August 27

Example 1 *From the text book: the principal arrives in a city that is new to him and hires a taxi to drive him to his hotel. The taxi driver is the agent. The best outcome for the principal is the shortest trip to the hotel. The best outcome for the taxi driver is a large wage at the end of his day. Does the system of fares provide the driver with the incentives to use his expertise to find the quickest route to the hotel? Or does he instead have the incentive to take a circuitous or indirect route?*

1. a. i. components of the problem: a linear fare schedule

$$fare = f + rx,$$

where f is a fixed fee upon setting the taxi meter, r is a rate, and x is either distance or time. It is also presumed that the driver bears a fixed cost of $c > 0$ between passengers. (I'm using different notation here than the text.)

ii. two extremes: $f = 0$ vs. $r = 0$.

(a). $f = 0$: The driver has the incentive to make x large in order to decrease the number of fixed costs c that he bears during his workday. This payment scheme can also induce drivers to refuse rides to people going to particular destinations.

(b). $r = 0$: The driver receives a fixed fare for picking up a passenger. His goal is therefore to minimize x so that he can pick up another passenger. This is optimal for the passenger, and it is often used in cities (i.e., a "fixed fare" between the airport and a common destination). It can be hard on the driver, however, if driving times may vary considerably.

(c). What about $f < c$, $f = c$, and $f > c$?

$f = c$: In this case, the fixed fare f exactly compensates for the expected cost of finding a new passenger, and so the driver has no reason to make x either large or small.

$f > c$: The driver in this case makes a profit of $f - c$ by dropping off one passenger and picking up another. He therefore benefits from turnover in passengers; lots of small trips are to his advantage.

$f < c$: The driver in this case bears a loss of $f - c$ by dropping off one passenger and picking up another. He is therefore better off by keeping a passenger in his taxi for a long time.

iii. the possibility of c large on a slow day, or other reasons for a long wait between passengers.

iv. Should the passenger's utility be a function of both the fare and x? I.e., both the travel time and the cost?

$$u(\textit{fare}, x) = u(f, r, x)$$

Example 2 *Acid Rain (Ch. 1, sec. 3 of text)*

A government wishes to decrease the amount of sulfur dioxide that is emitted in the country by its power plants. There are 10 privately-owned power plants of different age and technology. The cost per unit of decreasing emissions by one unit for each of the 10 firms are as follows:

firm	1	2	3	4	5	6	7	8	9	10
cost	300	120	200	100	150	180	130	300	160	140

The costs can vary because the firms may use different types of coal, they may differ in their proximities to supplies of coal, there are different methods of reduction (conversion to natural gas, scrubbers, different types of coal, etc.) that may be suitable at one plant but not another. There is an assumption here that the total cost of reduction in linear in the total amount reduced (i.e., constant marginal cost).

Each firm privately knows its own cost of reducing its sulfur dioxide emissions. The government could simply order each firm to reduce its emissions by x%, where x is chosen to reach the government's targeted reduction. This is simple and straightforward procedure that is in some sense "fair". It is, however, inefficient in the sense that it doesn't achieve the desired reduction at the lowest possible cost. The government would like to identify the low cost firm (which turns out to be firm 4) and have it provide the desired reduction, with perhaps some form of compensation (either a monetary subsidy or a tax break).

We can anticipate a problem for the government, namely, a firm may choose to misrepresent or exaggerate its costs to avoid the burden of making the reduction. Of course, if the compensation is sufficiently large, then a firm might underreport their costs in an effort to get the assignment of reducing emissions. If either form of lying occurs, the government may fail to accomplish its goal of identifying the low cost firm.

Consider, however, the following procedure. The government asks each firm to report its costs. The firm that reports the lowest cost per unit will be ordered to reduce its emissions. It will be paid, however, at a rate of the second highest reported cost for each unit of reduction. Firms that do not report the lowest cost are not required to reduce emissions and receive no compensation from the government. In the case of ties in the lowest report, a fair lottery determines which firm makes the reductions and receives the compensation. This is an example of the Vickrey-Clark-Groves (or VCG) mechanism.

Claim: Regardless of whether or not the other firms report honestly, and regardless of the true costs of the other firms, each individual firm's best interest is to honestly report his true cost.

If each firm follows its self-interest, the government can then determine that firm 4 is in fact the low-cost firm.

Consider firm i and let c_i denote its cost. It considers reporting c_i^*. Let c_{-i}^* denote the smallest cost reported by the other firms. Notice that c_{-i}^* is not assumed to necessarily the lowest cost of the other firms or even a true cost of any of them. The consequences of the VCG mechanism for firm i depend upon the relative sizes of c_i^* (its report) and c_{-i}^*:

1. *if $c_i^* < c_{-i}^*$, then firm i's reported cost is the lowest reported cost and c_{-i}^* is the second lowest reported cost. Firm i is therefore selected to reduce its emissions and earns $c_{-i}^* - c_i$ for each unit of reduced emissions. Here, c_i denotes the true cost of its per unit reduction in emissions and c_{-i}^* denotes the per unit compensation it receives from*

the government according to the VCG mechanism.

2. *if $c_i^* > c_{-i}^*$, then some other firm has reported the lowest cost. Under the rules of the VCG mechanism, the government selects some other firm to make the reductions. Firm i pays no costs of reduction and receives no subsidy, and so it earns 0.*

3. *if $c_i^* = c_{-i}^*$, then let k denote the number of the other firms that reported c_{-i}^*. Firm i is selected to make the reduction with probability*
$$\frac{1}{k+1},$$
in which case it earns
$$c_{-i}^* - c_i.$$
Again, c_{-i}^* is in this case the second smallest reported cost among all the reported costs (including firm i), and c_i is the true cost per unit that firm i pays when it reduces its emissions. Its expected payoff or earnings in this case is therefore
$$\frac{1}{k+1} \cdot \left(c_{-i}^* - c_i\right).$$

Not knowing what the other firms are going to report, what is the best possible report for firm i? My claim is that $c_i^* = c_i$ (i.e., honest reporting) always maximizes firm i's profit for whatever value of c_{-i}^* is determined by the reports of the other firms. We can see this from the following table:

row	*report*	*earnings*
1	$c_i^* < c_{-i}^*$	$c_{-i}^* - c_i$
2	$c_i^* > c_{-i}^*$	0
3	$c_i^* = c_{-i}^*$	$\frac{1}{k+1} \cdot \left(c_{-i}^* - c_i\right)$

Which of the the three numbers in the right-hand column is largest? Firm i would like to choose c_i to insure that it gets the largest of these 3 values.

1. *If $c_i < c_{-i}^*$, then row 1 is the best possible outcome and honest reporting $c_i^* = c_i$ insures that firm i gets this outcome.*

2. *If $c_i > c_{-i}^*$, then row 2 is the best possible outcome and honest reporting $c_i^* = c_i$ insures that firm i gets this outcome.*

3. *If $c_i = c_{-i}^*$, then all 3 outcomes provide earnings of 0. Honest reporting $c_i^* = c_i$ insures that firm i gets this outcome.*

Regardless of the value of the lowest cost $\mathbf{c}_{-i}^*$ reported by the other firms, honest reporting by firm i insures that it receives the highest possible earnings among the options available to it given the value of c_{-i}^*.

This is a very strong sense it which each firm i "has an incentive" to report honestly: it is best for firm i to report honestly regardless of whether or not the other firms report honestly, regardless of what their true costs actually are, and regardless of whether the other firms are smart or stupid! We made no assumptions about the relationship between c_{-i}^* and the true costs of the other firms. In the terminology of game theory, it is a dominant strategy for each firm i to report honestly. This means that it is optimal for firm i regardless of the strategies used by the other firms to select their reports based upon their *costs (whatever those costs may be). Firm i may not be happy with the options presented by the other firms in their determination of c^*_{-i}, but still, reporting honestly remains firm i's best option.*

Notice that we haven't used the assumption that there are 10 firms (the number of firms doesn't matter).

Campbell refers to this as the Vickrey mechanism, after William Vickrey, a Columbia University economist who received the Nobel Prize. It also originates in the work of Ted Groves and Edward Clark, and so it is now commonly referred to as the Vickrey-Clark-Groves or VCG mechanism. ***It can be used to solve a lot of different problems and we'll discuss it on several occasions and in much greater generality as we proceed through the course.***

Example 3 *A second example of the VCG mechanism is the second-price auction. This was invented by Vickrey. Suppose a seller has an item that he wants to sell. One way to sell it is to collect bids from interested parties, find the highest bid, then sell the item to the bidder who submitted that bid and charge him the high bid as his price. This is the first price auction, and it is a common procedure for auctioning an item.*

If you are one of the potential bidders for the item, think about how you should choose your bid. Suppose the item is worth v to you (i.e., you're willing to pay up to v dollars for the item). Your profit when you get the item is $v - b$, where b is your bid. You know that you must submit a bid b that is strictly less than v, for otherwise, winning the item is not profitable for you. You may be wary of submitting too low of a bid, however, for that lowering your bid may decrease the likelihood that you win. The "likelihood that you win" depends upon how other people bid; you must think about what other potential bidders may do in choosing your own bid. We'll model this later in the course using some elementary probability theory.

Vickrey proposed the following alternative procedure. The seller receives bids submitted by interested bidders, he orders them in a list, and the highest bidder receives the item and pays the second highest bid as his price (rather than his own highest bid). This is the second price auction.

Let's show that submitting your true value v as your bid is always a best bid for you regardless of how the other bidders choose their bids and regardless of how much each of them values the item. Let $\overline{b}$ denote the largest bid submitted by the other bidders and keep b as your bid and v as your value. Your profit in the auction depends on how b relates to $\overline{b}$:

case		*your profit*
I	$b > \overline{b}$	$v - \overline{b}$
II	$b < \overline{b}$	0
III	$b = \overline{b}$	$\frac{1}{k+1}\left(v - \overline{b}\right)$

Here, k denotes the number of other bidders who submitted the bid $\overline{b}$. You choose b, and ideally this choice would get you the biggest of the three profit numbers. We consider three possibilities:

- $v - \overline{b} > \frac{1}{k+1}\left(v - \overline{b}\right) > 0$. *Here,* $v > \overline{b}$ *and the bid* $b = v > \overline{b}$ *insures that case I applies and you end up with* $v - \overline{b}$*, the largest of the three possible profits.*
- $0 > \frac{1}{k+1}\left(v - \overline{b}\right) > v - \overline{b}$. *Here,* $v < \overline{b}$ *and the bid* $b = v < \overline{b}$ *insures that case II applies and you end up with* 0*, the largest of the three possible profits.*
- $0 = \frac{1}{k+1}\left(v - \overline{b}\right) = v - \overline{b}$. *Here,* $v = \overline{b}$ *and the bid* $b = v = \overline{b}$ *insures that case III applies and you end up with* 0*, the only possible profit.*

The noteworthy virtue of the second price auction is that it is in your best interest to bid exactly what the item is worth to you, regardless of how the other bidders may the item or choose to select their bids. You don't even need to think about what they may choose to do. As in the Acid Rain example, the incentive to bid strategically is eliminated by making sure that each bidder can not influence the price that pays when he wins. The point of underbidding in some kinds of auctions is to lower the price that you would pay if you win; in the second price auction, the price that you pay when you win is determined by the bids of others, and you can not influence it.

Exercises: Problem 2 (p. 21)
Efficiency: p. 28-29: 1, 4, 5, 6

0.0.2 Pareto Efficiency (Sec. 4, Ch. 1 of text)

We discuss here a notion of efficiency that is rooted in the individual preferences of group members over choices that may be made for the group. The word "efficiency" has other meanings in economics, such as "cost minimization" or "optimization." In the above example, for instance, it is efficient for the government to assign the task of reducing emissions to the firm that can do so at the lowest cost.

Example 4 *Suppose there are 3 people (1,2,3) and 4 possible choices of a restaurant as a choice for dinner for the group of three people: I (Italian), C (Chinese), S (seafood), and T (Thai). The three people rank the 4 restaurants as follows:*

person	*ranking*
1	*I>C>S>T*
2	*I>S=T>C*
3	*C>S=I>T*

Here, ">" indicates strictly prefers while "=" indicates that the individual is indifferent between the two alternatives.

Q: What are the efficient choices of restaurants?

The point of answering this question is to illustrate what economists typically mean by "efficient."

Definition. *Suppose a group of people can choose from a set* A *of alternatives. Alternative* $a \in A$ *is efficient if it is not possible to switch to some other alternative* $b \in A$ *and in the process make some people in the group strictly better off and without making anyone strictly worse off.*

An alternative a with this property is often referred to as Pareto efficient or Pareto optimal, after the 19th century Italian economist Vilfredo Pareto, who came up with the idea. I personally prefer this terminology, which is more precise than simply "efficient". As noted above, "efficiency" has other meanings in economics. If an alternative $b \in A$ exists such that b is ranked at least as good as a by every person, and at least one person strictly prefers b to a, then b is said to Pareto dominate a.

1. *Is I efficient? Yes, because switching to either of C, S, or T would make person 1 (and person 2) strictly worse off.*
2. *Is C efficient? Looking at person 1's preferences, we can see that the only possible improvement from his perspective is switching from C to I. Switching to I from C, however, makes person 3 strictly worse off. It is therefore not possible to switch from C to some other alternative and make some people strictly better off without hurting some other person. Yes, C is efficient.*
3. *Is S efficient? Looking at person 1's preferences, we can see that the only possible moves that would not hurt him are I and C. Moving from S to I would make persons 1 and 2 strictly better off and it would not hurt person 3. No, S is not efficient.*
4. *Is T efficient? No, because every person is made strictly better off by switching from T to I. Incidentally, it is also also true that switching from T to S shows that T is not efficient.*

We conclude that I and C are efficient but that S and T are not. What does this mean?

First, it would be really dumb for the group to go out together to either the seafood (S) or the Thai (T) restaurants.

Second, efficiency does not help the group to choose between I and C. You might be tempted to say, "But 1 and 2 rate I as their best choice, and 3 rates I as second best; 2 in fact ranks C as his worst choice!" But this argument presumes either (i) a method by which the choice is made such as majority rule, or (ii) the assumption that each individual's well-being counts equally.

As to (ii), it bears emphasizing that this notion of efficiency respects the preferences of each individual. There is no weighing of one person's interests against anothers. Efficiency rules out the stupid options but leaves the problem of weighing one person's interests against another's in selecting among the efficient choices.