables". *J. of Econ. Theory* 48, 152-178 (1989).

ii. Shafer: Alternatives to Bayesian decision-making in modeling how traders select their bids and asks in a k-DA

iii. Does the emergence of price-taking behavior as the market increases in size fundamentally require that one be a Bayesian?

iv. minimax regret and maxmin: behavior invariant to the size of the market

v. culprit: this is true of any decision rule that satisfies the axiom of symmetry

vi. Γ-minimax regret, and Γ-maxmin; minimizing maximum expected regret

(f) J. H. Kagel and W. Vogt, "Buyer's Bid Double Auctions: Preliminary Experimental Results". Chapter 10 in *The Double Auction Market: Institutions, Theories and Evidence*, D. Friedman and J. Rust, eds. Addison Wesley (1993): Reading.

i. experimental design

A. $m = 2$ and $m = 8$ traders on each side

ii. few sellers played their dominant strategies, causing inefficiency

iii. buyers underbid by less than the equilibrium prediction

iv. change from $m = 2$ to $m = 8$ notable but not as much as predicted by theory

A. this is largely attributable to the fact that the underbidding by buyers in the case of $m = 2$ is not as extreme as theory predicts, leaving little room for improvement

v. opportunities for learning in BBDA

(g) Continuous Bid/Ask Market

i. R. Wilson, "Equilibria of Bid-Ask Markets," in: *Arrow and the Ascent of Economic Theory: Essays in Honor of Kenneth J. Arrow*, G. Feiwel (ed.); Chapter 11, pp. 375-414. London and New York: Macmillan Press and New York University Press, 1987.

ii. D. Easley and J. Ledyard, "Theories of Price Formation and Exchange in Double Oral Auctions," in: *The Double Auction Market: Institutions, Theories and Evidence*, D. Friedman and J. Rust, eds. Addison Wesley (1993): Reading.

iii. D. Friedman, "A Simple Testable Model of Double Auction Markets". *J. Econ. Behav. & Org.* (1991), 47-70.

iv. D. Friedman, "The Double Auction Market Institution: A Survey", in: *The Double Auction Market: Institutions, Theories and Evidence*, D. Friedman and J. Rust, eds. Addison Wesley (1993): Reading.

3. Asymptotics

(a) goals

i. identify the asymptotic distribution of the BBDA's price

ii. identify the asymptotic limits of the probabilities in a trader's FOC

iii. formulate the asymptotic FOCs (AFOCs) and solve

iv. compare the solutions to the AFOCs to computed equilibrium

v. AFOCs identify what is "first order" in a trader's decision problem

A. comparative statics in the distributions and the numbers of traders

(b) review

i. CPV environment

ii. informational model

iii. convergence results

iv. limit market: Let $q \equiv m/(m+n)$ and (ksi) $\xi_q \equiv G_\varepsilon^{-1}(q)$, the q^{th} quantile of distribution G_ε. The *limit market* in state μ consists of probability masses of measure q of buyers and measure $1-q$ of sellers with values/costs z, which conditional on μ are i.i.d. according to $G_\varepsilon(z-\mu)$.

v. REE: The unique REE price in the limit market in state μ is $p^{\text{REE}} \equiv \mu + \xi_q$.

(c) fix n, m; markets with ηm buyers and ηn sellers

i. $x(\eta) = s_{\eta m:\eta(m+n)-1}, y(\eta) = s_{\eta m+1:\eta(m+n)-1}, p^e\,(\eta) = s_{\eta m+1:\eta(m+n)}$

ii. $x(\eta)$, $y(\eta)$ and $p^{\mathrm{e}}(\eta)$ are asymptotically consistent, unbiased and normal estimators of the REE price in state μ:

$$x(\eta), y(\eta) \sim \mathcal{AN}\left(p^{\mathrm{REE}}, \frac{m\,n/(m+n)^2}{[\eta(m+n)-1]\, g_\varepsilon^2(\xi_q)}\right),$$
$$p^{\mathrm{e}}(\eta) \sim \mathcal{AN}\left(p^{\mathrm{REE}}, \frac{m\,n/(m+n)^2}{\eta(m+n) g_\varepsilon^2(\xi_q)}\right)$$

iii. figures that depict the distribution of $p^{\mathrm{e}}(\eta)$

(d) FOC:
$$(v-b) f_{x(\eta)|v}(b|v) - \Pr[x(\eta) \le b \le y(\eta)|v] = 0$$

i. asymptotic offset:

$$\lambda_{\text{approx}}(\eta) = \frac{1}{(m+n)\eta - 1} \frac{1}{g_\varepsilon(\xi_q)}$$

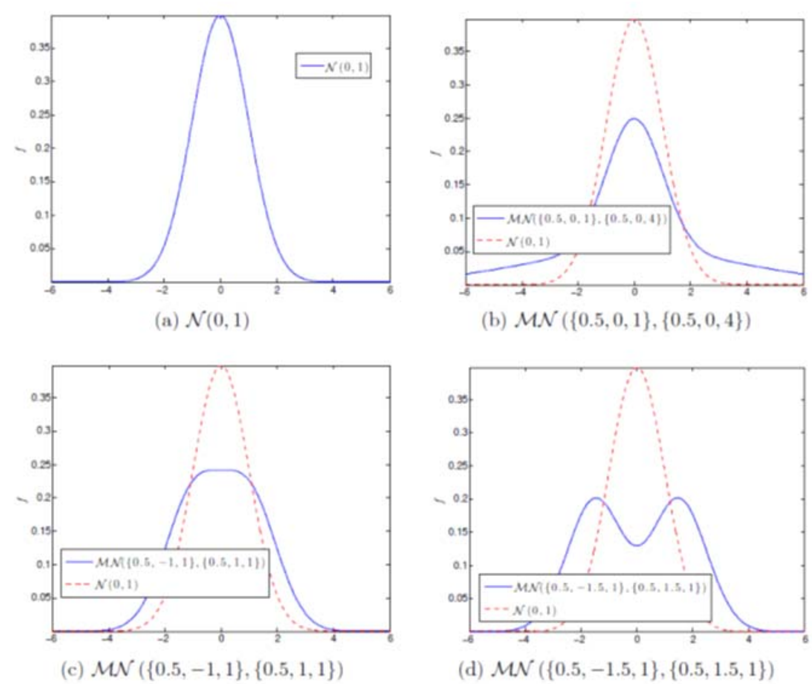
A. prices centered on $p^{\mathrm{REE}} \equiv \mu + \xi_q$

B. no distinction between m and n except in determining q

C. dependence on $g_\varepsilon(\xi_q)$

(e) Numerical Example

i.



Panel A: $\mathcal{N}(0,1)$, $\xi_q = 0$, $f(\xi_q) = 0.3989$.

η	λ	λ_{approx}	$\lvert\lambda_{\text{approx}} - \lambda\rvert$	$\frac{\lvert\lambda_{\text{approx}} - \lambda\rvert}{\lambda}$
2	0.6896	0.8355	0.1459	0.2116
4	0.3398	0.3581	0.0183	0.0539
8	0.1639	0.1671	0.0031	0.0195
16	0.0805	0.0809	0.0004	0.0050

Panel B: $\mathcal{MN}(\{0.5, 0, 1\}, \{0.5, 0, 4\})$, $\xi_q = 0$, $f(\xi_q) = 0.2992$.

η	λ	λ_{approx}	$\lvert\lambda_{\text{approx}} - \lambda\rvert$	$\frac{\lvert\lambda_{\text{approx}} - \lambda\rvert}{\lambda}$
2	0.9304	1.1141	0.1837	0.1974
4	0.4617	0.4775	0.0158	0.0342
8	0.2215	0.2228	0.0065	0.0293
16	0.1077	0.1078	0.0001	0.0009

Panel C: $\mathcal{MN}(\{0.5, -1, 1\}, \{0.5, 1, 1\})$, $\xi_q = 0$, $f(\xi_q) = 0.2420$.

η	λ	λ_{approx}	$\lvert\lambda_{\text{approx}} - \lambda\rvert$	$\frac{\lvert\lambda_{\text{approx}} - \lambda\rvert}{\lambda}$
2	1.0468	1.3776	0.3308	0.3160
4	0.5305	0.5904	0.0599	0.1129
8	0.2610	0.2755	0.0145	0.0556
16	0.1296	0.1333	0.0037	0.0285

Panel D: $\mathcal{MN}(\{0.5, -1.5, 1\}, \{0.5, 1.5, 1\})$, $\xi_q = 0$, $f(\xi_q) = 0.1295$.

η	λ	λ_{approx}	$\lvert\lambda_{\text{approx}} - \lambda\rvert$	$\frac{\lvert\lambda_{\text{approx}} - \lambda\rvert}{\lambda}$
2	1.4650	2.5737	1.1087	0.7568
4	0.7626	1.1030	0.3404	0.4464
8	0.3948	0.5147	0.1199	0.3037
16	0.2084	0.2491	0.0407	0.1953

A. note: comparison between table
B. accuracy of approximation

ii.

η	$\lambda_{1,2}$	$\lambda_{2,1}$	$\lvert\lambda_{1,2} - \lambda_{2,1}\rvert$	$\frac{\lvert\lambda_{1,2} - \lambda_{2,1}\rvert}{\lambda_{1,2}}$
2	0.5027	0.5085	0.0058	0.0115
4	0.2433	0.2441	0.0008	0.0033
8	0.1184	0.1185	0.0001	0.0008
16	0.0583	0.0583	0	0

Table 2: For different market sizes η and F standard normal, the equilibrium offset $\lambda_{1,2}$ fo case of $m = 1$ buyer, $n = 2$ sellers is compared to the equilibrium offset $\lambda_{2,1}$ for the case of n buyers, $n = 1$ seller.