Section 9.1 is a lengthy and fact-filled discussion of issues of information and effort that arise with insurance. In the remainder of section 9, Campbell presents a model in which an insured party may take an action that lessens the likelihood of a claim. These sections investigate the possibility of moral hazard, i.e., the possibility that the insured party will not exert the proper amount of care and effort to reduce the occurrence of accidents.

Section 9.2, p. 186. The Formal Model

There are two goods, wealth \( W \) and leisure \( l \geq 0 \). \( W \) is an amount of money and \( l \) is an amount of time. An individual has a utility function of the form

\[
U(W, l) = B(W) + l.
\]

We assume that \( B(W) \) is strictly concave down so that the individual is risk averse. The individual can apply some of his time to preventing accidents. Let \( e \geq 0 \) be the time he devotes in this way. We assume

\[
e + l = T,
\]

where \( T > 0 \) is his total amount of time. Substituting, we have

\[
U(W, e) = B(W) + T - e.
\]

We’ll often use this form of utility to emphasize the individual’s choice of \( e \). We will sometimes normalize this further and consider \( T = 1 \). The number \( e \) then represents the fraction of the individual’s total time that he devotes to accident prevention.

We have the following values of wealth:

Without insurance:
- \( a \): the individual’s wealth without insurance when the accident occurs;
- \( z \): the individual’s wealth without insurance when the accident does not occur.

With insurance:
- \( x \): the individual’s wealth with insurance when the accident occurs;
- \( y \): the individual’s wealth with insurance when the accident does not occur.

We have \( a \leq x \leq y \leq z \). We can recover the following values:
- \( z - a \): the loss in the accident;
- \( z - y \): the cost of the insurance policy;
- \( (x - a) + (z - y) \): the face value of the policy, i.e., the amount the insurance company pays out in the event of an accident. To see this, let \( K \) denote this payout. We have

\[
x = a + K - (z - y),
\]

where \( z - y \) is the cost of the policy. Solving this for \( K \) produces the value of the payout in the event of an accident.

Campbell clearly finds it convenient to work with \( x \) and \( y \) instead of the cost of the policy and its payout value, so we’ll go with the flow and follow his lead. The individual can choose \( x, y \) from his market opportunity line. For this reason, I allow the possibility of \( x = a \) and \( y = z \) (this models the purchase of zero insurance coverage by
the individual).

We assume that the individual can reduce the likelihood of an accident by exerting greater effort (i.e., choosing a larger $e$). Let $\pi (e)$ denote the likelihood of an accident. We assume it is decreasing in $e$.

From choices $(x, y)$ presented to him by his market opportunity line, the individual will choose $x$, $y$, and $e$ to maximize his expected utility

$$EU = \pi (e) \left[B(x) + T - e\right] + (1 - \pi (e)) \left[B(y) + T - e\right]$$

$$= \pi (e) B(x) + (1 - \pi (e)) B(y) + T - e.$$

### 0.0.12.2 9.3 The Binary Choice Model of Moral Hazard

We assume in this section that $e$ is either 1 or 0, i.e., the individual either acts to diminish the likelihood of the accident (1) or he does not (0). We assume $0 < \pi (1) < \pi (0) < 1$. The assumption $0 < \pi (1)$ means that accidents are possible even if the individual acts to prevent it. If $\pi (1) = 0$, then the insurance would know if an accident occurred that the individual had failed to act to prevent it. An insurance company is unwilling to insure against accidents occurring that are completely preventable. The assumption that $\pi (0) < 1$ is that the accident may or may not occur. If $\pi (0) = 1$, then it’s really not an accident in the sense that the loss will surely occur if the individual doesn’t act.

We’re going to show here that $e = 0$ is chosen by each individual in a competitive insurance market with many identical customers. We have several attributes of equilibrium and we show that they can only be achieved with $e = 0$.

We assume that the insurance market is perfectly competitive so that the market opportunity line is a fair odds line. Let’s discuss the market opportunity line. It has the form

$$\omega (e) = x = y = \pi (e) a + (1 - \pi (e)) z.$$

Now let’s examine expected utility:

$$\pi (e) B(x) + (1 - \pi (e)) B(y) + T - e.$$

The Complete Insurance Theorem states that for each value of $e$ the term

$$\pi (e) B(x) + (1 - \pi (e)) B(y)$$

is maximized on the market opportunity line at $x = y$ (complete insurance against loss). We can solve for this value of $x = y$ by substituting into the market opportunity line:

$$w(e) = x = y = \pi (e) a + (1 - \pi (e)) z.$$

The new notation $w(e)$ denotes the **certain wealth** of the insured individual when he
chooses \( x, y \) optimally given \( e \). The individual’s utility is then

\[
B(w(e)) + T - e.
\]

We now use the assumption of many identical consumers in a competitive market for insurance, all of whom choose the same level of insurance. Suppose everyone else chooses the level associated with \( e = 1 \). We note now that a selected individual would then benefit by switching to \( e = 0 \). The individual’s utility is

\[
B(w(1)) + T - e.
\]

It is clear that

\[
B(w(1)) + T - 0 > B(w(1)) + T - 1,
\]

and so given the market opportunity line, the individual maximizes his own utility by choosing \( e = 0 \). The point is that the only equilibrium market opportunity line is the one in which every identical individual chooses \( e = 0 \). For any market opportunity line determined by the behavior of all of the individual consumers, any individual who chooses \( e = 1 \) can profit by switching to \( e = 0 \). The only possible equilibrium is therefore the case in which everyone chooses \( e = 0 \).

Summary: No individual has an incentive to devote effort to prevention, and thus each individual’s utility will be

\[
B(\pi(0) a + (1 - \pi(0)) z) + T.
\]

Is it necessarily the case that

\[
B(\pi(0) a + (1 - \pi(0)) z) + T < B(\pi(1) a + (1 - \pi(1)) z) + T - 1,
\]

i.e., everyone would be better off if everyone choose \( e = 1 \)? In other words, is the market outcome necessarily inefficient? We have

\[
B(\pi(0) a + (1 - \pi(0)) z) < B(\pi(1) a + (1 - \pi(1)) z)
\]

because exerting effort increasing the expected outcome \( \pi(e) a + (1 - \pi(e)) z \). But

\[
T - 1 < T,
\]

and so the question of whether or not the market is inefficient will depend upon the parameters of each particular example. We’ll see that it can hold in the example below.

**Example 29 (9.1, p. 189: An inefficient effort supply at equilibrium)**  
Assumptions:

- \( B(w) = 2.4 \sqrt{w} \) and \( T = 1 \).

Therefore,

\[
U(w, e) = 2.4 \sqrt{w} + 1 - e.
\]

- \( \pi(1) = 1/3, \pi(0) = 1/2 \)
- \( a = 30 \) and \( z = 72 \)

**Analysis:** We characterize equilibrium above as \( e = 0 \) and each individual perfectly insuring himself against losses. When everyone chooses \( e = 0 \), the market opportunity line with fair odds is

\[
\frac{x}{2} + \frac{y}{2} = \frac{a}{2} + \frac{z}{2} = \frac{30}{2} + \frac{72}{2} = 51,
\]

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where we’ve used the fact that no insurance \( x = a, y = z \) is on the market opportunity line. The individual perfectly insures himself in this case, choosing \( x = y = 51 \). His utility is therefore

\[
2.4\sqrt{51} + 1 - 0 = 2.4\sqrt{51} + 1 \approx 18.14.
\]

If everyone choose \( e = 1 \), the market opportunity line would be

\[
x + \frac{2y}{3} = \frac{a}{3} + \frac{2z}{3} = \frac{30}{3} + \frac{2 \cdot 72}{3} = 58.
\]

An individual chooses the point \( x = y = 58 \). His utility is

\[
2.4\sqrt{58} + 1 - 1 = 2.4\sqrt{58} \approx 18.28.
\]

Everyone would be better off if everyone could agree to exert effort \( e = 1 \) to prevent accidents, but each individual would profit by shirking in this case to \( e = 0 \).

Now assume that insurance is not available for purchase. We first show that each individual will choose to exert effort \( e = 1 \). His expected utility with \( e = 0 \) is

\[
\frac{2.4\sqrt{30}}{2} + \frac{2.4\sqrt{72}}{2} + 1 \approx 17.75,
\]

and his expected utility with \( e = 1 \) is

\[
\frac{2.4\sqrt{30}}{3} + \frac{2 \cdot 2.4\sqrt{72}}{3} + 1 - 1 \approx 17.96,
\]

and so he’s better off by choosing \( e = 1 \). Notice, however, that 17.96 < 18.14, i.e., every individual is better with insurance and choosing \( e = 0 \) then he would be with no insurance and choosing \( e = 1 \). The no insurance, \( e = 1 \) outcome is not Pareto efficient.

### 0.0.13 9.4 A Continuum of Effort Levels

We now consider the case of \( e \in [0, 1] \) and \( \pi'(e) < 0 \). We look for an equilibrium in which:

- every individual chooses the same level of effort \( e \);
- the market prices insurance fairly given the probability \( \pi(e) \) of an accident.

As a consequence of these two points, we know that the risk averse individual will choose \( x = y \) on his market opportunity line,

\[
\pi(e) x + (1 - \pi(e)) y = \pi(e) a + (1 - \pi(e)) z.
\]

This provides him with a certain wealth of

\[
w(e) = \pi(e) a + (1 - \pi(e)) z.
\]

The individual’s utility after purchasing insurance is

\[
B(w(e)) + T - e.
\]

To determine the level of effort that characterizes equilibrium, we set \( w(e) = w \) because the individual’s choice of effort will not affect the market outcome. The individual then chooses \( e \) to maximize

\[
B(w) + T - e,
\]

which is obviously maximized at \( e = 0 \). Again, the only equilibrium will be when
everyone chooses $e = 0$. This is inefficient if there exists a value of $e$ such that

$$B(w(0)) + T < B(w(e)) + T - e.$$  

Again, whether or not this is true depends upon the parameters of the particular problem. It would not typically be true, however, that $B(w(e)) + T - e$ is maximized at $e = 0$. We would therefore expect in the continuum case that the competitive equilibrium is inefficient.

**Example 30 (9.2, p. 191: Calculating the efficient effort supply)**  
Assumptions:

- $B(w) = 4 \ln(w + 3), T = 1$;
- $a = 24, z = 96, \pi(e) = 1/2 - e/4, e \in [0, 1]$.

Equilibrium: $e = 0, \pi(0) = 1/2$. The individual perfectly insure himself the wealth

$$w(0) = \frac{24}{2} + \frac{96}{2} = 60,$$

and his utility is

$$4 \ln(63) + 1 = 17.57.$$

The efficient level of preventative care: If everyone chooses the same effort level $e$, and the market opportunity line adjusts to fair odds given this value of $e$, then every individual purchases the point $w(e) = x = y$, where

$$w(e) = \left(\frac{1}{2} - \frac{e}{4}\right) \cdot (24) + \left(1 - \left(\frac{1}{2} - \frac{e}{4}\right)\right) \cdot (96)$$

$$= (2 - e) \cdot (6) + (2 + e) \cdot (24)$$

$$= 60 + 18e.$$

An individual therefore has utility

$$4 \ln(60 + 18e + 3) + 1 - e = 4 \ln(63 + 18e) + 1 - e.$$

We choose $e$ to maximize this expression:

$$0 = 4 \cdot \frac{18}{63 + 18e} - 1 \iff 63 + 18e = 72$$

$$e = \frac{1}{2}.$$

By the second derivative test, this indeed is a maximum. It produces utility

$$4 \ln\left(\frac{63 + 18}{2}\right) + 1 - \frac{1}{2} = 4 \ln(72) + \frac{1}{2} = 17.61.$$

If insurance were not available at all, the individual would choose $e$ to maximize

$$\left(\frac{1}{2} - \frac{e}{4}\right) \cdot 4 \ln(24 + 3) + \left(1 - \left(\frac{1}{2} - \frac{e}{4}\right)\right) \cdot 4 \ln(96 + 3) + 1 - e$$

$$= (2 - e) \ln(27) + (2 + e) \ln(99) + 1 - e.$$  

The derivative with respect to $e$ is

$$- \ln(27) + \ln(99) - 1 = 0.299,$$
and so $e = 1$ is optimal. His expected utility in this case is

$$
(2 - 1) \ln(27) + (2 + 1) \ln(99) + 1 - 1
= \ln(27) + 3 \ln(99) = 17.08,
$$

which is less than his expected utility of 17.57 with no effort and complete insurance.

We can see that the competitive equilibrium outcome is not Pareto optimal, but it is still
der better than the situation with no insurance.