0.0.7 Games That Take Place Over Time

definitions: nodes, subgame, extensive form vs. normal form

strategy in a dynamic or extensive form game: A player’s strategy specifies his choice at every node assigned to him in the game. This includes nodes that are not actually reached or passed through when the game is played and the actions of the other players are taken into account. It is a "complete contingent plan", chosen before the game is played, that fully specifies how a player acts throughout the game against any possible strategies of the other players.

equilibrium path: the sequence of actions that are actually observed when the game is played. The issue of credible threats concerns behavior off the equilibrium path that is not actually observed when the game is played.

Example 21 Predation Game
A component of the chain store paradox, which will be discussed later.
E: entrant
I: incumbent

\[
\begin{array}{c|cc}
  & \text{Fight} & \text{Accomodate} \\
\hline
\text{Out} & 0,2 & 0,2 \\
\text{In} & -3,-1 & 2,1 \\
\end{array}
\]

\[
\begin{tikzpicture}
  \node (E) at (0,0) {E};
  \node (IN) at (1,-1) {I};
  \node (OUT) at (-1,-1) {Out};
  \node (IN) at (1,-1) {I};
  \node (OUT) at (-1,-1) {Out};
  \node (0,2) at (1,-2) {0,2};
  \node (-3,-1) at (-1,-2) {-3,-1};
  \node (2,1) at (1,-2) {2,1};
  \draw (E) -- (OUT);
  \draw (E) -- (IN);
  \draw (OUT) -- (0,2);
  \draw (OUT) -- (-3,-1);
  \draw (IN) -- (2,1);
  \draw (IN) -- (0,2);
\end{tikzpicture}
\]

Draw as normal form:

Definition. A Nash equilibrium of an extensive form game is subgame perfect if its strategies define a Nash equilibrium in every subgame of the game.
This insures that all behavior in the equilibrium is rational (choosing more over less), even behavior off the equilibrium path. The above game has one subgame perfect Nash equilibrium: the Entrant chooses In and the Incumbent chooses Accomodate.

Example 22 The "Divide the Dollar" game in which the dollar is the interval \([0, 1]\) (i.e., it is infinitely divisible) has exactly one subgame perfect Nash equilibrium: 1 proposes \(x = 1\), and 2 accepts any offer. Notice that the definition of equilibrium specifies how 2 responds to every possible offer by player 1, not just the one that 1 proposes in equilibrium. This emphasizes a subtlety of a player’s strategy in an extensive form game: it specifies how he responds in any situation that could arise, not just those that do arise when the game is played because of the choices of the other players.

If the dollar is divisible into pennies or cents, then there are two subgame perfect Nash equilibria:

1: \(x = 1\), 2: accept any offer, and

1: \(x = 0.99\), 2: accept the offer if and only if \(1 - x \geq 0.99\).

In both the case in which the dollar is infinitely divisible and the case in which it is divisible into cents, it is credible that player 2 turns down an offer that gives him 0 (2 gets 0 if he rejects 1’s offer, and he therefore doesn’t suffer from rejecting an offer that gives him 0). If the dollar is infinitely divisible, however, we cannot define a best response of player 1 to this strategy of 2 (there is no "smallest positive amount" in the unit interval that 1 can offer 2). We therefore cannot complete the construction of an equilibrium in this case. If the dollar is divisible into cents, however, and player 2 rejects any offer that doesn’t give him at least a penny, then 1’s best response is to propose 0.99 for himself and 0.01 for player 2 (which 2 then accepts).

Solving a finite extensive form game "backwards induction", which effectively derives a subgame perfect equilibrium.

Example 23 The solution of a game by backwards induction (i.e., the determination of a subgame perfect Nash equilibrium), along with a second Nash equilibrium:
In the bottom game, player 1 chooses a payoff of 1 over a payoff of 2 at the node furthest to the right. With this action, 2’s best response is to choose left over right at the preceding node, earning him a payoff of 5. This is a Nash equilibrium: given the action of the other player, each player’s strategy maximizes his own payoff. It is not subgame perfect, however, because of 1’s choice at this particular node.

In finite games of complete and perfect information, subgame perfection is exactly the same as solving the game through backwards induction. Why do we bother with it if it is so obvious? It is a useful idea that has significance as a refinement beyond this particular class of games. It is easiest to introduce, however, in the context of this class of games.

Zermelo’s Theorem – Backwords induction shows that any finite extensive form game in which there are no “ties” among the payoffs at the terminal or bottom nodes has a unique subgame perfect Nash equilibrium. If there are ties among payoffs, then a player’s choice at any particular node may not be uniquely determined. Consequently, ties can allow the existence of more than one subgame perfect Nash equilibrium.

0.0.8 A Criticism of How We Study Dynamic Games

A strategy is a plan that specifies a player’s choices throughout the game however it is played by the other players. Our model is of players who don’t make up their minds as the game proceeds. As a game gets complicated, however, it is hard to imagine that human players can actually behave in this way. Consider the game of chess, or, even simpler, tic-tac-toe.
**Example 24**  *The Centipede Game (Rosenthal) (Section 5.6 of Campbell)*

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</thead>
<tbody>
<tr>
<td>1</td>
<td>C</td>
<td>2</td>
<td>C</td>
<td>1</td>
<td>C</td>
<td>...</td>
<td>2</td>
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<tr>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>97,100</td>
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<tr>
<td>1,1</td>
<td>0,3</td>
<td>2,2</td>
<td>97,100</td>
<td>99,99</td>
<td>98,101</td>
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</tbody>
</table>

Think about this game as follows: Each player starts with $1. When a player chooses C, $1 is taken from him and $2 is given to the opponent. If a player chooses S, then play stops and each player leaves with his accumulated money.

Notice the independence of history: a player makes a decision at a node in anticipation of its future consequences and without regard to the sequence of moves that have been made to place him at that node.

Unlike chess or tic-tac-toe, it is not hard to determine the unique subgame perfect Nash equilibrium of this game. Does it describe what you think would happen if this game were played in an experimental setting? How would you as a player interpret a choice of C by your opponent? If you had seen him choose C repeatedly, would your expectations of the future play of the game be determined solely by looking forward? Should players interpret the actions they have observed by others in the past as signals of their future actions?