Answers – Second Exam
Information Economics (490), Fall 2013

1. (12 points) Consider in this problem a marriage market with 4 men and 4 women. The strict preferences of the men and women are as follows:

$$P(m_1): w_1, w_2, w_3, w_4 \quad P(w_1): m_4, m_3, m_2, m_1$$
$$P(m_2): w_4, w_2, w_3, w_1 \quad P(w_2): m_2, m_3, m_4, m_1$$
$$P(m_3): w_1, w_2, w_4, w_3 \quad P(w_3): m_2, m_3, m_4, m_1$$
$$P(m_4): w_2, w_4, w_3 \quad P(w_4): m_1, m_4, m_3, m_2$$

   (a) (4 points) Determine the matching that is obtained through the deferred acceptance algorithm in which men make the proposals.

women	w_1	w_2	w_3	w_4
stage 1	m_1, m_3	m_4		m_2
stage 2	m_3	m_4, m_1		m_2
stage 3	m_3	m_4	m_1	m_2

$\Rightarrow$

women	w_1	w_2	w_3	w_4
stage 1	m_3	m_4		m_2
stage 2	m_3	m_4		m_2
stage 3	m_3	m_4	m_1	m_2

   (b) (4 points) Show in this example that the algorithm used in a. is not strategy-proof.

   w_4 ends up with her least favorite man m_2. Let's see what happens if she rejects him in stage 1:

women	w_1	w_2	w_3	w_4
stage 1	m_1, m_3	m_4		m_2
stage 2	m_3	m_4, m_2, m_1		
stage 3	m_3	m_2	m_1	m_4

$\Rightarrow$

women	w_1	w_2	w_3	w_4
stage 1	m_3	m_4		
stage 2	m_3	m_2		
stage 3	m_3	m_2	m_1	m_4

   We see that w_4 can switch from her least favorite man m_2 to her second favorite man m_4 by turning down an acceptable man in stage 1. The algorithm is not strategy proof.

   (c) (4 points) Determine the stable matching that is optimal for women.

   This stable matching is obtained through the deferred acceptance algorithm in which women make the proposals.

$$P(m_1): w_1, w_2, w_3, w_4 \quad P(w_1): m_4, m_3, m_2, m_1$$
$$P(m_2): w_4, w_2, w_3, w_1 \quad P(w_2): m_2, m_3, m_4, m_1$$
$$P(m_3): w_1, w_2, w_4, w_3 \quad P(w_3): m_2, m_3, m_4, m_1$$
$$P(m_4): w_2, w_4, w_3 \quad P(w_4): m_1, m_4, m_3, m_2$$

men	m_1	m_2	m_3	m_4
stage 1	w_4	w_2, w_3		w_1
stage 2	w_4	w_2	w_1, w_3	
stage 3	w_4	w_2	w_1	w_3

$\Rightarrow$

women	m_1	m_2	m_3	m_4
stage 1	w_4	w_2		
stage 2	w_4	w_2	w_1	
stage 3	w_4	w_2	w_1	w_3

2. (16 points) Consider in this problem a seller who sells an item through the following auction. There are n bidders. Each bidder submits a bid.

The bidder who submits the highest bid wins the item and pays the seller the average of his bid and the second highest bid. You can ignore the possibility of ties in your answer to this question.

The reservation values of the n bidders are identically and independently distributed according to the uniform distribution on $[0, 1]$. The purpose of this example is to work through the first few steps in deriving an increasing function $b : [0, 1] \to [0, 1]$ that defines an equilibrium in this auction: if every other bidder besides bidder i uses the function b to select his bid, then $b(v_i)$ maximizes bidder i's expected profit in the auction for each of his possible reservation values $v_i \in [0, 1]$. Select a bidder i, let v_i denote his reservation value, x his bid, and assume all other bidders use the function b to select their bids.

(a) (4 points) Let y denote the highest reservation value in the sample of the reservation values of the other $n - 1$ traders. What is the cumulative distribution function for y and what is its density?

$$\begin{aligned} F_{(n-1)}(y) &= y^{n-1} \\ f_{(n-1)}(y) &= (n-1)\, y^{n-2} \end{aligned}$$

(b) (4 points) What is bidder i's profit $u(v_i, x, y)$ as a function of his bid x, his reservation value v_i and the value of y?

$$u(v_i, x, y) = \begin{cases} v_i - \frac{x+b(y)}{2} \text{ if } x \geq b(y) \\ 0 \text{ if } x < b(y) \end{cases}$$

(c) (4 points) Using your answers to a. and b., what is bidder i's expected profit $U(v_i, x)$ as a function of his bid x and his reservation value v_i?

$$U(v_i, x) = \int_0^{b^{-1}(x)} \left(v_i - \frac{x + b(y)}{2}\right)(n-1)\, y^{n-1} dy$$

Hint: Your answer should be an intergral with respect to y.

(d) (4 points) Let $U(v_i) = U(v_i, b(v_i))$, where $b(v_i)$ is the strategy that we are hoping to derive. The function $U(v_i)$ specifies the expected utility of buyer i in equilibrium as a function of his reservation value v_i. Calculate $U'(v_i)$ using the Envelope Theorem.

$$U^{'}(v_i) = \frac{\partial}{\partial v_i} U(v_i, x)\bigg|_{x=b(v_i)} = \int_0^{b^{-1}(x)} (n-1)\, y^{n-2} dy = \left. y^{n-1}\Big|_0^{b^{-1}(x)} \right|_{x=b(v_i)} = v_i^{n-1}.$$

3. (8 points) Consider the following decision problem with three actions A_1, A_2, A_3 and three states s_1, s_2, s_3:

action/state	s_1	s_2	s_3
A_1	4	5	-3
A_2	2	1	0
A_3	5	-2	3

Which action does the decision maker choose if he uses the maxmin criterion? Which action does he choose if minimizes his maximum regret? Show your work.

Maxmin: A_1 guarantees at least -3, A_2 guarantees 0, and A_3 guarantees -2. The decision maker therefore chooses A_2 if he uses the maxmin criterion.

Mimizing maximum regret: The regret determined by each action and each state is as follows:

action/state	s_1	s_2	s_3
A_1	1	0	6
A_2	3	4	3
A_3	0	7	0

The maximum regret associated with each action is therefore

action/state	max. regret
A_1	6
A_2	4
A_3	7

and the action A_2 minimizes the maximum regret.

4. (14 points) There are many identical consumers, each with the utility of wealth function $U(w) = \sqrt{w}$. Each has initial wealth \$100 and each faces a loss of half of it with probability

$$\pi(e) = \frac{2}{3} - \frac{e}{3}.$$

The variable e here equals either 0 or 1; it is a preventative action that an individual can privately take at a cost to his wealth of $20e$ to reduce the likelihood of the loss.

 (a) (3 points) Assuming a competitive market for insurance, what is the market opportunity line if each consumer takes the action e?

$$\begin{aligned}
\pi(e)\cdot x + (1-\pi(e))\cdot y &= \pi(e)\cdot 50 + (1-\pi(e))\cdot 100 \\
\left(\frac{2}{3} - \frac{e}{3}\right)x + \left(\frac{1}{3} + \frac{e}{3}\right)y &= \left(\frac{2}{3} - \frac{e}{3}\right)50 + \left(\frac{1}{3} + \frac{e}{3}\right)100 \\
\left(\frac{2}{3} - \frac{e}{3}\right)x + \left(\frac{1}{3} + \frac{e}{3}\right)y &= \frac{200}{3} + \frac{50e}{3}
\end{aligned}$$

 (b) (3 points) What is the individual's certain wealth in the case of the competitive equilibrium (which is the case of $e = 0$)?

 We know that the competitive equilibrium involves $e = 0$, in which case the market opportunity line is

$$\frac{2x}{3} + \frac{y}{3} = \frac{200}{3}$$

Each individual is risk averse and so selects complete insurance,

$$x = y = \frac{200}{3}.$$

(c) (4 points) Use your answer to b. to determine the price p per dollar of coverage and the amount C of coverage for each individual in the competitive equilibrium.

$$\begin{aligned} x &= 50 + C - pC \\ y &= 100 - pC \end{aligned}$$

We have $x = y = \frac{200}{3}$, and so

$$\begin{aligned} 50 + C - pC &= 100 - pC \Rightarrow C = 50 \\ \frac{200}{3} &= 100 - 50p \\ 50p &= \frac{100}{3} \\ p &= \frac{2}{3}, C = 50 \end{aligned}$$

(d) (4 points) Is the competitive equilibrium efficient? Explain.

Each person's utility in the competitive equilibrium is $\sqrt{\frac{200}{3}}$. If everyone instead choose $e = 1$, the market opportunity line is

$$\left(\frac{1}{3}\right) x + \left(\frac{2}{3}\right) y = \frac{250}{3},$$

and each person's wealth would be

$$\sqrt{\frac{250}{3} - 20} = \sqrt{\frac{190}{3}} < \sqrt{\frac{200}{3}}.$$

Yes, the competititive equilibrium is efficient.