In all problems in which you are asked to apply an algorithm, be sure in your answer to show enough of your work that I can see that you are applying the algorithm correctly.

1. (20 points) There are 5 men \( \{m_1, m_2, m_3, m_4, m_5\} \) and 4 women \( \{w_1, w_2, w_3, w_4\} \) in a marriage market. The preferences of the men and women over the opposite sex are as follows:

\[
\begin{align*}
m_1 &: w_1, w_2, w_3 & w_1 &: m_4, m_3, m_5 \\
m_2 &: w_1, w_3, w_4, w_2 & w_2 &: m_2, m_4, m_5, m_1, m_3 \\
m_3 &: w_3, w_2, w_4 & w_3 &: m_1, m_3, m_2, m_4, m_5 \\
m_4 &: w_2, w_3, w_1 & w_4 &: m_5, m_3, m_2, m_1 \\
m_5 &: w_1, w_3, w_2, w_4
\end{align*}
\]

(a) (10)

- (5) Determine the woman-optimal stable matching.
- (5) Show that in this particular example the deferred acceptance algorithm in which women make the proposals is strategy proof for men.
- (5) Determine the man-optimal stable matching.
- (3) Show that in this particular example the deferred acceptance algorithm in which men make the proposals is strategy proof for women.
- (2) What does the Rural Hospitals Theorem imply in this marriage market?

(b) (8)

1. (21 points) There are three colleges \( (C_1, C_2, \text{and } C_3) \) and 5 students \( (s_1, s_2, s_3, s_4, s_5) \). Each college has the capacity for two students. The preferences of the colleges and students are as follows:

<table>
<thead>
<tr>
<th></th>
<th>C_1</th>
<th>C_2</th>
<th>C_3</th>
<th>s_1</th>
<th>s_2</th>
<th>s_3</th>
<th>s_4</th>
<th>s_5</th>
</tr>
</thead>
<tbody>
<tr>
<td>s_1</td>
<td>s_2</td>
<td>s_4</td>
<td>C_1</td>
<td>C_2</td>
<td>C_3</td>
<td>C_1</td>
<td>C_1</td>
<td>C_1</td>
</tr>
<tr>
<td>s_2</td>
<td>s_1</td>
<td>s_5</td>
<td>C_2</td>
<td>C_3</td>
<td>C_2</td>
<td>C_2</td>
<td>C_2</td>
<td>C_2</td>
</tr>
<tr>
<td>s_3</td>
<td>s_4</td>
<td>s_3</td>
<td>C_3</td>
<td>C_4</td>
<td>C_1</td>
<td>C_3</td>
<td>C_3</td>
<td>C_3</td>
</tr>
<tr>
<td>s_4</td>
<td>s_3</td>
<td>s_1</td>
<td>C_4</td>
<td>C_3</td>
<td>C_4</td>
<td>C_1</td>
<td>C_3</td>
<td>C_3</td>
</tr>
<tr>
<td>s_5</td>
<td>s_5</td>
<td>s_1</td>
<td>C_5</td>
<td>C_3</td>
<td>C_5</td>
<td>C_3</td>
<td>C_3</td>
<td>C_3</td>
</tr>
</tbody>
</table>

(a) (13)

- (5) Determine the assignment of students to colleges using CODA (college optimal deferred acceptance algorithm).
- (5) Show that the algorithm is strategy proof for students in this particular example.
- (3) Is the matching that you derived in 2. a. Pareto efficient? Explain.

(b) (8)

- (5) Determine the assignment of students to colleges using the Boston mechanism.
- (3) Is the Boston mechanism strategy-proof for students in this particular example? Explain.
A decision maker can choose among 3 actions $A_1$, $A_2$, and $A_3$. The utility consequences of his actions depend upon the unknown state, which can assume 3 values $s_1$, $s_2$ and $s_3$. These utility consequences are listed in the following table:

<table>
<thead>
<tr>
<th>action/state</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>4</td>
<td>5</td>
<td>−5</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0</td>
<td>−3</td>
<td>2</td>
</tr>
<tr>
<td>$A_3$</td>
<td>−5</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

(a) (3) What action will the decision maker choose if he assigns probability 1/3 to each state and maximizes his expected utility?

(b) (3) What action will the decision maker choose if he uses the maxmin decision rule?

(c) (3) What action will the decision maker choose if he minimizes the maximum regret?