The answers below are my effort to respond to the questions. You may have answered the questions in different ways, which is fine! Please alert me to errors and typos in these answers and in the notes.

p. 21, problem 2
In this problem, let \( c_i \) denote the cost of firm \( i \), let \( c_i^* \) denote its report, and let \( c_{-i}^* \) denote the lowest cost reported by the other 4 firms.

A. Suppose \( c_i = 10 \) and \( c_{-i}^* = 20 \). If it reported truthfully, then firm \( i \) would receive a payment of 10.50 for a profit of 0.50. By reporting \( c_i^* = 15 \), for instance, it would still be selected by the government as the low cost producer but would instead receive 15.75, for a profit of 5.75. I am not claiming that \( c_i^* = 15 \) is optimal for firm \( i \); all I’m doing is demonstrating a situation in which firm \( i \) clearly benefits by not reporting honestly. Clearly, honest reporting is not a dominant strategy.

Here’s some intuition that might be helpful. As a general rule, you should be suspicious about being honest if it is possible to influence the price in one’s favor. The ability to affect the price typically provides an incentive to over-report for the sake of driving up the price received when selected. Alternatively, if one cannot influence the price, then under-reporting may in some cases be beneficial as a means of improving one’s chances of being selected. These are only general guidelines, however; in the VCG mechanism, for instance, a firm cannot affect the price it receives but it has no reason to under-report.

B. If \( c_i^* < c_{-i}^* \), then the low cost firm receives a payment of \( 1.05 \cdot c_{-i}^* \). Suppose firm \( i \)’s true cost \( c_i \) satisfies

\[
    c_{-i}^* < c_i < 1.05 \cdot c_{-i}^*.
\]

If firm \( i \) reports honestly, then it is not selected by the government and receives nothing. If it reports \( c_i^* < c_{-i}^* \), then it is the low cost reporting firm and it receives a profit of \( (1.05 \cdot c_{-i}^*) - c_i \). Again, we have demonstrated a situation in which it benefits firm \( i \) to misreport its true cost.

C. If \( c_i^* < c_{-i}^* \), then firm \( i \) receives a per unit profit of

\[
    \frac{c_i^* + c_{-i}^*}{2} - c_i.
\]

Suppose \( c_i = 10 \) and \( c_{-i}^* = 20 \). Any report \( c_i^* \) in the interval \( (10, 20) \) insures that firm \( i \) is selected, and each such report provides firm \( i \) with a strictly higher price that the honest report of \( c_i = 10 \). Honestly reporting is therefore not a dominant strategy for firm \( i \).
D. Let $h^*_{-i}$ denote the highest cost reported by the other 4 firms. We have $c^*_{-i} \leq h^*_{-i}$. If $c^*_i < c^*_{-i}$, then firm $i$ is selected and receives a profit of 

$$\frac{h^*_{-i}}{2} - c_i.$$ 

Can we concoct an example in which 

$$\frac{h^*_{-i}}{2} < c_i < c^*_{-i} \leq h^*_{-i}?$$ 

In this case, firm $i$ is selected if it reports honestly ($c_i < c^*_{-i}$) but it then receives a negative profit ($\frac{h^*_{-i}}{2} - c_i < 0$). It would prefer in this case to overreport, not be selected, and therefore receive a profit of 0. How about $c_i = 3$, $c^*_{-i} = 4$ and $h^*_{-i} = 5$? This works. It is clear in this case that firm $i$ would profit by reporting $c^*_i = 5$ (for instance), thereby insuring that it is not selected.

E. Suppose $c_i < c^*_{-i}$. Could firm $i$ prefer to receive 10% of $c_{-i}$ by reporting $c^*_i > c^*_{-i}$ instead of the profit of $c^*_{-i} - c_i$ that it gets by reporting honestly? Is it possible that 

$$0.10 \cdot c^*_{-i} > c^*_i - c_i?$$

This is equivalent to 

$$c_i > (0.9) \cdot c^*_{-i},$$

and recall that we also need $c_i < c^*_{-i}$. How about $c_i = 9.5$ and $c^*_{-i} = 10$? By reporting honestly, firm $i$ receives a profit of 

$$c^*_{-i} - c_i = 10 - 9.5 = 0.5,$$

but by reporting $c^*_i > 10$, it receives a payment of $(0.1) \cdot 10 = 1$ for doing nothing. Clearly, honest reporting is not a dominant strategy for firm $i$.

**Page 28-29.**

**Problem 1.** The efficient outcomes are (Opera, Opera) and (Hockey, Hockey). Incidentally, the textbook author is Canadian, which may explain the "Hockey" in this example. Let’s watch for other evidence of Campbell’s "Candianess" in the text.

**Problem 4.**

Christine: $A > B > C > D > E$

Jay: $E > D > C > B > A$

Christy-Ann: any two pairs are equivalent

The problem is not providing you with all possible information concerning preferences. For instance, how does Christy-Ann rank $A$ in comparison to $B$, and how does Christine compare the pair $(A, D)$ to $(B, C)$? It is stated, however, that each item is positively valued by each person. We thus know that providing a person with additional items makes him/her strictly better off.
Is it possible to improve the welfare of one person without hurting at least one of the other 2 people? If not, then the allocation is efficient. I propose allocations below and then argue that a person can be made better off only at the expense of some other person.

A. We list what Jay and Christine receive:

outcome 1:
Jay: $A, B, C, D, E$
Christine: nothing

In general, one person receiving everything is Pareto efficient because moving to some other allocation would make him worse off. This suggests

outcome 2:
Jay: nothing
Christine: $A, B, C, D, E$

outcome 3:
Christine: $A, B$
Jay: $E, D, C$

Christine receives her favorite pair and Jay receives his favorite triple. If one were to be made better off, he/she would have to receive an additional item, which means that the other would be made worse off. Consequently, this is Pareto efficient.

outcome 4:
Christine: $A, B, C$
Jay: $E, D$

The same logic as in outcome 3 applies.

outcome 5:
Christine: $A, B, C, D$
Jay: $E$

Christine receives her top 4 items and Jay receives his favorite item. Either would need to receive more items to be made better off. Neither one can therefore be made better off without the other one being hurt.

B. outcome 1:
Christine: $A, B, C, D, E$
Christy-Ann: nothing

outcome 2:
Christine: nothing
Christy-Ann: $A, B, C, D, E$
outcome 3:
Christine: $A, B, C$
Christy-Ann: $D, E$
Christine receives her top 3 items and so she can be made better off only by receiving an additional item. This, however, would hurt Christy-Ann. Christy-Ann may be happier with any set of 3 items, but this requires Christine to have only 2 items, which makes her strictly worse off.

outcome 4:
Christine: $A, B$
Christy-Ann: $D, E, C$
Christine receives her favorite pair. She can be made better off only by receiving 3 items, which would make Christy-Ann worse off. Christy-Ann may be happier with some other bundle of 3 items or with 4 or more items. Obtaining 4 or more items would make Christine worse off, while switching Christy-Ann to some other bundle of 3 items would move Christine to some pair that she thinks is strictly worse than $A, B$. Therefore, this is Pareto efficient.

outcome 5:
Christine: $A$
Christy-Ann: $D, E, C, B$
Christine is made worse off by switching to either no items or any other single item. Consequently, a potential improvement requires that she receive at least 2 items, which would make Christy-Ann worse off. Christy-Ann may be happier with some different set of 4 items or with 5 items, but this would require that Christine receive an item other than $A$ or no item at all, making her worse off.

C. outcome 1:
Christine: $A$
Jay: $E$
Christy-Ann: $B, C, D$
Christine receives her favorite item. Making her better off requires giving her two or more items, which necessarily hurts Jay or Christy-Ann. A similar argument shows that it is not possible to improve Jay’s outcome without hurting someone else. Christy-Ann might be better off with some other triple of items or with more than 3 items. This, however, would hurt either Christine or Jay.

outcome 2:
Christine: $A, B$
Jay: $E$
Christy-Ann: \( C, D \)
Christine receives her favorite pair. Making her better off requires giving her three or more items, which necessarily hurts Jay or Christy-Ann.
A similar argument shows that it is not possible to improve Jay’s outcome without hurting someone else.
Christy-Ann can be made better off only with 3 or more items. This, however, would hurt either Christine or Jay.

outcome 3:
Christine: \( A \)
Jay: \( E, D \)
Christy-Ann: \( C, B \)
Christine receives her favorite item. Making her better off requires giving her two or more items, which necessarily hurts Jay or Christy-Ann.
Jay receives his favorite pair. Making him better off requires that he receive 3 or more items, which would hurt Christine or Christy-Ann.
Christy-Ann can be made better off only with 3 or more items. This, however, would hurt either Christine or Jay.

outcome 4:
Christine: \( A, B \)
Jay: \( E, D \)
Christy-Ann: \( C \)
Christine receives her favorite item. Making her better off requires giving her 3 or more items, which necessarily hurts Jay or Christy-Ann.
Jay receives his favorite pair. Making him better off requires that he receive 3 or more items, which would hurt Christine or Christy-Ann.
Christy-Ann might be better off with some other item or with more than 1 item. Giving her more than 1 item or giving her a different single item would necessarily hurt either Christine or Jay.

outcome 5:
Christine: \( A, B, C \)
Jay: \( E \)
Christy-Ann: \( D \)
Christine receives her top 3 items. She can be made better off only with 4 or more items, which would make Jay or Christy-Ann worse off.
Jay receives his favorite item. He can be made better off only with 2 or more items, which would leave 3 or fewer items for Christine and Christy-Ann. At least one of them would be worse off.
Christy-Ann might be happier with some other item. This, however, would hurt Christine or Jay. She might also be happier with 2 or more items, but this would also hurt Christine or Jay.

Problem 4 is a difficult problem. Let me know if you suspect any flaws in my answer.

Problem 5. The sum is maximized by choice Y. Choice Z, however, is efficient, because Kyle strictly prefers it to Y or X. The choice X is efficient because it is Mia’s strict favorite, while choice Y is efficient because it is Jackson’s unique favorite.

Problem 6. Which outcomes are efficient? F, M, K
Some of you did not identify K as efficient (I also made this mistake in first draft of the answers).
When the new outcome is added, it becomes the only efficient outcome, eliminating both F and M as efficient.

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Problem 1.
A. Person 1 should maximize
\[ U_1(e_1, e_2) = 100(e_1 + e_2) - 150e_1 \]
taking \( e_2 \) as a fixed constant. We have
\[ U_1(e_1, e_2) = 100e_2 - 50e_1 \]
and so \( e_1 = 0 \) maximizes 1’s utility for each \( e_2 \).

B. The unique Nash equilibrium is \( e_1 = e_2 = 0 \). Notice that it is in fact a dominant strategy equilibrium: the choice of 0 is optimal for each person regardless of the choice of the other person.

Problem 3.
If every other voter votes for Y, then no voter can change the outcome by unilaterally changing his vote from Y to X. If everyone votes for Y, then no one can make himself better off by changing his vote. Therefore, this is a Nash equilibrium.

It is a dominant strategy for each voter to vote for X. Therefore, each voter voting for X defines a dominant strategy equilibrium, which is also necessarily a Nash equilibrium.

I find the dominant strategy equilibrium more plausible than the Nash equilibrium in which everyone votes for Y. Generally, a dominant strategy equilibrium when it exists is very believable.

Problem 4.
If the other person submits 10, then I can receive 10 by submitting 10. Alternatively, I could submit \( x > 10 \) and receive \( 10 - 5 = 5 \). Consequently, 10 is a best response to 10 by the other person, and so 10, 10 is a Nash equilibrium.
Are there other Nash equilibria? Suppose the other guy submits \( x > 10 \). My best response is to submit \( x - 1 \), in which case I get \( x - 1 + 5 = x + 4 \). Consequently, any Nash equilibrium in which one person submits a number \( x > 10 \) must have the other submitting \( x - 1 \). There is no pair of numbers \((x_1, x_2)\) such that \( x_1 > 10, x_2 > 10 \), and \( x_1 = x_2 - 1, x_2 = x_1 - 1 \). If there is another Nash equilibrium other than \( 10,10 \), it must therefore involve one person submitting \( 10 \) and the other submitting \( 11 \). But these reports are not a Nash equilibrium because the person reporting \( 11 \) would improve his payment by reporting \( 10 \) instead. Therefore, \( 10,10 \) is the only Nash equilibrium.