0.0.9.2 An Application in Industrial Organization

One form of collusive behavior among firms is to restrict output in order to keep the price of the product high. This is a goal of the OPEC oil cartel, for instance: member countries have output quotas that are mutually negotiated within the cartel with an eye toward keeping the price of oil at a desired level. Economists have often claimed that cartels and collusive behavior are fundamentally unstable and hence unlikely to endure for the long-term. The argument essentially is that collusive agreements are not Nash equilibria and member firms or countries have the incentive to cheat on the common agreement in pursuit of their own self-interests. This causes the cartel to break down. A footnote to this is that agreements within a nation among firms may violate antitrust laws. The firms therefore have no legal means of contracting among themselves or appealing to the courts for punishment if one or more firms fails to live up to its obligations to the cartel. Similar concerns apply for cartels of nations who have no over-arching government that can enforce the mutually beneficial arrangement.

How then do we explain the fact that OPEC has for the most part succeeded for over 40 years in influencing the global price of oil? Or how do we explain the documented existence of cartels among industries (such as railroads in the U.S. in the late 19th century)? Our model above suggests an answer. A collusive arrangement provides each firm with a larger profit than the competitive outcome. The collusive arrangement is a noncooperative equilibrium in a long-term relationship, provided that each firm cares enough about future profits.

Example 27 (Cournot Duopoly) We illustrate the point in a simple example. There are two identical firms that produce the same product. Let $q_i$ denote the output of firm $i$. The market price for the aggregate output $q = q_1 + q_2$ is determined by the inverse demand function

$$ p(q) = 14 - q. $$

The cost function of each firm $i$ is

$$ C(q_i) = \frac{q_i^2}{4}. $$

The profit function of firm $i$ is therefore

$$ P_i(q_i, q_{-i}) = (14 - (q_1 + q_2)) q_i - \frac{q_i^2}{4}. $$

The Nash equilibrium outputs. We solve for a Nash equilibrium by setting $\frac{\partial P_i}{\partial q_i} = 0$ for each firm $i$:

$$ \frac{\partial P_1}{\partial q_1} = (14 - (q_1 + q_2)) - q_1 - \frac{q_1}{2} = 0, $$

$$ \frac{\partial P_2}{\partial q_2} = (14 - (q_1 + q_2)) - q_2 - \frac{q_2}{2} = 0. $$
or
\[
14 - \frac{5q_1}{2} - q_2 = 0,
\]
\[
14 - q_1 - \frac{5q_2}{2} = 0.
\]
This implies
\[
14 - \frac{5q_1}{2} - q_2 = 14 - q_1 - \frac{5q_2}{2}
\]
or \(q_1 = q_2\). Substitution into either equation implies
\[
14 - \frac{5q_1}{2} - q_1 = 0
\]
\[
14 = \frac{7}{2}q_1 \Rightarrow
\]
\[
q_1 = q_2 = 4
\]
From above we see that
\[
\frac{\partial P_i}{\partial q_i}(q_i, q_{-i} = 4) = (14 - (q_i + 4)) - \frac{3q_i}{2}
\]
\[
= 10 - \frac{5q_i}{2}
\]
which changes from positive to negative at \(q_i = 4\). This verifies that \(q_1 = q_2 = 4\) is a Nash equilibrium. The Nash equilibrium profit for each firm is
\[
(14 - (8)) 4 - \frac{16}{4} = 24 - 4 = 20.
\]

**A better outcome for the firms.** We now calculate the outputs \(q_1, q_2\) that maximize the sum of the profits for the two firms:
\[
P_1(q_1, q_2) + P_2(q_1, q_2) = (14 - (q_1 + q_2)) (q_1 + q_2) - \frac{q_1^2}{4} - \frac{q_2^2}{4}.
\]
\[
0 = \frac{\partial P_1}{\partial q_1} = (14 - (q_1 + q_2)) - (q_1 + q_2) - \frac{q_1}{2}
\]
\[
0 = 14 - \frac{5q_1}{2} - 2q_2,
\]
\[
0 = \frac{\partial P_2}{\partial q_2} = 14 - 2q_1 - \frac{5q_2}{2}
\]
Again, we have \(q_1 = q_2\). Solving using either partial derivative implies
\[
0 = 14 - \frac{9}{2}q_i
\]
or \(q_1 = q_2 = \frac{28}{9}\).

I don’t want to work with such awful fractions. These numbers suggest that both firms would obtain a higher profit than in the Nash equilibrium by choosing \(q_1 = q_2 = 3\) (which is close to \(28/9\)). Let’s check:
\[
(14 - 6) 3 - \frac{9}{4} = 24 - \frac{9}{4} = \frac{87}{4} > 20.
\]

*If firm \(-i\) produces 3, then how much profit can firm \(i\) obtain by deviating?* Firm \(i\)
maximizes

\[ P_i(q_i, 3) = (14 - (q_i + 3)) q_i - \frac{q_i^2}{4} \]
\[ = (11 - q_i) q_i - \frac{q_i^2}{4} \]
\[ = 11q_i - \frac{5q_i^2}{4} \]
\[ 0 = \frac{\partial P_i}{\partial q_i} = 11 - \frac{5q_i}{2} \implies q_i = \frac{22}{5} \]

The marginal profit changes sign at \( q_i = \frac{22}{5} \), and so it indeed maximizes profit. The maximally profitable deviation therefore produces a profit of

\[ P_i \left( \frac{22}{5}, 3 \right) = \frac{22}{5} \left( 11 - \frac{5}{4} \cdot \frac{22}{5} \right) \]
\[ = \frac{22}{5} \left( 11 \cdot \frac{2}{2} \right) \]
\[ = \frac{121}{5} \cdot \frac{2}{2} \]

**Implementation of the superior outcome.** Each firm \( i \) adopts the following strategy: Choose \( q_i = 3 \) to start the game and as long as each firm produces 3 units in each stage game. If any output is observed by either firm other than 3, then switch to \( q_i = 4 \) for every stage in the remainder of the game.

The Nash equilibrium profit is

\[ \pi_i = 20, \]

a "collusive" outcome produces the profit

\[ \pi^*_i = \frac{85}{4}, \]

and each firm can deviate from the collusive output of 3 to obtain

\[ d_i = \frac{121}{5} \]

The use of this trigger strategy by each firm defines a Nash equilibrium in the infinitely repeated Cournot duopoly game if each firm \( i \)'s discount factor \( \delta_i \) satisfies

\[ \delta_i \geq \frac{d_i - \pi_i^*}{d_i - \pi_i} = \frac{121}{5} - \frac{87}{4} = \frac{121}{5} - \frac{87}{4} = \frac{121}{5} - \frac{21}{4} = \frac{484 - 435}{84} = \frac{49}{84} = \frac{7}{12} \]

**0.0.9.3 Several Observations:**

- The relevance of this result in the theory of repeated games to explaining how collusion occurs despite the absence of legal structures to enforce the collusive agreement was first noted by Jim Friedman.
- Are trigger strategies realistic? We in fact see cartels sustaining their collusive
behavior through mutual punishment if anyone member cheats on the collusive agreement. As the member of OPEC with the greatest reserves and capacity, Saudi Arabia plays the role of enforcer in the following sense. Suppose some nation cheats by producing beyond its OPEC-negotiated quota. Saudi Arabia opens its taps and floods the world market with oil, punishing all members with a lower price for oil and correspondingly low profits. After a period of punishment, the cartel gets its act together and reinstitutes a collusive agreement. Such flooding of the market has happened several times in the history of OPEC. It is the tool or threat that Saudi Arabia has to keep the member countries in line.

- The preceding story about punishment, however, does not correspond to an equilibrium in trigger strategies. Notice that: (i) in an equilibrium with trigger strategies, the firms collude and never revert to the Nash equilibrium outputs; (ii) if the firms ever did switch to the Nash equilibrium outputs, they would do so forever and would never reestablish the collusive outcome. These issues have been addressed in a paper by Ed Green and Rob Porter.¹ Each firm in this paper observes the market price and not the output of the other firm. Moreover, there is a random or stochastic element to market demand in their model; a decline in the market price may therefore be caused by an increase in production by a firm or simply by a random increase to demand. Green and Porter construct equilibria of the infinitely repeated game in which:
  - Each firm starts out producing at a collusive level;
  - A market price that falls below a target \( \bar{p} \) causes the two firms to enter a punishment phase in which they each choose larger and less profitable outputs for \( n \) stages;
  - After the \( n \) stages of the punishment phase, each firm returns to its collusive output level.

The target price \( \bar{p} \) and the length \( n \) of the punishment phase are part of the construction of the equilibrium. Their equilibrium has the property that (i) periods of intense competition through overproduction between the two firms occur with positive probability during the infinitely repeated game, and (ii) after a punishment phase, the firms reestablish their collusive agreement (that is, until it breaks down again). In equilibrium, no firm ever deviates from the collusive output in non-punishment stages; the punishment phases occur with positive probability, however, because of random declines in the market price.

Don’t worry about any more of section 7 of Campbell’s book than I have discussed here!

We’ll jump forward next to sections 6 and 7 of Chapter 2 in Campbell’s book. The first 5 sections of Chapter 2 are mainly a math review. I presume you don’t need this. If mathematical topics come up that you haven’t seen, then just let me know and we’ll go over them.

Chapter 2, Section 6: Decision Making Under Uncertainty

Section 6 discusses how we make decisions when we are faced with uncertain or random events and section 7 concerns insurance against random events. A basic issue in section 6 is, "Why do people buy insurance when the insurance company prices the policies so that they on average make a profit (and on average, the policy holder loses)? For simplicity, most of Campbell’s book is restricted to the case of binary outcome, i.e., "high or low", "good or bad", or "succeed or fail".

The basic model is as follows. A person receives wealth $x$ with probability $\pi$ and wealth $y$ with probability $1 - \pi$.

Examples:
1. A person has wealth $y$ and faces the possibility of a loss $y - x$ due to robbery, fire, etc. with probability $\pi$.
2. A person has wealth $z$, and buys an asset (or makes an investment) that results in a loss of size $z - x$ with probability $\pi$ or a gain $y - z$ with probability $1 - \pi$.

asset = investment or security

expected monetary value of the asset (EMV)  Campbell, page 112

Example 28 This is inspired by Example 6.1 in Campbell, p. 113. A person has wealth $100. She may be robbed of $60 with probability 0.3. Here expected wealth is therefore $0.3 \cdot 40 + 0.7 \cdot 100 = 82$.

How much should she be willing to pay for an insurance policy that completely compensates her for a loss if it occurs? Knowing the probability of a robbery, at what price could an insurance company profitably sell such a policy to the person?

Let’s forgo the first question for a moment. The insurance company has to pay out a $60 claim with probability 0.3. The expected payout of the insurance company is therefore $0.3 \cdot 60 = 18$. It would have a positive expected profit at any price that exceeds $18 for the policy. At the price of $18, the company has an expected profit of 0. As a general rule, firms avoid lines of businesses in which they don’t expect to make any money. We have also ignored the costs of the insurance company (rent, paying the salesman, the management, etc.). It needs to make money on its policies, not simply break even on them in an expected value sense.

Would the woman be willing to pay more than $18 dollars for a policy? If she buys the policy at $18, then her wealth will surely be $100 - 18 = 82$. She would buy the insurance at $18 + \varepsilon$ only if she preferred the certain outcome of $82 - \varepsilon$ to the expected (but uncertain) outcome of $82$. We want to model this idea that a certain outcome may be preferred to a larger but uncertain expected outcome.

Definition. A person is risk averse if, for all $m$, he prefers the monetary outcome of $m$ to any asset that provides him with an expected monetary outcome of $m$ but with positive probability gives him an outcome strictly less than $m$.

Another characteristic of risk averse behavior: Consider the following two assets:
asset 1: results in wealth $x$ with probability $1/2$ and $y$ with probability $1/2$, where $x < y$;

asset 2: for some $\delta > 0$, results in wealth $x - \delta$ with probability $1/2$ and in wealth $y + \delta$ with probability $1/2$

The two assets have the same expected value (the $1/2$ plays a role here – it isn’t true for arbitrary probabilities). The greater "spread" in the payoffs in asset 2 makes a risk averse person choose asset 1 over asset 2.

The risk averse person chooses asset 1 over asset 2 because there is "greater uncertainty" in asset 2.

We want to model a person’s decision making over uncertain events to capture risk aversion. We do this by assuming that a person has a utility function of money or monetary outcomes, $u(m)$. It is obvious that $u(\cdot)$ is increasing in $m$.

If the person makes his choices according to expected utility, then we need

$$u(\pi x + (1-\pi) y) > \pi u(x) + (1-\pi) u(y)$$

to indicate his choice of the certain payoff of $\pi x + (1-\pi) y$ over the uncertain payoff of $\pi u(x) + (1-\pi) u(y)$. This is depicted above, and suggests the following shape for the graph of $u$: 
The expected utilities from assets that pay off with either $x$ dollars or $y$ dollars are represented by the dotted line. The horizontal coordinate of any point on this line is the expected monetary value of the asset. The vertical coordinate is the expected utility. We want the utility of the expected monetary value to exceed the expected utility so that the individual prefers the "sure thing" over the randomized return. The graph of the utility of a risk averse person therefore has the property that any line segment between any two points on the graph lies below the graph, i.e., the graph is concave down. We therefore model risk aversion by assuming that the utility of money is strictly concave down.

We can also see that the individual weighs asset 1 over asset 2, where:

- asset 1: results in wealth $x$ with probability $1/2$ and $y$ with probability $1/2$, where $x < y$;
- asset 2: for some $\delta > 0$, results in wealth $x - \delta$ with probability $1/2$ and in wealth $y + \delta$ with probability $1/2$. 
Risk neutral behavior is modeled by a linear utility of money, i.e., \( u(m) = km \) for some \( k > 0 \). It's typical to take \( k = 1 \). The risk neutral person compares assets purely based upon their expected monetary values.

Comment. Modeling risk aversion using a concave down utility function of money is the work of Oscar Morgenstern and John Von Neumann. Von Neumann was one of the greatest mathematical minds of the 20th century. His accomplishments include starting the field of game theory with the economist Oscar Morgenstern, important contributions to probability theory and analysis, work on the Manhattan Project, and contributions to the foundations of computing and artificial intelligence.

Problem 1, p. 122. The individual’s utility-of-wealth function is \( U(w) = \sqrt{w} \) and current wealth is $10,000. Is this individual risk averse? What is the maximum premium that this individual would pay to avoid a loss of $1900 that occurs with probability \( \frac{1}{2} \)? Why is this maximum premium not equal to half of the loss?

The utility function is strictly concave down and therefore models risk averse behavior. Let \( p \) denote the maximum premium. The value of \( p \) equates the expected utility from having insurance to the expected utility from not having insurance; at a higher premium, he would be worse off from having insurance, and at a lower premium, he would be better off to have insurance. Therefore, we can solve this as

\[
\frac{\sqrt{8100}}{2} + \frac{\sqrt{10000}}{2} = \sqrt{10,000 - p} \\
\frac{90}{2} + \frac{100}{2} = \sqrt{10,000 - p} \\
95 = \sqrt{10,000 - p} \\
9025 = 10,000 - p \\
p = 975.
\]

This is strictly more than the expected loss of \( 1900/2 = 950 \), which is a reflection of the individual’s risk aversion.

Problem 2, 122. An individual has a utility-of-wealth function \( U(w) = \ln(w + 1) \)
and a current wealth of $20. Is this individual risk averse? How much of this wealth will the person use to purchase an asset that yields zero with probability 1/2 and with probability 1/2 returns $4 for every dollar invested?

Yes, he’s risk averse. We can check by noting that

\[ U'(w) = \frac{1}{1 + w} \]

and therefore

\[ U''(w) = -\frac{1}{(1 + w)^2} < 0. \]

The function \( U(w) \) is therefore strictly concave down. There are two common and tractable risk averse utility functions – \( \ln \) and \( \sqrt{\cdot} \). Campbell relies on variations of these two examples quite a bit.

Let \( C \) be the amount that the individual invests in the asset. His expected utility from acquiring the asset is

\[ \frac{1}{2} \ln (1 + 20 - C) + \frac{1}{2} \ln (1 + 20 - C + 4C) \]

\[ = \frac{1}{2} \ln (21 - C) + \frac{1}{2} \ln (21 + 3C). \]

He should choose \( C \in [0, 20] \) to maximize this expression. Setting its derivative with respect to \( C \) equal to 0 produces

\[ 0 = \frac{1}{2} \cdot \frac{1}{21 - C} (-1) + \frac{1}{2} \cdot \frac{1}{21 + 3C}. \]

Multiplying through by \((21 - C)(21 + 3C)\) produces

\[ 0 = -(21 + 3C) + 3(21 - C) \]

\[ 0 = 42 - 6C \Rightarrow C = 7. \]

The second derivative of his expected utility is

\[ -\frac{1}{(21 - C)^2} - \frac{9}{(21 + 3C)^2} \]

which is negative. The solution \( C = 7 \) therefore maximizes the individual’s expected utility.

Notice that the expected return on each dollar invested is $2, for an expected profit of $1 for every dollar invested. Risk aversion is the reason that the individual does not invest all of his money in the asset (as a risk neutral person would do).