We have to this point assumed an independent, private value model in the sense that each bidder before he bids privately observes his reservation value for the item, and these reservation values are independent. We could relax the assumption of independence and consider a correlated private value model in which each bidder’s observation of his own reservation value affects his beliefs about the distribution of the reservation values of others. It is quite plausible that reservation values are positively dependent, i.e., if bidder $i$ values the item for sale at a high reservation value, than he may believe that the other bidders are more likely to also value it highly.

We’ll explore a different issue than statistical dependence among private reservation values in this section. We focus on relaxing the private value assumption, which is the assumption that a bidder fully knows the value that he places on the item at the time he makes his bid. We want to model the idea that he may have some uncertainty about the ultimate value of the item to him, and he may also better estimate the ultimate value of the item by observing the bidding behavior of other bidders.

A classic example is the auction by a government of a tract of land for oil drilling. Each oil company can do its own research to estimate the amount of oil in the tract; this is only an estimate, however, because the actual amount of oil is not determined until the oil is actually drilled. A particular oil company might change its estimate if it could gain access to the research of its competitors. Without gaining access to this research, it might change its estimate based upon the observed bidding behavior of other firms (say, in the English or the Dutch auctions).

Let’s take a general perspective for the moment now. We now assume that bidder $i$ privately observes a signal $\sigma_i \in \mathbb{R}$ before he participates in the auction. We assume that the value of the item to bidder $i$ is

$$v_i(\sigma_i, \sigma_{-i}, x).$$

Here, $\sigma_{-i}$ is the $(n-1)$-tuple of signals of the other $n-1$ bidders, and $x$ represents any residual or remaining uncertainty about the value of the item even given all the signals of the $n$ bidders. The case in which

$$v_i = \sigma_i$$

is a private value model. At the other extreme, the case in which

$$v_i(\sigma_i, \sigma_{-i}, x) = v_j(\sigma_j, \sigma_{-j}, x) \text{ for all } 1 \leq i, j \leq n$$

is a pure common value model: the item ultimately has the exact same value to each of the $n$ bidders, though at the time they bid, no single bidder knows this ultimate value. The intermediate cases are examples of interdependent values in the sense that any single bidder’s value may depend on information held privately by others. The general case of interdependent values allows for the possibility that bidder $i$’s value is more affected by his own signal than by the signals of others (his own signal, for instance, may represent his own idiosyncratic preferences).

The rights to drill for oil on a tract of land is commonly taken as an example of a common value: there is a fixed amount of oil underneath the tract that is same for every
possible bidder. Even in this case, however, the oil companies may differ in their abilities to extract the oil from the land, in how close the tract is to each company’s refineries, etc.. In other words, the situation in reality may not truly be a pure instance of a common value. The common value model is extreme and overly simplistic. This is so typical of economic analysis: we consider extreme cases such as perfect competition, figure things out, and then try to understand how close the extreme case is to a situation in reality.

The common value model is especially relevant to the field of finance. The academic finance community defines the value of a share of stock in a company as the corresponding share of the discounted expected value of all future earnings of the company. The value of a firm is therefore is the expected amount of money that it will make in the future, discounted back to today’s dollars. Individual investors and analysts have different beliefs about the future earnings of a company, but this definition reflects the idea that there is some fundamental stream of earnings that will be realized regardless of who owns the share of stock. A share of stock is in this sense an example of common value.

We can see, however, that this definition assumes away many aspects of reality. Different potential owners may value the same company differently, because the owners may differ in the earnings that they may extract from owning the company. There may be complements, for instance, between the company’s business and some potential owner’s other businesses, and potential owners also differ in their management abilities. This is why takeovers can occur. Different potential investors may also value the same share of stock differently: I may want to buy a share in the Hong Kong Shanghai Bank (HSBC) because I feel that my portfolio will be more diversified if I gained more exposure to Asian economies, while you may want to sell your share in this bank to rebalance your portfolio so it has less exposure to Asia. Investors and potential acquiring firms can thus value the same company in different ways.

0.0.19 Chapter 7: Voting and Preference Revelation

We consider in this chapter a group of people who have to make a choice among a finite set of alternatives (say X, Y, and Z). Each individual has preferences over the alternatives. We explore in this section different voting methods and, in particular, whether the individuals will reveal their true preferences through their voting.

Let’s keep in mind the following examples of voting:

- choices by a committee.
- polling as a means of aggregating information. For instance, it is common in the world of sports to select winners of awards such as "Player of the Year," "Number One Team," or "Most Valuable Player" by polling sportswriters, coaches, or players.

0.0.19.1 1.1 Majority Rule

Theorem 8 Majority rule induces truthful preference revelation if there are only two alternatives.
This of course does not hold if there are three or more alternatives, in which case one may not want to cast one’s vote for one’s top choice. We now recognize, however, that the problem of group choice is most interesting when there are more than two alternatives. Most economic problems involve more than two choices (for instance, the tax burden for a public good can be funded in a multitude of possible ways). It is tempting to try to garner the virtues of majority rule. We can imagine, for instance, selecting two alternatives to run against each other. In doing so, however, we might unwittingly exclude a better or more efficient alternative.

Alternatively, we have a series of majority rule elections, each pairing one alternative off against another.

**Example 42 (The Condorcet triple)** Suppose there are three voters (1, 2, and 3) and three alternatives (X, Y, and Z). The preferences of the voters are as follows:

<table>
<thead>
<tr>
<th>voter</th>
<th>ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X &gt; Y &gt; Z</td>
</tr>
<tr>
<td>2</td>
<td>Y &gt; Z &gt; X</td>
</tr>
<tr>
<td>3</td>
<td>Z &gt; X &gt; Y</td>
</tr>
</tbody>
</table>

This constitutes a majority rule cycle: in a pairwise runoff, X defeats Y, Y defeats Z, and Z defeats X. If the three voters were to try to select an alternative by first running one against another, and then running the winner of the first election against the remaining alternative, then the outcome would depend completely upon the order of the elections. This suggests the importance of being able to set the agenda in meetings.

For instance, suppose you’re voter 1 and you’d like to see X elected. You might want to propose that the three voters choose between Y and Z, with the winner going against X. If the voters in each election vote according to their true preferences, then this results in the election of X. The symmetry of the problem shows that any of the three alternatives may be chosen by properly choosing the order of election.

Would the voters still vote according to their true preferences in such a process? Let’s consider the runoff of Y against Z, with the winner facing X. If the voters follow their true preferences, then X is elected. Voter 2 therefore sees the alternative that he deems the worst selected as the group choice. Can he change this outcome? Suppose in the election of Y against Z he votes for Z, resulting in Z being selected in the first round. In the second election of Z against X, no voter has any reason not to vote according to his true preferences and so Z wins. By voting for Z instead of Y in the first round, voter 2 can thereby produce an outcome that he deems better. Voter 2’s behavior in this case is referred to as manipulation of the voting method. The moral of the story is that voting according to one’s true preferences is not a dominant strategy in a repetition of pairwise majority voting. We thus lose this virtue of majority rule when we try to apply it in pairwise elimination of a sequence of alternatives.

"Manipulation" is difficult to formally define at this point because it depends upon the rules of the voting procedure. We mean to imply that a voter votes in a manner that does not reflect his true preferences, assuming that the other voters do vote according to their true preferences, and with the goal of influencing the outcome toward an alternative that he deems more favorable.
We will prove a theorem in this chapter known as the Gibbard-Satterthwaite Theorem. It states that when there are three or more alternatives and each individual voter may rank the alternatives in any possible order, then the only voting procedures that cannot be manipulated are dictatorial in the sense that one particular voter gets to pick his favorite. Let’s look now at some other common procedures for voting and the problems that they may have with manipulation.

0.0.19.2 Plurality Voting.

Each voter has one vote that he may assign to any one of the alternatives. Imagine, for instance, a presidential election in which there is a Republican candidate, a Democratic candidate, and a third-party candidate. It is actually quite common in American history to have a significant third party candidate (e.g., Nader in 2000, Perot in 1992, Anderson in 1980,...). It is clear that you may not want to vote for your favorite candidate. For instance, if your favorite candidate is likely to rank third among the likely vote totals, then you may want to vote for one of the other two candidates so that your vote "counts" (i.e., has some probability of affecting the outcome). It is in fact a major challenge to third candidates to convince voters that it is worthwhile to vote for them.

0.0.19.3 The Borda Count.

Suppose there are \( m \) alternatives. Each voter can assign \( m \) votes to one alternative, \( m - 1 \) votes to a second, \( m - 2 \) to a third, ..., and 1 to a last alternative. Votes are tallied and the alternative that receives the most votes is selected as the winner. (Borda was a French mathematician.) This procedure and variations on it are commonly used in sports polling.

Example 43 (1.10, p. 395) Here’s an example that illustrates that the Borda count may be manipulated: there are 3 individuals and 4 alternatives, ranked as follows:

<table>
<thead>
<tr>
<th>voter</th>
<th>ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X &gt; Y &gt; Z &gt; W</td>
</tr>
<tr>
<td>2</td>
<td>Z &gt; X &gt; Y &gt; W</td>
</tr>
<tr>
<td>3</td>
<td>W &gt; Y &gt; X &gt; Z</td>
</tr>
</tbody>
</table>

We can tally the votes as follows, assuming each person votes according to his true preferences:

- \( X: 4+3+2 = 9 \),
- \( Y: 3+2+3 = 8 \),
- \( Z: 2+4+1 = 7 \),
- \( W: 1+1+4 = 6 \).

\( X \) is therefore chosen. Which voter might be dissatisfied with this outcome and might therefore try to manipulate the outcome by changing his votes? Voter 3 sees his third best outcome chosen, and he effectively wastes his top vote by using it on \( W \). Suppose voter 3 changes his vote to

\[
Y = 4, W = 3, Z = 2, X = 1.
\]

The totals are now

- \( X: 4+3+1 = 8 \),
- \( Y: 3+2+4 = 9 \).
Voter 3 in this way can improve the outcome from X to Y. If everyone votes according to his true preferences, then voter 1 sees his top choice elected (X). We do not have to worry about manipulation by voter 1 unless we imagine him anticipating manipulation by voter 3 and trying to double-cross voter 3’s efforts!

Given honest voting by voters 1 and 3, does voter 2 have any ability to manipulate the outcome? X is his second-best choice, and so the only reason that he would have to manipulate the outcome is if he can change the group choice from X to Z. Let’s have voter 2 try to trash both X and Y:

\[ \text{Z: } 2 + 4 + 2 = 8, \]
\[ \text{W: } 1 + 1 + 3 = 5. \]

The totals are now
\[ \text{X: } 4 + 1 + 2 = 7, \]
\[ \text{Y: } 3 + 2 + 3 = 8, \]
\[ \text{Z: } 2 + 4 + 1 = 7, \]
\[ \text{W: } 1 + 3 + 4 = 8. \]

Voter 2 doesn’t want this outcome because he rates W as the worst possible choice. He therefore needs to move W down, which forces Y or X up. It appears that voter 2 cannot successfully manipulate the voting system to obtain the better outcome of Z instead of X.

0.0.19.4 Approval Voting.

This example is not in Campbell’s text. I include it because it is good to have a variety of examples in mind so that we are open to a wide range of possible voting methods.

There are typically about 30 members in the Economics Department. The department is governed by a Head (appointed by the Dean) and an advisory committee of seven faculty members, which is elected by the 30 faculty members from among their ranks. Until recently, the department used the following procedure for selecting its committee members. Each faculty member received a ballot listing all faculty members eligible for the committee for the upcoming year (a faculty member who would be away on leave, for instance, is ineligible, as is any faculty member who had served the preceding two years). As a voter, a faculty member is required to “approve” at least half of the eligible candidates. A faculty member can choose to choose every eligible candidate, but his ballot is thrown out if he fails to mark approval for at least half of the eligible candidates. Each approval counts as one vote for the candidate. The seven candidates receiving the highest vote totals are selected for the committee, with ties resolved by a fair lottery.

Question: Why was this procedure chosen by the department?

Question: The university allows each department to choose between two different ways of administering itself:

1. A Head is responsible for making all decisions, which he does in consultation with an Advisory Committee. The term "advisory" is self-explanatory: all decisions ultimately rest with the Head.

2. Alternatively, the department can have a Governing Committee along its Chair, where
the Chair administers the committee, which is responsible for all decisions. The Chair runs the committee meetings and then implements its decisions.

These two alternatives are specified in the University Statutes. Each department, however, can invent its own voting procedure for selecting its advisory or governing committee. If there is a "best" or "optimal" voting procedure, then why don’t the University Statutes require that all departments use this best procedure?

Exercises: p. 400–401, problems 4, 7 and 8

0.0.19.5 Restrictions on Preferences

The Gibbard-Satterthwaite Theorem assumes that (i) there are three or more alternatives, and (ii) the preferences of each voter are unrestricted in the sense that all possible rankings of the alternatives are feasible for each voter. In such a case, the only possible non-manipulable voting procedure is a dictatorship. In this section, we discuss cases in which the possible preferences of the group members may be restricted in some way. Preferences are often restricted in economic settings (e.g., everyone my prefer "more" to "less").

Example 44 Here’s an example that motivates restrictions on preferences. Let’s imagine three alternatives, L, M and H. L might correspond, for instance, to low taxes and small government, M to medium taxes and medium size government, and H to high taxes and large government. We can characterize Ron Paul’s preferences as

\[ L > M > H, \]

and Rachel Maddow’s as

\[ H > M > L. \]

A moderate democrat might have the preferences

\[ M > H > L, \]

and a moderate republican (do they exist any more?) might have the preferences

\[ M > L > H. \]

The moderates in each case rank M as the top choice, with the difference between moderate republicans and democrats concerning whether they would rather have L or H if they can not have M.

Who has the preferences

\[ H > L > M, \]

or

\[ L > H > M? \]

Would a person who cannot have his first choice of big government H really prefer small government over medium government? If these rankings simply do not occur among the population, then we say that preferences are restricted. It will turn out that it may be possible to accomplish things through voting when preferences are restricted that cannot be accomplished when preferences are unrestricted.

Note that the implausibility of these last two rankings is in reference to this particular interpretation of L, M, and H. If L represents "Luigi’s Italian Restaurant," M represents
"Manny’s BBQ." and H represents "Hunan Dynasty Chinese Food," and if the group members were trying to pick a place to go to dinner together, then we may not have any reason to rule out the last two preferences as unreasonable or unlikely.

Example 45 (Single Peaked Preferences) A common restriction on preferences is to assume that they are single peaked. This means that the alternatives can be ordered on a line and a numerical representation of preferences so that utility has a "single peak". For instance, suppose there are 4 alternatives $X$, $Y$, $Z$, $W$ that are ordered in this way from left to right. The following graph depicts two utility functions that represent single-peaked preferences:

The following two graphs depict utility functions that do not represent single-peaked preferences:
The reason for the name "single peaked" is clear from these pictures. In the preceding example, we exclude $M < H < L$, which might be represented graphically as

Excluding this case and $M < L < H$ is sufficient to insure that the remaining orderings are all single peaked.