1. First Exam  
Information Economics (490), Fall 2011

Rules for the exam:

• You can consult all course materials, including your notes, past homework assignments, and everything posted on the course webpage. Do not consult other books other than our course textbook and do not consult any other sources, such as online sites.

• Your work should be your own. Do not consult with anyone else.

• If you have questions, then ask me.

1. (20) Consider in this problem a seller who sells an item through the following auction. There are \( n \) bidders. Each bidder submits a bid. The bidder who submits the highest bid wins the item and pays the seller his bid (just like in the first price auction). The bidder who submits the lowest bid also pays his bid to the seller. Think of this as a punishment for being the low bidder; the seller might think that this will lead to an increase in bids. Ties are resolved with fair lotteries such as coin flips.

We assume that the reservation values of the \( n \) bidders are identically and independently distributed according to the uniform distribution on \([0, 1]\). The purpose of this example is to derive an increasing function \( B : [0, 1] \to [0, 1] \) that defines an equilibrium in this auction: if every other bidder besides bidder \( i \) uses the function \( B \) to select his bid, then \( B(v_i) \) maximizes bidder \( i \)'s expected profit in the auction for each of his possible reservation values \( v_i \in [0, 1] \). Select a bidder \( i \), let \( v_i \) denote his reservation value, \( b \) his bid, and assume all other bidders use the function \( B \) to select their bids.

(a) (3) What is the probability that bidder \( i \) wins the item with the bid \( b \)? What is the probability that his bid \( b \) is the lowest bid?

(b) (2) What is bidder \( i \)'s expected profit function \( U(v_i, b) \) as a function of his reservation value \( v_i \) and his bid \( b \)?

(c) (5) For equilibrium, we want \( U(v_i, b) \) to be maximized at \( b = B(v_i) \).

Let \( U(v_i) \) denote this maximized value of expected profit, \( U(v_i) = U(v_i, B(v_i)) \). Apply the Envelope Theorem to calculate \( U'(v_i) \). Use your answer to derive a formula for \( U(v_i) \).

(d) (5) Using your answers to b. and c., solve for the strategy \( B(v_i) \). (The answer may not be pretty). Verify that \( B(v_i) \) is indeed an increasing function by showing that its derivative is positive. In the case of \( n = 2 \), this auction is simply the all-pay auction. You may want to check your answer against the equilibrium we calculated in class for the all-pay auction.
(e) (5) Does the seller benefit by charging the low bidder his bid in addition to the high bidder? Answer the question using the Revenue Equivalence Theorem.

2. (10) Bob’s utility of wealth function is \( U(w) = \sqrt{w} \). He has $1000 and he faces a possible loss of $600 with probability 1/4. An insurance company offers him a policy that provides $1 of coverage for $0.30. Let \( C \) denote the dollar value of the coverage that Bob purchases.
   
   (a) (5) What is the market opportunity line? Is it a fair odds line?
   
   (b) (5) What is Bob’s expected utility if he buys \( C \) dollars of coverage? How much coverage does he choose to buy?

3. (20) There are many identical consumers, each with the utility of wealth function \( U(w) = \ln(w + 1) \). Each has initial wealth $100 and each faces a loss of half of it with probability \( \pi(e) = \frac{1}{2} - \frac{e}{3} \).

   The variable \( e \) here equals either 0 or 1: it is a preventative action that an individual can privately take at a cost to his wealth of 10\( e \) to diminish the likelihood of the loss.

   (a) (5) Assuming a competitive market for insurance, what is the market opportunity line if each consumer takes the action \( e \)?
   
   (b) (5) What is the individual’s certain wealth in the case of the competitive equilibrium?
   
   (c) (5) Use your answer to b. to determine the price \( p \) per dollar of coverage and the amount \( C \) of coverage for each individual in the competitive equilibrium.
   
   (d) (5) Is the competitive equilibrium efficient? Explain.