November 6

**Theorem 15 (Gibbard-Satterthwaite Theorem)**  If \( A \) has at least 3 elements and \( f : \mathcal{L}^N \to A \) is onto and strategy-proof, then \( f \) is dictatorial.

**Corollary 16**  If \( A \) has at least 3 elements and \( f : \mathcal{L}^N \to A \) is Pareto optimal and is implemented by a dominant strategy equilibrium of some game \((\prod S_i, \tau)\), then \( f \) is dictatorial.

**Theorem 17 (Sen’s Majority Rule Theorem, p. 397)**  If the number of individuals is odd, the number of alternatives is finite, preference rankings are strict (no indifference among alternatives), and preferences are value-restricted, then there exists a unique alternative that defeats any other alternative in majority rule. We call this alternative the unique majority winner.

### 0.0.22 Matching

We’re jumping now to Chapter 9 and the topic of matching. This is very hot topic in economic research right now. The problem in general can be described as follows. There are two sets, \( A \) and \( B \). The elements of set \( A \) must be assigned to the elements of the set \( B \). The elements of the set \( A \) are people who have strict preferences over the elements of \( B \) to which they may be assigned. Examples include:

- assigning students to dormitory rooms;
- admitting students to colleges and universities;
- pairing a graduate student with an academic advisor.

Campbell distinguishes between assignment, admissions and the marriage problems:

- an assignment problem is distinguished by the fact that the elements of the set \( B \) do not have preferences over the elements of the set \( A \). Dormitory rooms, for instance, do not have preferences over the students who may be assigned to live in them.
- an admissions problem is distinguished by the fact that multiple elements of the set \( A \) may be assigned to a single element of the set \( B \).
- a marriage problem is distinguished by the fact that elements of the two sets are either paired with each other (one to one), or they are not matched at all.

With reference to the marriage problem, it is often common in a matching problem to have a default option of being unmatched (i.e., remaining single, or not going to a
We will discuss the procedures by which matchings are made. We typically want a matching to be *individually rational* in the sense that no one is matched with an alternative that he deems worse than being unmatched. If the matching is to be based upon the preferences of the individuals, then we will also want the matching to be *incentive compatible* in the sense that no individual can benefit by misrepresenting his true preferences.

Here are some current and topical matching problems:

- **The matching of new physicians to medical residency programs of hospitals.** New physicians (the elements of set A) typically spend 2-3 years in a medical residency program of a hospital (set B) during which time they receive additional training in a specialty. The hospitals are dependent on their residents as a form of relatively cheap labor that works long hours. Both sides have preferences over the other, and hospitals typically hire a number of doctors for each of its specialties. The National Residency Matching Program is a system implemented by the hospitals to organize the matching process; we’ll provide more details its operation later on. One might imagine instead have a free and competitive market in which doctors apply to hospitals and the hospitals hire who they want. This is in fact how the process worked before 1945. A problem, however, was *unraveling* of the market: because of competitive pressures, hospitals began signing up doctors earlier and earlier during their medical training (e.g., in the middle of their medical school education, not in the final year). The properties of the resulting matching were questionable. Economists are always wary of market intervention, but in this case it seems to have improved the situation.

- **The matching of new lawyers to judicial clerkships.** There is no structure to this process. Judges receive applicants from new lawyers and they make offers. Offers are sometimes even "exploding" in the sense a judge may offer a new lawyer a position in his office on the condition that the offer expires if it is not accepted immediately. This can cause regret among the lawyers if they sense that they did not receive the best possible clerkship.

- **Kidney exchange for transplant: matching donors to recipients.** One possible donor if one needs a kidney for transplant is a cadaver. Each of us has two kidneys, however; if one is removed, then the remaining kidney can easily handle the role previously played by the pair. This leaves open the possibility of a living person donating a kidney to someone who needs a transplant. Typically, a loved one or relative may volunteer. The problem, however, is that there are many medical "markers" that determine the suitability of a particular kidney for transplant (e.g., the blood types of the potential donor and recipient). The person who volunteers his kidney for another may not be a suitable match. This creates the possibility of exchange: person A1 volunteers to give a kidney to person B1, but the match is poor. Person A2 volunteers to give a kidney to person B2, but the match is also poor. Suppose that A1 is a good match for B2 and A2 is a good match for B1. A trade could be arranged. In fact, you can imagine such trades occurring among larger numbers of donor-recipient pairs; these are called "chains".

- **School choice.** It is increasingly common for school districts to allow students to
have some ability to choose among the schools in the district. A student, for instance, may want the school closest to his home, or his parent may want the student a school close to his or her place of work. A student may also wish to attend a school with particular honors programs or areas of expertise (e.g., math and science, or the arts). Unlike universities and colleges, schools typically do not allow schools to express preferences over students (except for matters of qualification, such as minimal test scores). The school district instead typically specifies a priority system for each school in determining which students receive the available slots. A student typically receives priority, for instance, if he lives within walking distance of the school or if he has a sibling who already attends the school. A variety of procedures are used in practice around the United States to match students to schools, including the "Boston mechanism" and the "Student Optimal Stable Mechanism."

A salient property of each of these four examples is that money is either immaterial or second order in its importance in comparison with the match: doctors have regional preferences over hospitals (e.g., "I want to be matched near my spouse’s match"), but their priority is typically the quality and prestige of the residency program. The point of the residency is not to make money; it is to gain worthwhile experience that may in turn provide a lifetime of opportunities and benefits. The same issues apply in judicial clerkships. While money may change hands in gaining admission to private schools, it is generally not part of the public school assignment problem. As to kidney exchange, it is illegal in United States to involve money in organ transplants. One can argue about whether this is a good idea or a bad idea, but currently, you cannot buy and sell organs in the United States.

**Remark 2** As a side note, here's a counter argument. Suppose I’m dying of kidney disease and I’m wealthy. A high quality match for me is identified in an underdeveloped country. I pay this person $100,000 for his kidney. After several days in bed, he recovers and now has a range of opportunities (e.g., business, education, travel) for his life that he didn’t have before he denoted his kidney. Is this wrong? Why?

We can model these problems as two distinct sets such that we wish to match elements of one set to elements of the other. The elements of the two sets have preferences and so the matching creates welfare. It is clear that some matchings may be better from a welfare perspective than others. Another feature of these problems is money is either insignificant or can play no role in allocation of welfare.

Roth and Niederle have identified three types of market failure in which a matching algorithm or clearinghouse may be an appropriate remedy:

1. **Unraveling** so that offers were being made at earlier and at dispersed times. This is a form of "thinness" in the market.

2. **Congestion** so that employers found that they did not have the time to make all of the offers that they wished to make.
3. Strategic behavior in the sense that participants are concerned that they cannot act straightforwardly based upon their true preferences.

The use of a matching algorithm is a form of market intervention. Economists are generally wary of intervening in markets, though the problems that Roth and Niederle motivate intervention. What are the attributes of a market in which a matching algorithm may be helpful? I can list four possible attributes:

1. The goods that are traded are extremely heterogeneous. For instance, doctors vary by both specialities and their intellects.

2. There is extreme excess demand for some goods, which is what causes the market unraveling.

3. Money is "second-order" in preferences. Lawyers who seek judicial clerkships, for instance, bear the opportunity cost of extremely large salaries in corporate law. The point of a clerkship is both the high-level experience and its signaling value on one’s resume throughout the remainder of one’s career. Perhaps the new lawyers even have lexicographic preferences where the quality of the clerkship counts first and salary counts second (i.e., there is no trade-off between the quality of the clerkship and salary). It is difficult for the price mechanism to successfully allocate goods in a situation in which the participants are not particularly concerned about the prices.

4. Roth emphasizes that traders on both sides of many of these markets have strong preferences over who they are matched with. Buyers typically care only about the good and sellers only about the price. A person selling a house, for instance, is concerned mainly about the price and not who buys the house.

We understand how externalities cause market failure and thus motivate market intervention, and we understand how the attributes of a public good cause underprovision of that good. The four attributes above are perhaps a step towards similarly understanding in a formal theoretical sense when a matching algorithm may be needed to improve upon a market’s allocation.